We now turn our attention to the concept of pure substances and the presentation of their data.

**Simple System**

A simple system is one in which the effects of motion, viscosity, fluid shear, capillarity, anisotropic stress, and external force fields are absent.

**Homogeneous Substance**

A substance that has uniform thermodynamic properties throughout is said to be homogeneous.

**Pure Substance**

A pure substance has a homogeneous and invariable chemical composition and may exist in more than one phase.

Examples:

1. Water (solid, liquid, and vapor phases)
2. Mixture of liquid water and water vapor
3. Carbon dioxide, CO₂
4. Nitrogen, N₂
5. Air is treated as a pure substance as long as there is no change of phase, even though it is a mixture of gases.

**State Postulate**

Again, the state postulate for a simple, pure substance states that the equilibrium state can be determined by specifying any two independent intensive properties.
The $P-V-T$ Surface for a Real Substance

- $P-V-T$ Surface for a Substance that contracts upon freezing

- $P-V-T$ Surface for a Substance that expands upon freezing
Looking at the PvT surfaces from different views we get very useful diagrams: T-v, P,v and p-T (often called phase-diagram):

It is important to memorize the basic features of those 2-D diagrams (names, regions, how do the constant pressure and constant temperature lines look like, in a T-v diagram draw a constant pressure line, and then you should be able to locate a constant pressure line that corresponds to a higher pressure and to a lower pressure). Make sure you can draw a diagram and indicate on it those basic features.
You can see in the P-T (or phase diagram) and on the PVT surface that we make a difference between substances that expand on freezing (e.g. water) and those that contract.

The CRITICAL POINT is a limiting state above which there is no clear distinction between the liquid and vapor phases.

At the TRIPLE POINT all the phases (liquid, solid and vapor) coexist.
The triple point for water is $0.01^\circ$C, 0.6113kPa and the Critical point for water is $374.14^\circ$C, 22.09 MPa.

Exercise 1: (Make sure you can do it)

Plot the following processes on the $P$-$T$ diagram for water (expands on freezing) and give examples of these processes from your personal experiences.

1. process a-b: liquid to vapor transition
2. process c-d: solid to liquid transition
3. process e-f: solid to vapor transition
Draw on a P-v diagram the following processes: 1. constant pressure, 2. constant volume, 3. constant temperature. 4. Assume you have a CLOSED system inside a RIGID tank, can you sketch how will the processes look like on the diagram?

Draw a T-v diagram. Indicate the critical point, indicate all the regions.

**Using the tables:**

The following steps will help you locate the appropriate table.

1. What substance?
2. draw a P-v or T-v diagram, and locate the information you have.
3. Usually you will go next to the saturation table (remember there are usually two presentations: one ordered by temperature (“temperature table”) and the other ordered by pressure “pressure table”).
4. The right table to use can be determined by comparing the known properties to properties in the saturation region (saturated table). For example if pressure and specific volume are specified (P\_given, v\_given), we go to the saturation table (pressure table) and read v\_f and v\_g. and compare with the v\_given.

   If v\_given < v\_f we are in the compressed liquid region.

   If v\_f < v\_given < v\_g we are in the saturation region (L+V). We calculate the quality, x, to evaluate the remaining desired properties.

   If v > v\_given we are in the superheated region.
QUALITY:

We defined the quality of a liquid, vapor mixture (a state in the saturation dome) as:

\[ x = \frac{\text{mass}_{\text{vapor}}}{\text{mass}_{\text{total}}} \]

This allows us to calculate any specific property on the saturation region knowing the values of the property for the saturated liquid and saturated vapor:

\[ \phi = \phi_f + x(\phi_g - \phi_f) \]

or calling

\[ \phi_{fg} = \phi_g - \phi_f \]

we have

\[ \phi = \phi_f + x\phi_{fg} \]

Examples and notes:

1. The quantity (1-x) is sometimes called moisture.
2. \( 0 < x < 1 \) always. But sometimes we can be given the quality in percentage (85% quality means \( x = 0.85 \))
3. The specific volume \( v \) of a mixture inside the saturation region can be calculated from the formula above as:

\[ v = v_f + x(v_g - v_f) \]

