Example

If M1 operates at a speed of 960 rpm, what is the operating condition of M2 in order to establish unity power factor at the transformer? Calculate all relevant quantities such as line currents, voltages, and field current.

Example (Solution)

Approach this problem in steps
1) What is the voltage on the motor side?
2) Find the current the induction motor M1 draws
3) Find the rated operating condition of M2
4) Adjust the field current of M2 to reach unity PF at the transformer

1) The turns ratio and the YΔ-connection yields the no-load secondary voltage at the transformer to be

\[ V_{2,LL} = \frac{V_{1,LL}}{N_H} = \frac{4160}{\sqrt{3}} \frac{1}{\sqrt{3}} \frac{5.4586}{5.4586} = 440 V \]

Since the voltage drop will only be 3% for 1.5MVA reactive (full load, 90° phase angle of load current) it will be negligible for the present load situation (approx. 0.2 MW at PF=1). Therefore, the voltage at M1 and M2 can be assumed to be constant 440 V for this problem.
Example (Solution)

2) All the impedances for the equivalent circuit of M1 are given. The only information missing is the slip \( s \).

\[
\begin{align*}
\frac{n_{\text{sync}}}{\frac{n_m}{P}} &= \frac{120 \cdot 50}{6} = 1000 \text{ rpm} \\
\frac{n_{\text{sync}}}{n_m} &= \frac{1000 - 960}{1000} = 0.04
\end{align*}
\]

With \( V_\phi = 440/\sqrt{3} = 254 \) V the line current of M1 at this condition is given by

\[
I_{M1} = \frac{V_\phi}{R_1 + jX_1 + \frac{R_2}{s} + jX_2 + jX_m} = \ldots = 141 A \angle -24.3^\circ
\]

Example (Solution)

3) M2 has to be operated such that it consumes a positive reactive current compensating for the negative reactive current of M1

\[
\text{Im}(I_{M2}) = -\text{Im}(I_{M1}) = -141 \sin(-24.3^\circ) = 57 A
\]

The rated operating condition of M2 is given by

\[
I_{M2} = \frac{P_{\text{OUT}}}{\eta V_{LL,2} \sqrt{3}} = \frac{100 \cdot 10^3}{0.9 \cdot 440 \sqrt{3}} = 145.8 A \angle 0^\circ
\]

\[
S_{M2} = \frac{P_{\text{OUT}}}{\eta} = \frac{100}{0.9} = 111.11 \text{ kVA}
\]

From the equivalent circuit of the SM we find the induced voltage

\[
E_A = \sqrt{V_\phi^2 + (I_{M2} X_S)^2} = \sqrt{\left(\frac{440}{\sqrt{3}}\right)^2 + (145.8 \cdot 1.4)^2} = 254.9 V
\]

with the synchronous reactance in Ohm

\[
X_S = \frac{V_{LL,2}^2}{S_{M2}} = \frac{440^2}{111.11 \cdot 10^3} = 1.4 \Omega
\]
Example (Solution)

The phasor diagram of M2 looks like

In order to establish a positive reactive current, \( I_{M2} \) has to be rotated CCW while keeping the same magnitude (in order not to exceed the rated current magnitude). The new phasor diagram will look like

\[
E_{A,new} = \sqrt{(\text{Im}(I_{M2})X_S + V_\phi)^2 + (\text{Re}(I_{M2})X_S)^2} = \sqrt{(57 \cdot 1.4 + 254)^2 + (134.2 \cdot 1.4)^2} = 383 \text{ V}
\]

\[
\text{Re}(I_{M2}) = \sqrt{I_{M2}^2 - (\text{Im}(I_{M2}))^2} = \sqrt{145.8^2 - 57^2} = 134.2 \text{ A}
\]

Example (Solution)

This internal voltage can be achieved by adjusting the field current to

\[
I_{F,new} = 3 \frac{E_{A,new}}{E_A} = 3 \frac{383}{254.9} = A
\]

Since the M2 stator current magnitude stays the same, the losses will stay the same. Therefore, the power of M2 at the shaft will be

\[
P_{\text{new}} = 3V_\phi \cdot \text{Re}(I_{M2}) - P_{\text{loss}} = 3 \cdot 254 \cdot 134.2 - 11.11 \cdot 10^3 = 91.2 \text{ kW}
\]