Piezohydraulic Actuator Development for Microjet Flow Control

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ABSTRACT
This paper describes the development of a new piezohydraulic actuator for broadband microjet flow control applications. The actuator utilizes a lead zirconate titanate (PZT) stack actuator and a hydraulic amplification design to achieve relatively large displacements in a compact actuator to control flow through a microjet orifice. Displacement amplification of 81 times the stack actuator displacement was achieved using a new dual-diaphragm design. The nonlinear and hysteretic field-coupled material behavior, structural dynamics, and fluid dynamics of the actuator are modeled using a system dynamic model and compared with experimental results. The nonlinear and hysteretic piezohydraulic actuator characteristics are shown to be strongly dependent on the nonlinear deformation of the rubber diaphragm and minor loop hysteresis of the PZT stack actuator. The modeling technique provides a design tool for broadband performance predictions and system optimization to facilitate implementation of the actuator in a flow environment.

1. INTRODUCTION
Flow control theory and actuator development have been the subject of intense research for more than a decade for applications on various aircraft structures including fixed wings, cavity flow, rotor blades, and impinging jets. A number of actuation techniques have been proposed which include passive systems, active open-loop, and closed-loop flow control designs. Passive systems such as micro vortex generators create undesirable drag and suffer from robustness over a broad range of operating conditions [1]. Active open-loop actuators are often limited in bandwidth and require large power. Closed-loop control designs can reduce the amount of power required [2], but complexities associated with achieving broadband capabilities have limited its implementation in many applications.

Current open-loop and closed-loop systems typically utilize actuators that provide steady mass flow, pulsed mass flow, or zero-net mass injection (synthetic jets). Maximum flow control is typically
achieved by strategically placing the actuator(s) at the initial point of flow separation to interact with the shear boundary layer so that the minimum amount of control effort is expended. Steady mass injection increases the shear boundary and reduces undesirable vortices whereas pulsed mass injection includes a steady flow component and an oscillatory component that is believed to enhance flow control [3], although the physical mechanism(s) are still unclear. Synthetic jets, typically actuated with piezoelectric diaphragms, produce no net mass flow and control flow through second order effects [4].

The major drawbacks of these systems are the lack of actuation capability and bandwidth limitation. Pulsed mass flow actuators such as oscillating fluidic actuators [5] can operate at high frequency (low kHz regime) but are limited to a narrow frequency range. Similarly, plasma or spark jet actuators [6] can operate at high frequency, but the pulsed flow is limited to the combustion process and heat transfer rate. These narrow bandwidth controllers often lead to splitting the resonant peaks (spillover) [3]. Piezoelectric composite actuators used in synthetic jets have provided reasonable flow control in the subsonic regime; however, the force output and narrow bandwidth limit raises questions about its robustness in a broad range of operating conditions.

In contrast to pulsed flow, steady flow microjets have demonstrated significant improvement in reducing undesirable flow characteristics on a broad range of aircraft structures including cavities, inlet designs, and impinging jets. In these systems, an array of orifices (each with diameter of ~400 µm) is strategically placed on an aircraft surface and steady blowing from the orifices reduces flow separation and acoustic emissions. Implementation of steady flow microjets have demonstrated significant reductions in the overall sound pressure level by 11 dB in a supersonic cavity flow application [7]. However, different cavity geometries required different microjet mass flow rates to obtain similar reductions in the SPL. Similarly, Lou et al. [8] have demonstrated that steady-flow microjets can significantly reduce acoustic fields near an impinging jet for short take-off and vertical landing (STOVL) aircraft, but flow control using the microjets varied as the aircraft approached the ground. This has motivated the need for active microjet actuators that can be applied to aircraft structures operating under different external flow conditions and can pulse the flow over a broad frequency range to enhance efficiency by reducing mass flux without sacrificing flow separation control or acoustic emission reduction.

To address this issue, a new piezohydraulic actuator is developed for integration into a microjet flow control system. The design presented here is focused on developing a benchtop, broadband flow control actuator that can be used to further understand the effect of broadband pulsed microjet actuation on various aircraft structures. Piezoelectric materials are well known for their broadband electro-mechanical actuation characteristics which have been utilized in a number of compact actuator devices [9-12]. The large forces and small displacements afforded by piezoelectric stack actuators are ideal for
broadband nanopositioning applications [13]. In applications where larger displacement (>1 mm) is desired, amplification techniques or frequency rectification is often employed [9, 11, 14-18]. As one example, a hydraulic amplification technique is utilized here to provide a compact throttling mechanism that can control a microjet with an orifice diameter on the order of 400 µm.