4. Water, saturated mixture, \( T \) (or \( P \)) given and \( v \) given. How can you calculate the quality?

With \( T \) go to saturation table, read \( v_f \) and \( v_g \) and then from the formula above:
\[ x = \frac{V_{\text{known}} - V_f}{v_g - V_f} \]

or

\[ x = \frac{v - V_f}{v_{fg}} \]

\[ x = \frac{y - y_f}{y_{fg}} \]

Here \( y \) represents any specific property for the saturated mixture (liquid+vapor). You can see the relation between the use of the quality on thermodynamics and the use of the “lever rule” on two phase regions studied in material science.
Property Tables
In addition to the temperature, pressure, and volume data, Tables A-4 through A-8 contain the data for the specific internal energy $u$ the specific enthalpy $h$ and the specific entropy $s$. The enthalpy is a convenient grouping of the internal energy, pressure, and volume and is given by

$$H = U + PV$$

The enthalpy per unit mass is

$$h = u + Pv$$

We will find that the enthalpy $h$ is quite useful in calculating the energy of mass streams flowing into and out of control volumes. The enthalpy is also useful in the energy balance during a constant pressure process for a substance contained in a closed piston-cylinder device. The enthalpy has units of energy per unit mass, kJ/kg. The entropy $s$ is a property defined by the second law of thermodynamics and is related to the heat transfer to a system divided by the system temperature; thus, the entropy has units of energy divided by temperature. We will deal with entropy later.

Saturated Water Tables
Since temperature and pressure are dependent properties using the phase change, two tables are given for the saturation region. Table A-4 has temperature as the independent property; Table A-5 has pressure as the independent property. These two tables contain the same information and often only one table is given.
<table>
<thead>
<tr>
<th>Sat. Temp. press</th>
<th>Specific volume</th>
<th>( P_{sat} )</th>
<th>m³/kg</th>
<th>Sat. liquid</th>
<th>Sat. vapor</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>kPa</td>
<td>( T )</td>
<td>( \rho_{sat} )</td>
<td>( v_f )</td>
<td>( v_g )</td>
</tr>
<tr>
<td>85</td>
<td>57.83</td>
<td></td>
<td></td>
<td>0.001033</td>
<td>2.828</td>
</tr>
<tr>
<td>90</td>
<td>70.14</td>
<td></td>
<td></td>
<td>0.001036</td>
<td>2.361</td>
</tr>
<tr>
<td>95</td>
<td>84.55</td>
<td></td>
<td></td>
<td>0.001040</td>
<td>1.982</td>
</tr>
</tbody>
</table>

