\[ U = \alpha_1 G b^3 - \tau b^2 l \] (15.5)

**Figure 15-7** (a) Bowing out of dislocations between super jog to produce (b) dislocation dipoles; (c) formation of dislocation loops from dipole.
Figure 15-8. Schematic representation of the operation of a Frank-Read source.

\[ \tau \approx \frac{Gb}{l} \]  
(15.6)
Figure 15-9. Diagram of low-angle grain boundary. (a) Two grains having a common [001] axis and angular difference in orientation of $\theta$. (b) Two grains joined together to form a low-angle grain boundary made up of an array of edge dislocations.
Dislocation Pile-Ups

- Dislocations frequently pile up on slip planes at barriers such as grain boundaries, second phases, or sessile dislocations.

- The leading dislocation in the pile-up is acted on not only by the applied shear stress but also by the interaction force with the other dislocations in the pile-up.

- This leads to high concentration of stress in the pile-up, the stress on the dislocation at the head of a pile-up can approach the theoretical shear stress of the crystal.
• This high stress either can initiate yielding on the other side of the barrier, or in other instances, it can nucleate a crack at the barrier.

• Dislocations piled up against a barrier produce a back stress acting to oppose the motion of additional dislocations along the slip plane in the slip direction.

• The dislocations in the pile-up will be tightly packed together near the head of the array and more widely spaced toward the source (Fig. 5-27).
• The distribution of dislocations of like sign in a pile-up along a single slip plane has been studied by Eshelby, Frank, and Nabarro.

• The number of dislocations that can occupy a distance $L$ along the slip plane between the source and the obstacle is

$$n = \frac{k\pi g \tau_s L}{Gb}$$  \hspace{1cm} (15.7)
• Where $\tau_s$ is the average resolved shear stress in the slip plane and $k$ is a factor close to unity.

• For an edge dislocation $k=1-v$, while for a screw dislocation $k=1$.

• When the source is located at the center of a grain of diameter $D$, the number of dislocations in the pile-up is given by

$$n = \frac{k\pi g \tau_s D}{4Gb}$$  \hspace{1cm} (15.8)
• Isolated solute atoms and vacancies are centers of elastic distortion just as are dislocations.

• Therefore, point defects and dislocations will interact elastically and exert forces on each other.

• To a good approximation the strains around a point defect distort the lattice spherical hole of radius $a$ in an elastic continuum. The resulting strain is $\varepsilon = (a' - a)/a$.

• If the point defect is a vacancy, the radius $a$ is the radius of the atom normally at the lattice site, while if the defect is an interstitial atom, $a$ corresponds to the average radius of an empty interstitial site.
• The volume change produced by the point effect is given by

\[ \Delta V = 4\pi a^3 \varepsilon \quad (15.9) \]

• Because we are concerned only with spherical distortions, the interaction only occurs with the hydrostatic component of the dislocation stress field.

• The elastic interaction energy between the dislocation and the point effect is given by

\[ U_i = \sigma_m \Delta V \quad (15.10) \]
- The hydrostatic stress of a positive edge dislocation at \( r, \theta \) (Fig. In Pres. 14) from the dislocation is

\[
\sigma_m = \frac{(1+\nu)Gb \sin \theta}{3\pi(1-\nu)r}
\]  \hspace{1cm} (15.11)

- so that the interaction energy is

\[
U_i = \frac{4(1+\nu)Gb^3 \varepsilon \sin \theta}{3(1-\nu)r}
\]  \hspace{1cm} (15.12)

- However, this expression includes only the energy external to the point defect.
• When the strain energy includes due to elastic distortion of the solute atom is considered, the complete expression for interaction energy is

\[ U_i = 4Gba^3 \varepsilon \frac{\sin \theta}{r} = A \frac{\sin \theta}{r} \]  

(15.13)
Figure 15-10. Dislocation pile-up at an obstacle.