The introduction of hydraulic fluid and mechanical sealing elements within the piezohydraulic actuator requires a careful assessment of the underlying field-coupled mechanics and dynamics that contribute to the overall actuator response. System dynamic modeling has been previously used to estimate the dynamic performance of similar piezohydraulic pump systems [15, 16, 18]. These models are useful for identifying subsystem performance and efficiencies that contribute to the overall dynamic response and bandwidth of the actuator. Here, a similar modeling framework is implemented and extended to include nonlinear and hysteretic ferroelectric behavior using a rate-dependent homogenized energy modeling framework [19] and hyperelastic behavior of the rubber diaphragm seal [20]. The integration of rate-dependent nonlinear and hysteretic behavior illustrates key performance attributes associated with the overall performance of the piezohydraulic actuator. The design and experimental results of the actuator are first presented. The nonlinear system dynamic model is then described, compared to the experimental results in the quasi-static regime, and predictions of dynamic behavior up to 1 kHz are modeled. Discussion and concluding remarks are given in the final sections.

2. PIEZOHYDRAULIC DESIGN

The piezohydraulic actuator consists of a piezoelectric stack actuator, piston, two rubber diaphragms, hydraulic fluid, and aluminum structural housing. Hydraulic amplification of this device is achieved using a converging nozzle design as shown in Figure 1(a). The piezoelectric stack actuator (Kinetic Ceramics) used in this device is 22 mm long and has a circular cross section with a diameter of 19 mm. The nominal free displacement and blocked force of this actuator is approximately 20 µm and 10 kN, respectively. The rubber dual-diaphragm design seals a low viscosity hydraulic fluid (Dow Corning 200, viscosity—0.65 cSt) inside the cylinder head which provides a relatively simple mechanical design for charging the system with fluid and evacuating entrained air. When an electric field is applied to the piezoelectric stack actuator, fluid is forced through an orifice at the top of the cylinder head and subsequently deforms the external diaphragm that seals the fluid and controls air flow to a microjet. A list of the components and material properties of the design that is illustrated in Figure 1 is listed in Table 1.
Figure 1. (a) A cross section of the piezohydraulic actuator.  (b) Prototype of the piezohydraulic actuator. A bleed valve is located on the top left for purging air. The rubber diaphragm is located underneath the top plastic microjet interface.

<table>
<thead>
<tr>
<th>Table 1. Prototype device parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Piezoelectric stack actuator</strong></td>
</tr>
<tr>
<td>Overall length</td>
</tr>
<tr>
<td>Diameter</td>
</tr>
<tr>
<td>Blocking force</td>
</tr>
<tr>
<td><strong>Piston</strong></td>
</tr>
<tr>
<td>Thickness</td>
</tr>
<tr>
<td>Diameter</td>
</tr>
<tr>
<td><strong>Hydraulic fluid – silicon oil</strong></td>
</tr>
<tr>
<td>Density</td>
</tr>
<tr>
<td>Viscosity</td>
</tr>
<tr>
<td><strong>Cylinder head</strong></td>
</tr>
<tr>
<td>External Orifice Diameter</td>
</tr>
</tbody>
</table>

3. EXPERIMENTAL SETUP

The experimental set-up consists of a hydraulic circuit and data acquisition system used for characterizing the displacement of the piezohydraulic actuator. A schematic of the hydraulic circuit is shown in Figure 3 followed by a schematic of the drive electronics and data acquisition system in Figure 4. The hydraulic system was set up to charge the piezohydraulic actuator with fluid, control the bias
pressure, and purge air from the device. A hydraulic accumulator was used to control bias pressure in the piezohydraulic actuator and a fluid reservoir was connected to a vacuum pump to purge air from the system. Shut-off valves were used to pressurize the system and evacuate entrained air using a vacuum pump. During operation, the shut-off valve between the fluid reservoir and the piezohydraulic actuator was closed and the shut-off valve to the accumulator was left open. Hydraulic fluid was prevented from flowing back into the accumulator by including a check valve in the hydraulic circuit.

Figure 3. The schematic of the hydraulic system used to charge the piezohydraulic actuator with fluid and remove entrained air.

The piezohydraulic actuator was characterized at different bias pressures and voltage amplitudes using the drive electronics and data acquisition set-up in Figure 4. A dSpace data acquisition system (DS1005 DSP board), controlled by Matlab and Simulink, was used to collect actuator displacement data. A 1000V/7A switching power supply (PEIZOMechanik) amplified the voltage applied to the piezoelectric stack actuator. A Lion Precision capacitor probe with a sensitivity of 10 V/mm was used to detect displacement of a light-weight pin (1 g) that was placed in contact with the external diaphragm. The amount of displacement amplification was estimated by measuring the free displacement of the piezoelectric stack actuator and these results were compared with displacement measurements of the external rubber diaphragm of the piezohydraulic actuator. The same experimental set-up shown in Figure 4 was used to measure free displacement of the stack actuator.