**Specific temperature**

**Specific volume of saturated liquid**

**Corresponding saturation pressure**

**Specific volume of saturated vapor**

### TABLE A-4

Saturated water—Temperature table

<table>
<thead>
<tr>
<th>Temp., press., ( P_{sat} ), MPa</th>
<th>Specific volume, m³/kg</th>
<th>Internal energy, kJ/kg</th>
<th>Enthalpy, kJ/kg</th>
<th>Entropy, kJ/(kg·K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ), °C</td>
<td>( \rho_{sat} )</td>
<td>( v_f )</td>
<td>( u_f )</td>
<td>( h_f )</td>
</tr>
<tr>
<td>0.00</td>
<td>0.6113</td>
<td>0.001000</td>
<td>206.14</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>0.8721</td>
<td>0.001000</td>
<td>147.12</td>
<td>20.97</td>
</tr>
<tr>
<td>10</td>
<td>1.2276</td>
<td>0.001000</td>
<td>106.38</td>
<td>42.00</td>
</tr>
<tr>
<td>15</td>
<td>1.7051</td>
<td>0.001001</td>
<td>77.93</td>
<td>62.99</td>
</tr>
<tr>
<td>20</td>
<td>2.3339</td>
<td>0.001002</td>
<td>57.79</td>
<td>83.95</td>
</tr>
<tr>
<td>25</td>
<td>3.1693</td>
<td>0.001003</td>
<td>43.26</td>
<td>104.88</td>
</tr>
<tr>
<td>30</td>
<td>4.2468</td>
<td>0.001004</td>
<td>32.89</td>
<td>125.76</td>
</tr>
<tr>
<td>35</td>
<td>5.6286</td>
<td>0.001006</td>
<td>25.22</td>
<td>146.67</td>
</tr>
<tr>
<td>40</td>
<td>7.3646</td>
<td>0.001008</td>
<td>19.52</td>
<td>167.56</td>
</tr>
<tr>
<td>45</td>
<td>9.5839</td>
<td>0.001010</td>
<td>15.26</td>
<td>188.44</td>
</tr>
<tr>
<td>50</td>
<td>12.349</td>
<td>0.001012</td>
<td>12.03</td>
<td>209.32</td>
</tr>
<tr>
<td>55</td>
<td>15.758</td>
<td>0.001015</td>
<td>9.568</td>
<td>230.21</td>
</tr>
<tr>
<td>60</td>
<td>19.940</td>
<td>0.001017</td>
<td>7.671</td>
<td>251.11</td>
</tr>
<tr>
<td>65</td>
<td>25.03</td>
<td>0.001020</td>
<td>6.197</td>
<td>272.02</td>
</tr>
<tr>
<td>70</td>
<td>31.19</td>
<td>0.001023</td>
<td>5.042</td>
<td>292.95</td>
</tr>
<tr>
<td>75</td>
<td>36.56</td>
<td>0.001026</td>
<td>4.131</td>
<td>313.90</td>
</tr>
<tr>
<td>80</td>
<td>47.39</td>
<td>0.001029</td>
<td>3.407</td>
<td>334.86</td>
</tr>
<tr>
<td>85</td>
<td>57.83</td>
<td>0.001033</td>
<td>2.828</td>
<td>355.84</td>
</tr>
<tr>
<td>90</td>
<td>70.14</td>
<td>0.001036</td>
<td>2.361</td>
<td>376.85</td>
</tr>
<tr>
<td>95</td>
<td>84.55</td>
<td>0.001040</td>
<td>1.982</td>
<td>397.88</td>
</tr>
</tbody>
</table>

Sat. press., MPa

| 100 | 0.10135 | 0.001044 | 1.6729 | 418.94 | 2087.6 | 2506.5 | 419.04 | 2257.0 | 2675.1 | 1.3069 | 6.0480 | 7.3549 |
| 105 | 0.12082 | 0.001048 | 1.4194 | 440.02 | 2072.3 | 2512.4 | 440.15 | 2243.7 | 2683.8 | 1.3630 | 5.9328 | 7.2958 |

For the complete Table A-4, the last entry is the critical point at 374.14 °C.
For the complete Table A-5, the last entry is the critical point at 22.09 MPa.

Saturation pressure is the pressure at which the liquid and vapor phases are in equilibrium at a given temperature.

Saturation temperature is the temperature at which the liquid and vapor phases are in equilibrium at a given pressure.

In Figure A, states 2, 3, and 4 are saturation states.

The subscript \( fg \) used in Tables A-4 and A-5 refers to the difference between the saturated vapor value and the saturated liquid value region.
\[ u_{fg} = u_g - u_f \]
\[ h_{fg} = h_g - h_f \]
\[ s_{fg} = s_g - s_f \]

The quantity \( h_{fg} \) is called the enthalpy of vaporization (or latent heat of vaporization). It represents the amount of energy needed to vaporize a unit of mass of saturated liquid at a given temperature or pressure. It decreases as the temperature or pressure increases, and becomes zero at the critical point.

**Superheated Water Table**

A substance is said to be *superheated* if the given temperature is greater than the saturation temperature for the given pressure.
In the superheated water Table A-6, T and P are the independent properties. The value of temperature to the right of the pressure is the saturation temperature for the pressure. The first entry in the table is the saturated vapor state at the pressure.

