Thermal Strains and Element of the Theory of Plasticity
Thermal Strains

- Thermal strain is a special class of Elastic strain that results from
  - expansion with increasing temperature, or
  - contraction with decreasing temperature
- Increased temperature causes the atoms to vibrate by large amount. In isotropic materials, the effect is the same in all directions.
- Over a limited range of temperatures, the thermal strains at a given temperature $T$, can be assumed to be proportional to the change, $\Delta T$.

$$\varepsilon = \alpha(T - T_0) = \alpha(\Delta T) \quad (A8-1)$$
where $T_0$ is the reference temperature ($\varepsilon = 0$ at $T_0$). The coefficient of thermal expansion, $\alpha$, is seen to be in units of $1/\degree C$, thus making strain dimensionless.

- Since uniform thermal strains occur in all directions in isotropic material, Hooke’s law for 3-D can be generalized to include thermal effects.

\[
\varepsilon_x = \frac{1}{E} \left[ \sigma_x - \nu(\sigma_y + \sigma_z) \right] + \alpha(\Delta T) \\
\varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu(\sigma_x + \sigma_z) \right] + \alpha(\Delta T) \\
\varepsilon_z = \frac{1}{E} \left[ \sigma_z - \nu(\sigma_x + \sigma_y) \right] + \alpha(\Delta T)
\]
• The theory of plasticity is concerned with a number of different types of problems. It deals with the behavior of metals at strains where Hooke’s law is no longer valid.

• From the viewpoint of design, plasticity is concerned with predicting the safe limits for use of a material under combined stresses. i.e., the maximum load which can be applied to a body without causing:
  – Excessive Yielding
  – Flow
  – Fracture

• Plasticity is also concerned with understanding the mechanism of plastic deformation of metals.
• Plastic deformation is not a reversible process, and depends on the loading path by which the final state is achieved.

• In plastic deformation, there is no easily measured constant relating stress to strain as with Young’s modulus for elastic deformation.

• The phenomena of *strain hardening*, *plastic anisotropy*, *elastic hysteresis*, and *Bauschinger effect* can not be treated easily without introducing considerable mathematical complexity.
Figure 8-1(a). Typical true stress-strain curves for a ductile metal.

Hooke’s law is followed up to the yield stress $\sigma_0$, and beyond $\sigma_0$, the metal deforms plastically.
Figure 8-1b. Same curve as 8-1a, except that it shows what happens during unloading and reloading - Hysteresis. The curve will not be exactly linear and parallel to the elastic portion of the curve.
Figure 8-1c. Same curve as 8-1a, but showing Bauschinger effect.

It is found that the yield stress in tension is greater than the yield stress in compression.
Figure 8-2. Idealized flow curves. (a) Rigid ideal plastic material
Figure 8-2b. Ideal plastic material with elastic region

\[ \sigma \]

\[ \epsilon \]

\[ \sigma_0 \]
Figure 8-2c. Piecewise linear (strain-hardening) material.
• A true stress-strain curve is frequently called a flow curve, because it gives the stress required to cause the metal to flow plastically to any given strain.

• The mathematical equation used to describe the stress-strain relationship is a power expression of the form:

\[ \sigma = k\varepsilon^n \]  \hspace{1cm} (8-1)

where \( K \) is the stress at \( \varepsilon = 1.0 \) and \( n \), the strain-hardening coefficient, is the slope of a log-log of Eq. 8-1.

That is, \( \log \sigma = \log K + n \log \varepsilon \) \hspace{1cm} (8-2)
FAILURE CRITERIA: FLOW/YIELD and FRACTURE

- The Flow, Yield or Failure criterion must be in terms of stress in such a way that it is valid for all states of stress.
- A given material may fail by either yielding or fracture depending on its properties and the state of stress.
- A number of different failure criteria are available, some of which predict failure by yielding, and others failure by fracture.
The terms \textit{flow criterion}, \textit{yield criterion} and \textit{failure criterion} have different meanings.

– Yield criterion applies mainly to materials that are in the annealed condition (usually ductile materials).

– Failure criterion applies to both ductile and brittle materials. However, it is mainly used for brittle materials (fracture criterion), in which the limit of elastic deformation coincides with failure.

– Flow criterion applies to materials that have been previously processed via work hardening (usually ductile materials).
In applying a yielding criterion, the resistance of a material is given by its yield strength.

In applying a fracture criterion, the ultimate tensile strength is usually used.

Failure criterion for isotropic materials can be expressed in the following mathematical form:

\[ f(\sigma_1, \sigma_2, \sigma_3) = \sigma_c \]  \hspace{1cm} (8-3)

where failure (yielding or fracture) is predicted to occur when a specific mathematical function \( f \) of the principal normal stresses is equal to the failure strength of the material, \( \sigma_c \), from a uniaxial tension test.