Figure 4. The schematic of the drive electronics and data acquisition system used to characterize the piezohydraulic actuator.
4. EXPERIMENTAL RESULTS

The piezoelectric stack actuator and hydraulic actuator displacements were characterized to correlate behavior between the PZT stack actuator displacement and the rubber diaphragm deformation. The PZT stack actuator was characterized at different voltage amplitudes and frequencies under free displacement boundary conditions. This data was used for modeling rate dependent ferroelectric hysteresis and predicting broadband piezohydraulic actuation presented in subsequent sections. The piezohydraulic actuation was measured under quasi-static conditions at different voltage amplitudes. Different bias pressures were applied to the piezohydraulic system to optimize displacement. Sinusoidal wave forms were applied to the stack actuator under free displacement conditions and to the piezohydraulic actuator by applying voltages ranging up to 1000V (2 MV/m). A summary of the free displacements of the piezoelectric stack actuator and piezohydraulic actuator displacements is given in Table 2. The hydraulic amplification gain is determined from these measurements which ranged between 58 and 81. This is a reasonable estimate for hydraulic gain since the biased pressures applied to the hydraulic fluid resulted in a compressive stress of approximately 2 MPa on the PZT stack actuator. These loads are expected to have a minor effect on reducing stack actuator displacements during operation since the bias load is ~6% of the stack actuator blocked force.

<table>
<thead>
<tr>
<th>Voltage</th>
<th>400V</th>
<th>600V</th>
<th>800V</th>
<th>1000V</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT displacement</td>
<td>9 µm</td>
<td>14 µm</td>
<td>18 µm</td>
<td>23 µm</td>
</tr>
<tr>
<td>Diaphragm displacement</td>
<td>730 µm</td>
<td>1047 µm</td>
<td>1209 µm</td>
<td>1339 µm</td>
</tr>
</tbody>
</table>

The nonlinear and hysteric response of the piezohydraulic actuator displacement is illustrated in Figure 5. Optimum performance at each voltage amplitude was obtained by applying small changes in the bias pressure as listed in Figure 5. The mechanisms contributing to nonlinear and hysteric displacement are shown to be dependent on the nonlinear deformation of the rubber diaphragm and minor hysteresis of the piezoelectric stack actuator. This behavior is quantified and compared in the following section using a nonlinear system dynamic model.
Figure 5. Response of the piezohydraulic actuator for a sinusoidal voltage input at 1 Hz. The legend denotes the bias pressure used at different voltage amplitudes.

5. SYSTEM DYNAMIC MODEL

The displacement of the piezohydraulic actuator was modeled to quantify the underlying mechanisms contributing to the device performance illustrated in Figure 5. This is implemented using a lumped parameter approach [21, 22]. Nonlinear and hysteretic behavior associated with the PZT stack actuator and rubber diaphragm are integrated into the model and coupled with fluid dynamic behavior in the cylinder head. Each subsystem is described and then formulated as a coupled set of equations to predict piezohydraulic actuation.

The state space model is developed with reference to Figure 6. The stack actuator is modeled as a damped oscillator which includes linear piezoelectric coupling and nonlinear and hysteretic ferroelectric behavior. During the application of an applied field, fluid is forced through the cylinder head which is modeled as fluid capacitive elements with flow resistance. The fluid forces the rubber diaphragm to deform which is described by a nonlinear damped oscillator. The effective nonlinear stiffness of the diaphragm is quantified using a hyperelastic constitutive relation. The governing equations describing the piezoelectric stack actuator, fluid coupling, and rubber diaphragm are presented and then combined into a matrix of equations for numerical implementation.
5.1 PIEZOELECTRIC STACK ACTUATOR SUBSYSTEM

The dynamic behavior of the piezoelectric stack actuator/piston subsystem was modeled as a damped oscillator actuated by an applied voltage. Linear piezoelectricity is first modeled and then the model is extended to include ferroelectric hysteresis. The stack actuator drives a piston mass that is coupled to the hydraulic fluid as illustrated in Figure 6. Kelvin-Voigt damping is included in the subsystem model to account for internal friction. The resulting equation of motion is

\[ m_p \ddot{x}_i + c_p \dot{x}_i + k_p x_i = c V - P_1 A_{pi} \]  

(1)

where \( m_p \) is the effective mass of the piston and stack actuator, \( c_p \) is the damping coefficient, and \( k_p \) is the stiffness of the stack actuator. The forces include piezoelectric coupling given by the coefficient \( c \) and voltage input \( V \) and the force from the fluid pressure \( P_1 \) acting over the piston area \( A_{pi} \).