![Table A-6](image)

Superheated water

<table>
<thead>
<tr>
<th>T, °C</th>
<th>( v ), m³/kg</th>
<th>( u ), kJ/kg</th>
<th>( h ), kJ/kg</th>
<th>( P = 0.1 \text{ MPa (99.63°C)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat.</td>
<td>1.6940</td>
<td>2506.1</td>
<td>2675.5</td>
<td>1.6940</td>
</tr>
<tr>
<td>100</td>
<td>1.6958</td>
<td>2506.7</td>
<td>2676.2</td>
<td>1.6958</td>
</tr>
<tr>
<td>150</td>
<td>1.9364</td>
<td>2582.8</td>
<td>2776.4</td>
<td>1.9364</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>1300</td>
<td>7.260</td>
<td>4683.5</td>
<td>5409.5</td>
<td>7.260</td>
</tr>
</tbody>
</table>

| \( P = 0.5 \text{ MPa (151.86°C)} \) |
|------|-------------|-------------|-------------|-----------------|
| Sat. | 0.3749      | 2561.2      | 2748.7      | 0.3749          |
| 200  | 0.4249      | 2642.9      | 2855.4      | 0.4249          |
| 250  | 0.4744      | 2723.5      | 2960.7      | 0.4744          |
Compressed Liquid Water Table

A substance is said to be a **compressed liquid** when the pressure is greater than the saturation pressure for the temperature.

Data for water compressed liquid states are found in the compressed liquid tables, Table A-7. Table A-7 is arranged like Table A-6, except the saturation states are the saturated liquid states. Note that the data in Table A-7 begins at 5 MPa or 50 times atmospheric pressure.

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>v (m³/kg)</th>
<th>u (kJ/kg)</th>
<th>h (kJ/kg)</th>
<th>s (kJ/(kg·K))</th>
<th>P = 5 MPa (263.99°C)</th>
<th>v (m³/kg)</th>
<th>u (kJ/kg)</th>
<th>h (kJ/kg)</th>
<th>s (kJ/(kg·K))</th>
<th>P = 10 MPa (311.06°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat.</td>
<td>0.0012859</td>
<td>1147.8</td>
<td>1154.2</td>
<td>2.9202</td>
<td>0.0014524</td>
<td>1393.0</td>
<td>1407.6</td>
<td>3.3596</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.0009977</td>
<td>0.04</td>
<td>5.04</td>
<td>0.0001</td>
<td>0.0009952</td>
<td>0.09</td>
<td>10.04</td>
<td>0.0002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.0009995</td>
<td>83.65</td>
<td>88.65</td>
<td>0.2956</td>
<td>0.0009972</td>
<td>83.36</td>
<td>93.33</td>
<td>0.2945</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.0010056</td>
<td>166.95</td>
<td>171.97</td>
<td>0.5705</td>
<td>0.0010034</td>
<td>166.35</td>
<td>176.38</td>
<td>0.5686</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>0.0010149</td>
<td>250.23</td>
<td>255.30</td>
<td>0.8285</td>
<td>0.0010127</td>
<td>249.36</td>
<td>259.49</td>
<td>0.8258</td>
<td></td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>0.0010268</td>
<td>333.72</td>
<td>338.85</td>
<td>1.0720</td>
<td>0.0010245</td>
<td>332.59</td>
<td>342.83</td>
<td>1.0688</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.0010410</td>
<td>417.52</td>
<td>422.72</td>
<td>1.3030</td>
<td>0.0010385</td>
<td>416.12</td>
<td>426.50</td>
<td>1.2992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>0.0010576</td>
<td>501.80</td>
<td>507.09</td>
<td>1.5233</td>
<td>0.0010549</td>
<td>500.08</td>
<td>510.64</td>
<td>1.5189</td>
<td></td>
<td></td>
</tr>
<tr>
<td>140</td>
<td>0.0010768</td>
<td>586.76</td>
<td>592.15</td>
<td>1.7343</td>
<td>0.0010737</td>
<td>584.68</td>
<td>595.42</td>
<td>1.7292</td>
<td></td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>0.0010988</td>
<td>672.62</td>
<td>678.12</td>
<td>1.9375</td>
<td>0.0010953</td>
<td>670.13</td>
<td>681.08</td>
<td>1.9317</td>
<td></td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>0.0011240</td>
<td>759.63</td>
<td>765.25</td>
<td>2.1341</td>
<td>0.0011199</td>
<td>756.65</td>
<td>767.84</td>
<td>2.1275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>0.0011530</td>
<td>848.1</td>
<td>853.9</td>
<td>2.3255</td>
<td>0.0011480</td>
<td>844.5</td>
<td>856.0</td>
<td>2.3178</td>
<td></td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>0.0011866</td>
<td>938.4</td>
<td>944.4</td>
<td>2.5128</td>
<td>0.0011805</td>
<td>934.1</td>
<td>945.9</td>
<td>2.5039</td>
<td></td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>0.0012264</td>
<td>1031.4</td>
<td>1037.5</td>
<td>2.6979</td>
<td>0.0012187</td>
<td>1026.0</td>
<td>1038.1</td>
<td>2.6872</td>
<td></td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>0.0012749</td>
<td>1127.9</td>
<td>1134.3</td>
<td>2.8830</td>
<td>0.0012645</td>
<td>1121.1</td>
<td>1133.7</td>
<td>2.8699</td>
<td></td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>0.0013216</td>
<td>1220.9</td>
<td>1234.1</td>
<td>3.0548</td>
<td>0.0013216</td>
<td>1220.9</td>
<td>1234.1</td>
<td>3.0548</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>0.0013972</td>
<td>1328.4</td>
<td>1342.3</td>
<td>3.2469</td>
<td>0.0013972</td>
<td>1328.4</td>
<td>1342.3</td>
<td>3.2469</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

