Ideal gases

\[ p_v = RT \]

\[ u = f(T) \quad \text{only} \]
\[ h = f(T) \quad \text{only} \]

for a closed system, ideal gas \( u, T \) const

then \( p_v = \text{const} \) \[ u = \text{const} \]
\[ h = \text{const} \]

from last time,

\[ C_u = \left( \frac{\partial u}{\partial T} \right)_v \quad C_p = \left( \frac{\partial h}{\partial T} \right)_p \]

for ideal gases

\[ du = C_v \, dT \]
\[ dh = C_p \, dT \]
\[ dU = m \, C_v \, dT \]
\[ dH = m \, C_p \, dT \]

Remember,

\[ h = u + pv \]

for I.G. \( pv = RT \)
\[ h = u + RT \]
\[ dh = du + RdT \]

\[ C_{pdT} = C_v \, dT + RdT \]

\[ C_p = C_v + R \quad \text{or} \quad C_p - C_v = R \quad \text{for I.G.} \]

\[ \text{Likewise, } C_p - C_v = R \quad \text{for I.G.} \]
First Law as a rate equation:

\[ \delta Q = \delta E + \delta W \]
\[ \delta Q = dU + d(KE) + d(PE) + \delta W \]
\[ \frac{\delta Q}{\delta t} = \frac{dU}{dt} + \frac{d(KE)}{dt} + \frac{d(PE)}{dt} + \frac{\delta W}{\delta t} \]

In the limit as \( \delta t \to 0 \)

\[ \dot{Q} = \frac{dU}{dt} + \frac{d(KE)}{dt} + \frac{d(PE)}{dt} + \dot{W} \]

\[ \dot{Q} = \frac{dE}{dt} + \dot{W} \]

--- powers \( \frac{kW}{s} \) or \( \dot{W} \); \( \frac{ft \cdot lb}{s} \) or \( \text{hp} \)

--- rate of energy change

--- rate of heat transfer \( \frac{kW}{s} \) or \( \dot{W} \); \( \frac{ft \cdot lb}{s} \) or \( \text{hp} \)
Open Systems:

Mass Conservation

\[ \Delta m_i \rightarrow m_t \rightarrow \Delta m_e \]

A control volume (C.V.)
(region in space)

In a time interval \( \Delta t \), for the C.V., \( m \rightarrow m_i + \Delta t \)

the accumulation

\[ (\Delta m_i - \Delta m_e) = m_i + \Delta t - m_e \]

\[ \frac{\Delta m_i - \Delta m_e}{\Delta t} = \frac{m_i + \Delta t - m_e}{\Delta t} = \frac{dm_{C.V.}}{dt} \]

\[ \frac{dm_{C.V.}}{dt} = \dot{m}_i - \dot{m}_e \]

where \( \dot{m} = \frac{dm}{dt} \) mass flow rate \( \frac{ka}{S} \frac{lb}{s} \)

or more generally

\[ \frac{dm_{C.V.}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e = \dot{m}_{in} \]
Consider flow through a pipe:

\[ \mathbf{A} \xrightarrow{\text{flow}} \]

Look at a plug of fluid:

\[ \delta m = \frac{A \, dx}{V} \quad \frac{A \, dx}{V} = \frac{m^3}{2g} = \text{kg} \]

If plug crosses plane A in time \( \delta t \):

\[ \frac{\delta m}{\delta t} = \frac{A \, dx}{V} \quad \dot{m} = \frac{A \, \dot{V}}{V} \]

where \( \dot{m} \) is mass flow rate.

\( \dot{V} \) is the flowing substance.

Also:

\[ m = \frac{V}{\rho} \]

\[ \frac{dm}{dt} = \frac{dV}{dt} \quad \dot{m} = \frac{\dot{V}}{\rho} \]

\[ \dot{V} = \dot{m} \rho \]

Comparing \( \dot{m} = \frac{A \, \dot{V}}{V} \) and \( \dot{m} = \frac{\dot{V}}{\rho} \):

or \[ \dot{V} = \frac{A \, \dot{V}}{V} = \frac{V}{A} \]

or \[ \frac{\dot{V}}{\dot{m}} = \rho \]

or \[ \dot{V} \]
Steady state steady Flow (SSSF)

No accumulation \( \Rightarrow m_{cv, i} = 0 \)

\[ \sum m_i - \sum m_e = 0 \Rightarrow \sum m_i = \sum m_e \]

\[ \frac{dE}{dt} = 0 \] (there is no change in the energy of the control volume with time)

(but there may be a change in energy for the fluids)

Energy associated with a flowing fluid

\[ W_{fw} \] = the flow work

\[ \delta W_{fw} = -p_dV \] (work required to move the fluid across the control boundary of the control volume)

\[ W_{net} = -p_u V \]

\[ \delta W_{fw} = -p_u V \]

\[ W_{fw, i} = -p_u V_i \]

\[ W_{fw, e} = +p_u V_e \] (because work is being done by system to push fluid out)

\[ W_{fw} = \sum p_u V_e - \sum p_u V_i \] (just the sum of all of the flow works)

For inlet, outlet, this reduces to

\[ W_{fw} = p_u V_e - p_i V_i \]

and for SSSF \( m_i = m_e = m \)

\[ \frac{W_{fw}}{m} = \frac{p_u V_e - p_i V_i}{m} \]

\( \frac{W}{m} \) specific work = \( \frac{W}{m} = \frac{W}{m} \)