The constants \( c \) and \( k_p \) were derived from the linear constitutive law of the piezoelectric material

\[ S_{ij} = s^E_{ijkl} T_{kl} + d_{ij} E_k \]  

(2)

where \( S_{ij} \) is infinitesimal strain, \( s^E_{ijkl} \) is compliance at constant field, \( T_{kl} \) is stress, \( d_{ij} \) is the piezoelectric tensor, and \( E_k \) is the electric field.

In the uniaxial case considered here, (2) reduces to

\[ S_{33} = s^E_{3333} T_{33} + d_{33} E_3 \]  

(3)
where $S_{33}$ is the axial strain component, $T_{33}$ is the axial stress component, $E_{3}$ is electric field component, $s_{3333}^E$ is the compliance component at fixed electric field, and $d_{333}$ is the piezoelectric coefficient. Equation (3) is written in terms of force, displacement, and voltage as

$$F = \frac{n A_{PZT} e_{333}}{L} V - \frac{A_{PZT} E}{L} x_i = c V - k_p x_i$$

(4)

where the constant $c$ is equivalent to $n A_{PZT} e_{333}/s_{3333}^E L$, with the number of layers of the stack actuator ($n$), its cross-sectional area ($A_{PZT}$), and the total length of the stack actuator ($L$). The spring constant, $k_p$, from (1), is equivalent to $A_{PZT}/s_{3333}^E L$. The equivalent modulus of the stack actuator was estimated using published short circuit modulus of PZT. The effective piezoelectric constant for the stack actuator was obtained by measuring stress free displacement at each electric field as previously given in Table 2. The parameter values used in the linear stack actuator model are given in Table 3.

<table>
<thead>
<tr>
<th>Table 3. The effective stack actuator/piston subsystem parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT short circuit modulus</td>
</tr>
<tr>
<td>Number of PZT layers, n</td>
</tr>
<tr>
<td>Effective piezoelectric constant</td>
</tr>
<tr>
<td>Normalized stiffness ($k_p/m_p$)</td>
</tr>
<tr>
<td>Normalized damping ($c_p/m_p$)</td>
</tr>
</tbody>
</table>

5.2 NONLINEAR FERROELECTRIC BEHAVIOR

A ferroelectric homogenized energy model was introduced into the system dynamic model and compared to the linear piezoelectric model to quantify its effect on piezohydraulic actuation. Ferroelectric nonlinearities and hysteresis are primarily due to inhomogeneities such as domain structures, crystal anisotropy, grain boundary effects, etc. During electro-mechanical loading, this can lead to minor loop hysteresis at moderate to large field inputs. The introduction of this behavior in the
system dynamic model is shown to play an important role in predicting the piezohydraulic actuation previously shown in Figure 5.

The homogenized energy modeling framework is based on stochastic distributions of a reduced set of parameters describing material inhomogeneities typically present in polycrystalline ferroelectric materials; see [19, 23] for details on the modeling framework. A local variation in the internal field is represented by a normal probability distribution. A local variation in the average coercive field is assumed to be distributed about a log-normal probability distribution. Ferroelectric switching is determined by comparing these distributions of effective fields and coercive fields such that when a local effective field is larger than a local coercive field, local polarization switching occurs. Macroscopic polarization is computed by homogenizing the local polarization over the probability distributions of the effective fields and coercive fields.

Equation (1) is modified to include ferroelectric switching behavior as a function of polarization

\[ m_p \ddot{x}_i + c_p \dot{x}_i + k_p x_i = \tilde{a}_i \tilde{P}(E) - P_A p_i \]  

where \( \tilde{a}_i \) is the effective piezoelectric coefficient, \( \tilde{P} \) is the homogenized polarization in the direction of applied field. The homogenized polarization is defined by

\[ \tilde{P} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \nu(E_i, E_c) \tilde{P}(E + E_i; E_c, \xi) dE_i dE_c \]  

where \( \tilde{P}(E + E_i; E_c, \xi) \) is the local polarization, \( E \) is the applied field, \( E_i \) is an interaction field, \( E_c \) is the coercive field, and \( \xi \) is an initial set of variants defining the local polarization. The effective field is defined by the sum of the applied field and interaction field, \( E_{\text{eff}} = E + E_i \). Rate dependent hysteresis is included in the model by defining a time constant and thermal relaxation behavior; see [19, 23] for details. The probability distribution is denoted by \( \nu(E_i, E_c) \) and defined by

\[ \nu(E_i, E_c) = C e^{-E_i^2/2b^2} e^{-[\ln(E_i/E_c)]/2c_z^2} \]

where \( b \) is the variance of the normal distribution and \( c_z \) defines the variance of the log-normal distribution with maximum value at \( E_c \). The constant \( C \) ensures integration to unity. The values for the homogenized energy model are given in Table 4. The homogenized energy model is compared to the
linear piezoelectric model and stack actuator displacement measurements at different voltage amplitudes and frequencies as shown in Figure 6.