At pressures below 5 MPa for water, the data are approximately equal to the saturated liquid data at the given temperature. We approximate intensive parameter $y$, that is $v$, $u$, $h$, and $s$ data as

$$y \approx y_{f \@ T}$$
The enthalpy is more sensitive to variations in pressure; therefore, at high pressures the enthalpy can be approximated by

\[ h \approx h_f @ T + v_f (P - P_{sat}) \]

For our work, the compressed liquid enthalpy may be approximated by

\[ h \approx h_f @ T \]

**Example 1:**

Find the internal energy of water at the given states for 7 MPa and plot the states on T-v, P-v, and P-T diagrams.
1. \( P = 7 \) MPa, dry saturated or saturated vapor

Using Table A-5,

\[
u = u_g = 2580.5 \frac{kJ}{kg}\]

Locate state 1 on the \( T-v, P-v, \) and \( P-T \) diagrams.

2. \( P = 7 \) MPa, wet saturated or saturated liquid

Using Table A-5,

\[
u = u_f = 1257.6 \frac{kJ}{kg}\]

Locate state 2 on the \( T-v, P-v, \) and \( P-T \) diagrams.

3. Moisture = 5\%, \( P = 7 \) MPa

let moisture be \( y \), defined as
\[ y = \frac{m_f}{m} = 0.05 \]

then, the quality is
\[ x = 1 - y = 1 - 0.05 = 0.95 \]

and using Table A-5,
\[ u = u_f + x(u_g - u_f) \]
\[ = 1257.6 + 0.95(2580.5 - 1257.6) \]
\[ = 2514.4 \frac{kJ}{kg} \]

Notice that we could have used
\[ u = u_f + x u_{fg} \]

Locate state 3 on the \( T-v, P-v, \) and \( P-T \) diagrams.

4. \( P = 7 \) MPa, \( T = 600^\circ C \)

For \( P = 7 \) MPa, Table A-5 gives \( T_{sat} = 285.9^\circ C \). Since \( 600^\circ C > T_{sat} \) for this pressure, the state is superheated. Use Table A-6.
\[ u = 3260.7 \frac{kJ}{kg} \]

Locate state 4 on the \( T-v, P-v, \) and \( P-T \) diagrams.

5. \( P = 7 \) MPa, \( T = 100^\circ C \)

Using Table A-4, At \( T = 100^\circ C, P_{sat} = 0.10132 \) MPa. Since \( P > P_{sat} \), the state is compressed liquid.
Approximate solution:

\[ u \approx u_f @ T = 100^\circ C = 418.94 \frac{kJ}{kg} \]

Solution using Table A-7:

We do linear interpolation to get the value at 100 °C. (We will demonstrate how to do linear interpolation with this problem even though one could accurately estimate the answer.)

<table>
<thead>
<tr>
<th>P MPa</th>
<th>u kJ/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>417.52</td>
</tr>
<tr>
<td>7</td>
<td>u = ?</td>
</tr>
<tr>
<td>10</td>
<td>416.12</td>
</tr>
</tbody>
</table>

The interpolation scheme is called “the ratio of corresponding differences.”

