12.35 A series RLC circuit resonates at 1000 rad/s. If 
\( C = 20 \, \mu F \), and it is known that the impedance at 
resonance is 2.4 \( \Omega \), compute the value of \( L \), the \( Q \) of 
the circuit, and the bandwidth.

**SOLUTION:**

\[
Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}
\]

at resonance, 
\[
\omega_0 = \frac{1}{\sqrt{LC}} = 1000 \Rightarrow \quad L = 50 \, mH
\]

also 
\[
Z(j\omega_0) = R = 2.4 \Omega
\]

\[
Q = \frac{\omega_0 L}{R} = \frac{1000 \times (50 \times 10^{-3})}{2.4} \quad Q = 20.8
\]

\[
BW = \frac{\omega_0}{Q} \quad BW = 48 \, \text{rad/s}
\]
12.41 A variable-frequency voltage source drives the network in Fig. P12.41. Determine the resonant frequency, $Q$, BW, and the average power dissipated by the network at resonance.

![Figure P12.41](image)

**SOLUTION:**

\[
\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 = 2 \, \text{krad/s}
\]

\[
Q = \frac{\omega_0 L}{R} \Rightarrow Q = 1
\]

\[
BW = \frac{\omega_0}{Q} \Rightarrow BW = 2 \, \text{krad/s}
\]

At resonance, $Z = R = 100 \Omega$

\[
I = \frac{V}{R} = \frac{12}{100} = 0.12 \, \text{A}
\]

\[
P = \frac{1}{2} I^2 R \Rightarrow P = 720 \, \text{mW}
\]
12.45 A parallel $RLC$ circuit, which is driven by a variable-frequency 2-A current source, has the following values: $R = 1 \, \text{k}\Omega$, $L = 400 \, \text{mH}$, and $C = 10 \, \mu\text{F}$. Find the bandwidth of the network, the half-power frequencies, and the voltage across the network at the half-power frequencies.

**SOLUTION:**

\[
\frac{1}{BW} = \frac{1}{RC} \Rightarrow BW = 100 \, \text{r/s} \\
\omega_0 = \frac{1}{\sqrt{LC}} = 500 \, \text{r/s} \\
Q = \frac{\omega_0}{BW} = 5 \\
\omega_{HI} = \omega_0 \left[ \frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right] = 552 \, \text{r/s} \\
\omega_{LO} = \frac{\omega_0^2}{\omega_{HI}} \Rightarrow \omega_{LO} = 452 \, \text{r/s} \\
\]

\[
\begin{align*}
V &= I / \gamma = 2L_0 / \gamma \\
\gamma (j\omega_{HI}) &= 1 + j \cdot 1 \, \text{mS} \\
\gamma (j\omega_{LO}) &= 1 - j \cdot 1 \, \text{mS} \\
V (j\omega_{HI}) &= 1414 \angle -45^\circ \, \text{V} \\
V (j\omega_{LO}) &= 1414 \angle +45^\circ \, \text{V}
\end{align*}
\]
12.47 Consider the network in Fig. P12.47. If $R = 1 \, \text{k}\Omega$, $L = 20 \, \text{mH}$, $C = 50 \, \mu\text{F}$, and $R_S = \infty$, determine the resonant frequency $\omega_0$, the $Q$ of the network, and the bandwidth of the network. What impact does an $R_S$ of 10 k$\Omega$ have on the quantities determined?

![Figure P12.47](image)

**SOLUTION:**

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 = \frac{1000}{\text{radians/second}} \quad \text{and} \quad R_{eq} = R_S // R = 1\,\text{k}\Omega$$

$$Q = \omega_0 R_{eq} C = 50 \Rightarrow Q = 50 \quad \text{and} \quad \text{Bandwidth} = \frac{\omega_0}{Q} \Rightarrow \text{Bandwidth} = 20 \, \text{radians/second}$$

If $R_S = 10\,\text{k}\Omega$, $R_{eq} = 9.9\,\Omega$.

- $\omega_0$ is unchanged.  
  $\omega_0 = 1000 \, \text{radians/second}$
- $Q$ changes  
  $Q = 45.5$
- Bandwidth changes  
  $\text{Bandwidth} = 22.0 \, \text{radians/second}$
12.50 Determine the value of $C$ in the network shown in Fig. P12.50 for the circuit to be in resonance.

\[ 4 \cos 2t \text{ V} \]

\[ 4 \Omega \]

\[ 4 \text{ H} \]

\[ 6 \Omega \]

\[ C \]

**Figure P12.50**

**SOLUTION:**

\[ \omega = 2 \text{ r/s} \quad Z_L = j8 \Omega \quad Z_C = -j/2C \Omega \]

Let \( Z_1 = Z_C + R_2 \) and \( Z_2 = R_2 + Z_L = 6 + j8 \Omega \)

\[
\frac{Z}{Z_1 + Z_2} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{R_1 R_2 + \frac{1}{\omega C} + j(\omega L_1 - \frac{R_2}{\omega C})}{R_1 + R_2 + j(\omega L - 1/\omega C)}
\]

At resonance, \( Z \) is real. So, phase angle of numerator and denominator are equal.

\[
\frac{\omega L_1 - \frac{R_2}{\omega C}}{R_1 R_2 + \frac{1}{\omega C}} = \frac{\omega L - \frac{1}{\omega C}}{R_1 + R_2} \Rightarrow \frac{32 - 3/\omega C}{24 + 4/\omega C} = \frac{8 - 1/2C}{10}
\]

\[
\frac{32 - 3}{24 + 4} = \frac{16C - 1}{20C} \Rightarrow \frac{32C - 3}{6C + 1} = \frac{16C - 1}{5C} \Rightarrow 64C^2 - 25C + 1 = 0
\]

\[
C = \left\{ \begin{array}{c} 3.45 \text{ mF} \\ 4.6 \text{ mF} \end{array} \right. \]