Table 4. The constitutive model parameters used in the homogenized energy model. See [19, 23] for definitions of rate dependent parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{E}_c$</td>
<td>1 MV/m</td>
</tr>
<tr>
<td>$b$</td>
<td>0.8 MV/m</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.5 MV/m</td>
</tr>
<tr>
<td>$\tilde{a}_i$</td>
<td>6.0×10⁴ Nm²/C</td>
</tr>
<tr>
<td>$\eta$</td>
<td>1.0×10⁷</td>
</tr>
<tr>
<td>$P_r$</td>
<td>0.4 C/m²</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.10 m²s⁻¹kg⁻¹K⁻¹</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.02 sec</td>
</tr>
</tbody>
</table>

Figure 6:  (a) Comparison of the linear piezoelectric model, nonlinear ferroelectric homogenized energy model and experimental results at 1 Hz.  (b) Comparison of rate dependent actuation and homogenized energy model estimates for frequencies up to 300 Hz.

5.3 FLUID SUBSYSTEM

The lumped parameter fluid subsystem was developed for coupling the fluid with the actuator/piston and rubber diaphragm subsystems. The hydraulic fluid volume was divided into two subdomains to accommodate flow resistance in the cylinder head as the fluid flows through the converging nozzle as
shown in Figure 7. An effective fluid bulk modulus is defined for the cylinder head and for a second fluid element underneath the external diaphragm. The governing equations for each fluid element are given as follows.

Figure 7. The hydraulic cylinder head and diaphragm subsystem. The external diaphragm is shown in an equilibrium deformed state in the presence of an internal bias pressure.

The fluid dynamic relations are based on mass flow rate continuity and the definition of bulk modulus

\[
\sum \dot{m}_j = \frac{d(\rho_0 V)}{dt} \tag{8}
\]

\[
\beta_f = \rho_0 \frac{dP}{dp} \tag{9}
\]

where \(\rho_0\) is the nominal fluid density, \(\dot{m}\) is mass flow, \(V\) is the fluid volume and \(P_i\) is pressure in the cylinder head (\(i=1\)) and the fluid element in contact with the rubber diaphragm (\(i=2\)). The bulk modulus of the fluid is denoted by \(\beta_f\).

These equations can be combined to obtain a first order dynamic relation governing changes in the pressure in response to a mass flow

\[
\frac{\rho_0 V}{\beta_{\text{eff}}} \dot{P}_i = C_i \dot{P}_i = \sum \dot{m}_j = \frac{AP}{R_f} \tag{10}
\]

where the fluid capacitance is denoted by \(C_{\text{eff}}\) which is proportional to the nominal density and volume of the chamber \(V_0\) and indirectly proportional to an effective fluid bulk modulus \(\beta_{\text{eff}}\). The fluid resistance
is denoted by $R_f$ and is estimated based on losses due to the converging nozzle geometry in the cylinder head. The fluid resistance is approximated by an inviscid fluid by employing Bernoulli’s equation.

Based on an inviscid flow problem, the flow resistance in the cylinder head is determined from the slope of the pressure versus flow rate due to losses in a converging nozzle. This is described by

$$\Delta P = \frac{\rho_0 Q^2}{2} \left( 1 + K_L \frac{1}{A_r^2} - \frac{1}{A_p^2} \right)$$

(11)

where the pressure drop is associated with the change in area from the piston face ($A_p$) to the orifice area ($A_r$) and the volume flow rate is denoted by $Q$. A loss coefficient ($K_L$) of 0.5 was used to model a sharp corner [24].

The flow resistance based on (11) is therefore

$$R_f = \frac{d\left(\frac{\Delta P}{\rho_0}\right)}{dQ} = Q \left( 1 + K_L \frac{1}{A_r^2} - \frac{1}{A_p^2} \right)$$

(12)

The linear dependence on flow rate was found to be negligible in the system dynamic model for frequencies up to 1 kHz and was therefore set to a nominal value based on an average flow rate at 1 kHz for all simulations. Note that this neglects inertial effects and turbulence which may arise at these higher frequencies.

The fluid capacitance for the lumped fluid elements in Figure 7 was determined based on a nominal density and an effective bulk modulus. The effective bulk modulus of the fluid in the cylinder head was based on prior modeling of a piezoelectric pump which included experimental measurements of the effective bulk modulus of the fluid system. The effective bulk modulus was 70 MPa using the same silicon oil that was used in the experiments presented in [15]. The published bulk modulus of the fluid is 1150 MPa. The reduced value assumes a level of entrained air remains in the system after vacuuming or compliance in the internal rubber diaphragm seal. The capacitance is also dependent on the volume of the cavity. A nominal volume is used in the cylinder head since the displacement of the stack actuator is relatively small. The fluid volume in contact with the rubber diaphragm is updated on each load step due to finite volume changes within the external diaphragm seal shown in Figure 7.