Using the above table, form the following ratios.

\[ \frac{5 - 7}{5 - 10} = \frac{417.52 - u}{417.52 - 416.12} \]

\[ u = 416.96 \frac{kJ}{kg} \]

Locate state 5 on the \( T-v \), \( P-v \), and \( P-T \) diagrams.

6. \( P = 7 \) MPa, \( T = 460^\circ C \)

Since \( 460^\circ C > T_{sat} \) at \( P = 7 \) MPa, the state is superheated. Using Table A-6, we do linear interpolation to get \( u \).
<table>
<thead>
<tr>
<th>T °C</th>
<th>u kJ/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>450</td>
<td>2978.0</td>
</tr>
<tr>
<td>460</td>
<td>u = ?</td>
</tr>
<tr>
<td>500</td>
<td>3073.4</td>
</tr>
</tbody>
</table>

Using the above table, form the following ratios.

\[
\frac{460 - 450}{500 - 450} = \frac{u - 2978.0}{3073.4 - 2978.0}
\]

\[
u = 2997.1 \frac{kJ}{kg}
\]

Locate state 6 on the \(T-v\), \(P-v\), and \(P-T\) diagrams.

**Example 2-2**

Determine the enthalpy of 1.5 kg of water contained in a volume of 1.2 \(m^3\) at 200 kPa.

Recall we need two independent, intensive properties to specify the state of a simple substance. Pressure \(P\) is one intensive property and specific volume is another. Therefore, we calculate the specific volume.

\[
v = \frac{Volume}{mass} = \frac{1.2 \ m^3}{1.5 \ kg} = 0.8 \ \frac{m^3}{kg}
\]

Using Table A-5 at \(P = 200\) kPa,

\[
v_f = 0.001061 \ \frac{m^3}{kg}, \quad v_g = 0.8857 \ \frac{m^3}{kg}
\]

Now,
Is \( v < v_f \) ? No
Is \( v_f < v < v_g \) ? Yes
Is \( v_g < v \) ? No

Locate this state on a \( T-v \) diagram.

We see that the state is in the two-phase or saturation region. So we must find the quality \( x \) first.

\[
v = v_f + x(v_g - v_f)
\]

\[
x = \frac{v - v_f}{v_g - v_f}
\]

\[
x = \frac{0.8 - 0.001061}{0.8857 - 0.001061}
\]

\[
x = 0.903 \quad \text{(What does this mean?)}
\]

Then,

\[
h = h_f + x h_{fg}
\]

\[
h = 504.7 + (0.903)(2201.9)
\]

\[
h = 2493.3 \frac{kJ}{kg}
\]
Example 2-4

Consider the closed, rigid container of water shown below. The pressure is 700 kPa, the mass of the saturated liquid is 1.78 kg, and the mass of the saturated vapor is 0.22 kg. Heat is added to the water until the pressure increases to 8 MPa. Find the final temperature, enthalpy, and internal energy of the water. Does the liquid level rise or fall? Plot this process on a $P$-$v$ diagram with respect to the saturation lines and the critical point.
**System:** A closed system composed of the water enclosed in the tank

**Property Relation:** Steam Tables

**Process:** Volume is constant (rigid container)

For the closed system the total mass is constant and since the process is one in which the volume is constant, the average specific volume of the saturated mixture during the process is given by

\[ \nu = \frac{V}{m} = \text{constant} \]

or

\[ \nu_2 = \nu_1 \]

Now to find \( \nu_1 \) recall that in the two-phase region at state 1

\[ x_1 = \frac{m_{g1}}{m_{f1} + m_{g1}} = \frac{0.22 \text{ kg}}{(1.78 + 0.22) \text{ kg}} = 0.11 \]

Then, at \( P = 700 \text{ kPa} \)

\[ \nu_1 = \nu_{f1} + x_1 (\nu_{g1} - \nu_{f1}) \]

\[ = 0.001108 + (0.11)(0.2729 - 0.001108) \]

\[ = 0.031 \frac{m^3}{kg} \]

State 2 is specified by:

\[ P_2 = 8 \text{ MPa}, \quad \nu_2 = 0.031 \frac{m^3}{kg} \]

At 8 MPa,
\[ v_f = 0.001384 \, m^3/kg \quad v_g = 0.002352 \, m^3/kg \]

at 8 MPa, \( v_2 = 0.031 \, m^3/kg \); therefore, is

Is \( v_2 < v_f \) ? No

Is \( v_f < v_2 < v_g \) ? No

Is \( v_g < v_2 \) ? Yes

Therefore, State 2 is superheated.