The two equations describing the fluid behavior in each lumped fluid element was obtained from the continuity equation
where \( x_2 \) corresponds to radial deformation of the rubber diaphragm as described in the following section. This gives a change in fluid capacitance as the volume changes. The mass flow rate \( \dot{m}_i \) is determined by the velocity of the piston/stack actuator assembly.

### 5.4 RUBBER DIAPHRAGM SUBSYSTEM

The piezohydraulic actuator exhibited significant departures from linearity which is partially due to finite deformation of the external rubber diaphragm. To accommodate this behavior, a hyperelastic constitutive law was integrated into the system dynamic model.

The rubber diaphragm subsystem was modeled as a second-order mass-spring-damper with a driving force provided by the pressure of the hydraulic fluid. The governing equation is

\[
C_1 \dot{P}_1 = \dot{m}_1 - \dot{m}_2 = \dot{m}_1 - \frac{P_1 - P_2}{R_f}
\]

\[
C_2 (x_2) \dot{P}_2 = \dot{m}_2 - \rho A_r \dot{x}_2 = \frac{P_1 - P_2}{R_f} - \rho A_r \dot{x}_2
\]

which includes an effective mass \( (m_r) \), nonlinear stiffness \( (k_r(x_2)) \), and a linear Kelvin-Voigt damping coefficient \( (c_r) \). The displacement was produced by the pressure \( (P_2) \) of the hydraulic fluid applied to the rubber area \( (A_r) \). A second force was generated by the external ambient pressure \( (P_0) \).

Hyperelastic constitutive behavior is incorporated into the system dynamic model using Ogden’s model [8] to simulate the rubber diaphragm pressure versus stretch constitutive behavior as a function of pressure in the piezohydraulic actuator. The deformation is approximated by assuming a spherical shape and uniform deformation. Using this approximation, as described in [8] pp. 239-242, the constitutive relation between inflation pressure and circumferential stretch is

\[
P_2 = 2 \frac{H}{R} \sum_{p=1}^{3} \mu_p \left( \kappa^{a_{r,-3}} - \kappa^{-2a_{r,-3}} \right)
\]
where $\mu_p$ are shear moduli and $\alpha_p$ are dimensionless constants. The initial thickness of the rubber diaphragm is $H$, the initial radius (zero-pressure) of the rubber diaphragm is $R$, and $\lambda$ is the circumferential stretch which is defined by $\lambda = 1 + \frac{x_2}{R}$. Thus, $x_2$ is the radial displacement from the initial radius $R$. The values of the constants were obtained by fitting to the piezohydraulic actuator results for a field input of 2 MV/m at 1 Hz; see Table 5.

### Table 5. The rubber diaphragm subsystem parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber Diameter</td>
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</tr>
<tr>
<td>Rubber Thickness</td>
<td>0.6 mm</td>
</tr>
<tr>
<td>Rubber Density</td>
<td>1346 kg/m³</td>
</tr>
<tr>
<td>Rubber Damping</td>
<td>0.6 N-s/m</td>
</tr>
<tr>
<td>Dimensionless constants</td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.011</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-10</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-11.6</td>
</tr>
<tr>
<td>Shear moduli</td>
<td></td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1.07×10⁵ kPa</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>1.19 kPa</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>-0.72 kPa</td>
</tr>
</tbody>
</table>

The set of parameters in Table 5 give rise to the inflation pressure versus stress behavior illustrated in Figure 9. These parameters give an effective shear modulus computed using $\mu_{eff} = \frac{1}{2} \sum_{p=1}^{3} \alpha_p \mu_p = 587$ kPa, which is reasonable for a typical rubber material. A comparison of the constitutive behavior based on (17) is shown and compared to the dynamic pressure vs. stretch model prediction based on (16). In Figure 9(a), the stretch is incremented and the resulting pressure is computed independent of dynamic effects. This result differs from the fully coupled system dynamic model where the dynamic relation given by (16) is coupled to the fluid and piezoelectric stack actuator. In the dynamic case shown in Figure 9(b), the time dependent pressure response clearly illustrates unstable “snap buckling” behavior as
the pressure increases. More details on this behavior and the fully coupled model are given in the following section.

![Graph](image)

**Figure 9.** Inflation pressure $P_2$ versus stretch $\lambda$ based on parameters in Table 5. In (a), the hyperelastic constitutive law given by (17) was used to predict pressure as a function of stretch. In (b), the system dynamic response is shown by computing pressure vs. stretch using the fully coupled system dynamic model for a 1 Hz, 2 MV/m field input to the stack actuator.

The tangential stiffness is implemented in the system dynamic model based on (17). This stiffness is

$$ k_r(\lambda) = \frac{dP_2}{d\lambda} \left( \frac{A_r}{R} \right) $$

which changes at each operating point $\lambda_0$. The stiffness can be written as a function of the displacement $x_2$ using the relation $\lambda = 1 + \frac{x_2}{R}$.

### 5.5 Overall System Model

The fully coupled system dynamic model of the piezohydraulic actuator system consist of the equations of motion for the piezoelectric stack actuator/piston, fluid in the cylinder head, and rubber diaphragm. By utilizing (1), (14), (15), and (16), the state space model was developed as a sixth order system using the linear piezoelectric constitutive relation. The ferroelectric model is developed by replacing (1) with the homogenized energy model given by (5). The second-order differential equations for the stack actuator and rubber are written as two first-order differential equations for convenience in simultaneously solving the system of equations. The system of differential equations is written in state space form as
\[ \dot{x}(t) = A(x)x(t) + [B(u)](t) \]  
\[ y(t) = Cx(t) \]  

and subjected to a set of initial conditions \( x(0) = x_0 \). The state vector \( x \) includes the displacements and velocities of the stack actuator and rubber diaphragm and pressures within the cylinder head. The matrix \( A(x) \) is a function of the rubber diaphragm displacement due to the finite deformation of the external rubber diaphragm. The input operator \( B(u) \) is written as a nonlinear function of polarization as given in the Appendix.

Due to the nonlinearity of the differential equations, the system of equations is solved numerically using a temporal discretization at time steps denoted by \( i = 1, \ldots, N \) and time step \( \Delta t \). The solution of the state equations at the end of each time step is used as the initial condition for the next iteration. Using the central difference method, the discrete form of (19) is

\[ x_{i+1} = \left( I - \frac{\Delta t}{2} A_i \right)^{-1} \left[ \left( I + \frac{\Delta t}{2} A_i \right) x_i + \Delta t B(u_i) \right] \]  

where the subscripts denote each time step defined over the time interval \([t_0, t_f]\) with uniform mesh having a size \( \Delta t \) at points \( t_0, t_1, \ldots, t_N = t_f \). Note that this is an approximate central difference algorithm since the state matrix \( A_{i+1} \) would normally enter into the equation inside the inverse operator; however, it is not known \textit{a priori}.

The system level performance is simulated by comparing the linear piezoelectric model to the nonlinear ferroelectric model. The same bias pressure used in the experiments was applied to the model. All other parameters were held fixed. As shown in Figure 10, significant differences in piezohydraulic actuation is predicted when minor loop ferroelectric hysteresis is incorporated into the model. This is due to the hydraulic amplification of the device which amplifies the minor loop ferroelectric stack actuator displacement. Negligible minor loop hysteresis is predicted when linear piezoelectricity is modeled. Reasonable estimates are predicted by the ferroelectric model at 2 MV/m; however, the model did not predict lower voltages as well. The differences in model predictions and experimental results are discussed in the following section.
Figure 10. System dynamic model comparisons of the piezohydraulic actuator experimental results. Minor loop ferroelectric hysteresis of the stack actuator is shown to affect piezohydraulic actuation.

Additional simulations were conducted at frequencies up to 1 kHz using the rate-dependent homogenized energy model. Minimal attenuation in displacement is achieved which is promising for broadband actuation; however larger hysteresis is generated. It should also be noted that bias pressures, ranging up to 700 kPa, resulted in no attenuation in displacement. This is important for implementation of the device in a flow environment where microjets typically operate under a bias pressure of approximately 200 kPa.

Figure 11: Rate dependent predictions of the piezohydraulic actuator using the nonlinear system dynamic model coupled to the rate-dependent homogenized energy model.
6. DISCUSSION

The piezohydraulic actuator has provided significant displacement amplification necessary to throttle a microjet with a typical diameter on the order of 400 µm. Key underlying contributions to piezohydraulic actuation have been identified by formulating a system dynamic model that includes nonlinear and hysteretic electro-fluid-mechanical coupling and dynamic behavior. The underlying mechanisms were determined to be strongly dependent on the nonlinear and hysteretic response of the piezoelectric stack actuator and finite deformation of the external rubber diaphragm.