Interpolating in the superheated tables at 8 MPa gives,

\[
\begin{align*}
T_2 &= 362 \, ^\circ C \\
h_2 &= 3024 \, kJ/kg \\
u_2 &= 2776 \, kJ/kg \\
\end{align*}
\]

Since state 2 is superheated, the liquid level falls.

**Extra Problem**

What would happen to the liquid level in the last example if the specific volume had been 0.001 \( m^3/kg \) and the pressure was 8 MPa?
**Extra Assignment**

Complete the following table for properties of water

Sketch a $T$-$v$ or $P$-$v$ diagram for each state. Describe the phase as compressed liquid, saturated mixture, or superheated vapor. If the state is saturated mixture, give the quality.

<table>
<thead>
<tr>
<th>$P$ MPa</th>
<th>$v$ $m^3$/kg</th>
<th>$T$ °C</th>
<th>Phase Description and Quality if Applicable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.725</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>0.227</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>15.0</td>
<td>0.001124</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.221</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>0.3879</td>
<td>150</td>
<td></td>
</tr>
<tr>
<td>0.04</td>
<td>350</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.6</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>0.095</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>0.055</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td>0.580</td>
<td>135</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>0.4625</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
INTERPOLATION

We will use LINEAR INTERPOLATION:

Assume you have the following table:

<table>
<thead>
<tr>
<th>T, °C</th>
<th>v, m³/kg</th>
<th>u, kJ/kg</th>
<th>h, kJ/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>P = 0.1 MPa (99.63°C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sat. 100</td>
<td>1.6940</td>
<td>2506.1</td>
<td>2675.5</td>
</tr>
<tr>
<td>150</td>
<td>1.6958</td>
<td>2506.7</td>
<td>2676.2</td>
</tr>
<tr>
<td></td>
<td>1.9364</td>
<td>2582.8</td>
<td>2776.4</td>
</tr>
<tr>
<td>1300</td>
<td>7.260</td>
<td>4683.5</td>
<td>5409.5</td>
</tr>
<tr>
<td>P = 0.5 MPa (151.86°C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sat. 200</td>
<td>0.3749</td>
<td>2561.2</td>
<td>2748.7</td>
</tr>
<tr>
<td>250</td>
<td>0.4249</td>
<td>2642.9</td>
<td>2855.4</td>
</tr>
<tr>
<td></td>
<td>0.4744</td>
<td>2723.5</td>
<td>2960.7</td>
</tr>
</tbody>
</table>

and you have: P = 0.1 MPa and T=120 °C, and you want to find either v, or u, or h. (let's pick v). What can we do?

We make an estimation. That estimation is usually done assuming that the behavior is linear (and that is why we call it LINEAR interpolation, interpolation = “to estimate values of (a function) between two known values”).

The value we want
We can find the equation of the line (we know two points), and then plug in the given 120°C and find the value that we want.

The equation of the line passing by \((x_1, y_1)\) and \((x_2, y_2)\):

\[
y = y_1 + \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)
\]

so in our case:

\[
v_{\text{wanted}} = 1.6958 + \frac{1.9364 - 1.6958}{150 - 100} (T - 100)
\]

Notice that \(T\) has units of °C and the result \(v_{\text{wanted}}\) will have units of \(m^3/kg\).

For \(T = 120°C\) we get \(v = 1.79204 m^3/kg\)

You can also interpret the linear interpolation procedure from similar triangles: (Triangles ABC and ADE are similar), then,

\[
\frac{X_{\text{known}} - X_1}{X_2 - X_1} = \frac{Y_{\text{wanted}} - Y_1}{Y_2 - Y_1}
\]

or

\[
Y_{\text{wanted}} = Y_1 + \frac{Y_2 - Y_1}{X_2 - X_1} (X_{\text{known}} - X_1)
\]