The piezohydraulic actuation of this device is shown to be partially related to minor loop hysteresis of the piezoelectric stack actuator. Whereas the hydraulic amplification increases displacement, it also amplifies the ferroelectric hysteresis previously shown in Figure 6. However, various control designs such as nonlinear optimal control can be implemented to mitigate this effect [25, 26]. Furthermore, single crystal ferroelectric relaxor stack actuators, which exhibit smaller hysteresis under uni-polar fields [27], may also improve open loop performance by reducing the minor loop hysteresis. As shown in Figure 10, the actuator displacement was anhysteretic using the linear piezoelectric model.

The modeling results also illustrate that the rubber constitutive behavior is critical to achieve significant displacement amplification. The implementation of a hyperelastic constitutive model for the rubber shown in Figure 9 was used to predict a dynamic snap phenomenon of the rubber diaphragm. This leads to large deformation as the hydraulic fluid forces the rubber diaphragm to deform over an unstable pressure regime. The model predictions and experimental results compare well for a field input of 2 MV/m; however, lower input fields were not able to accurately predict the experimental results. This could be due to the hyperelastic constitutive parameters used or the approximation of the deformation as an ideal spherical shape change. In the device, the rubber diaphragm was constrained to deform up a cylindrical cavity as opposed to ideal spherical deformation. In addition, variations in the effective bulk modulus may have occurred because the hydraulic accumulator was recharged during the experiments. Increasing the effective bulk modulus in the model by 50% led to improved model predictions at lower field amplitudes. These issues are currently under investigation to ensure reliable integration of the piezohydraulic actuator into the microjet flow control system.

Calculations using the system dynamic model predict a bandwidth above 1 kHz which was primarily limited by the material properties of the rubber diaphragm. This was also verified using finite element analysis of the rubber diaphragm while assuming infinitesimal strain. Finite element results predicted the first vibration mode to be approximately 2 kHz. The material properties in Table 5 were used in this calculation. Despite this dynamic performance, the deformation of the rubber diaphragm may introduce
significant reliability challenges for pulsed microjet actuation at frequencies in the range of 1 kHz. Since previous microjet experiments have used an orifice that is 400 µm, smaller deformation will be required for flow control applications and this may reduce mechanical fatigue.

7. CONCLUDING REMARKS

A piezohydraulic actuator has been designed, characterized, and modeled for integration into a microjet flow control system. Significant displacement amplification has been achieved which is sufficient to throttle a microjet actuator. The device presented here is expected to provide an important testing device for integration into bench top and wind tunnel microjet experiments to elucidate how micropulsed actuation interacts with different aircraft control surfaces. Many of these problems are still poorly understood in the subsonic and supersonic flow regimes. New knowledge on broadband pulsed flow actuation is anticipated to help understand how to exploit microjet flow for enhanced separation control and noise reduction while simultaneous reducing mass flux requirements. This is expected to have broad implications on a number of aircraft control surfaces such as jet inlets, cavity bays, rotor blades, and impinging jets.

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APPENDIX

The expressions of x(t), A(x) and B(u) in (19) for the state space model are given as follows.

\[
x(t) = \begin{bmatrix}
    x_1 \\
    \dot{x}_1 \\
    P_1 \\
    P_2 \\
    x_2 \\
    \dot{x}_2
\end{bmatrix}
\]  

(22)
\[
A(x) = \begin{bmatrix}
0 & \frac{1}{k_{ep}} & 0 & 0 & 0 & 0 \\
-k_{ep} & m_p & 0 & 0 & 0 & 0 \\
\frac{1}{c_{e}} & \frac{1}{m_p} & 0 & 0 & 0 & 0 \\
0 & 0 & -\rho_{o} \frac{A_{w}}{C_{R_e}} & \frac{1}{C_{R_e}} & 0 & 0 \\
0 & 0 & \frac{1}{c_{e}(x_{e}) R_{e}} & -\frac{1}{c_{e}(x_{e}) R_{e}} & 0 & 0 \\
0 & 0 & 0 & \frac{A_{w}}{m_p} & 0 & 1 \\
0 & 0 & 0 & 0 & -\rho_{o} \frac{A_{w}}{C_{e}(x_{e})} & 0 \\
\end{bmatrix}
\]

(23)

\[
B(u) = \begin{bmatrix}
0 \\
\frac{c_{x}}{m_p} \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
+ \begin{bmatrix}
P_{0} \\
\end{bmatrix}
\]

(24)

In the case of the ferroelectric model, the voltage input is replaced by a field input which is a nonlinear function of the polarization

\[
B(u) = \begin{bmatrix}
0 \\
\frac{\alpha_{c}}{m_p} \\
0 \\
0 \\
0 \\
\end{bmatrix} \begin{bmatrix}
\vec{F}(E) \\
\end{bmatrix} + \begin{bmatrix}
P_{0} \\
\end{bmatrix}
\]

(25)

REFERENCES