Thermal Performance Measures

A general performance measure is expressed as:

\[
\text{Performance Measure} = \frac{\text{Desired result}}{\text{Required input to achieve desired result}}
\]

Heat engine/power cycles:

A schematic representation of a heat engine operating in a cycle:

- Hot thermal reservoir at \( T_H \)
- Cold thermal reservoir at \( T_C \)
- \( Q_H = Q_{in} \)
- \( W_{\text{NET}} = W_{\text{cycle}} = Q_{in} - Q_{out} \)
- \( Q_C = Q_{out} \)

The heat engine is a power cycle where the desired result of the cycle is the work transfer of energy to the surroundings during each cycle.

The power cycle receives heat transfer of energy into the system from some hot body (hot thermal reservoir) in the amount \( Q_{in} \), and rejects heat transfer of energy out in the amount \( Q_{out} \) to some cold body (cold thermal reservoir).

Clearly, in a power cycle \( Q_{in} > Q_{out} \). The desired result is \( W_{\text{cycle}} \), and the required input is \( Q_{in} \). The corresponding performance measure is called the thermal efficiency and is represented by the symbol \( \eta \).

\[
\eta = \frac{W_{\text{cycle}}}{Q_{in}}
\]

An alternative form is expressed as:
Thus, the thermal efficiency represents the extent to which the energy added by heat to the system $Q_{in}$ is converted into a net work output $W_{cycle}$.

Since energy is conserved (first law of thermodynamics), the thermal efficiency can never by greater than one. In actual power cycles the efficiency is always less than one.

Not all energy added to the system by heat energy is converted into work; a portion of the input energy must be rejected to a cold body/reservoir by heat transfer.

The second law of thermodynamics will provide a maximum efficiency (must be less than one) for cycles operating between two reservoirs.

**Refrigeration and heat pump cycles**

A refrigeration and heat pump cycles are “power cycles in reverse.” A schematic diagram of refrigeration and heat pump cycles is shown below.

In the refrigeration and heat pump cycles the energy transfer into the system by heat transfer, $Q_{in}$, is from a cold body/reservoir, and the energy transfer out of the system by heat transfer, $Q_{out}$ is to a hot body/reservoir. This transfer of heat energy is accomplished by a net work input, $W_{cycle}$, to the system.

Note that refrigeration and heat pump cycles differ only in their objectives.

The objective of a refrigeration cycle is to cool a refrigerated space or maintain a temperature of a house below the temperature of its surroundings.
The objective of a heat pump is to maintain the temperature within a house above the temperature of the surroundings.

Since the refrigerator and heat pump cycles have different objectives, their performance measures, called coefficients of performance, are defined differently.

**Refrigeration Cycles**

The desired result is the energy transfer to the system by heat transfer from the cold body/reservoir, $Q_{in}$, required input to achieve this result is the net work transfer into the system, $W_{cycle}$. Let the symbol for the refrigeration coefficient of performance be $\beta$, thus:

\[ \beta = \frac{Q_{in}}{W_{cycle}} \]

or alternatively:

\[ \beta = \frac{Q_{in}}{Q_{out} - Q_{in}} \]

In a refrigerator $Q_{out}$ is rejected to the room where the refrigerator is located, and $W_{cycle}$ is usually provided by the electric motor that drives the refrigerator compressor. Note that $Q_{in}$ represents the heat transfer in to the system and out of the refrigerated space in order to maintain the desired cool space at a temperature below the room temperature. To maintain this desired cool temperature, $Q_{in}$ must balance the heat gain by the cool space and its contents from the relatively warm room and the frequent opening and closing of the refrigerator door.

**Heat Pump Cycles**

The desired result is the energy transfer from the system by heat transfer from the hot body/reservoir, $Q_{out}$, required output to achieve this result is the net work transfer into the system, $W_{cycle}$. Let the symbol for the heat pump coefficient of performance be $\gamma$, thus:

\[ \gamma = \frac{Q_{out}}{W_{cycle}} \]

or alternatively:

\[ \gamma = \frac{Q_{out}}{Q_{out} - Q_{in}} \]

In a heat pump $Q_{in}$ is obtained from the surrounding atmosphere, and $W_{cycle}$ is usually provided by electricity. Note that $Q_{out}$ represents the heat transfer out of the system and
in to the heated space in order to maintain the desired warm space at a temperature above the ambient outdoor temperature. To maintain this desired indoor warm temperature, \( Q_{\text{out}} \) must balance the heat loss by the house to the surrounding cool atmosphere.

Note that the value of \( \gamma \) is never less than one.

**Maximum performance measures for cycles operating between two reservoirs**

The Carnot power cycle is the most efficient power cycle. Since it is an internally and externally reversible cycle its efficiency will always be greater than any irreversible cycle operating between the same two reservoirs. Note that in the Carnot cycle we showed that the ratio of the heat transfers is related to the absolute temperatures of the corresponding thermal reservoirs.

\[
\frac{Q_H}{Q_C} = \frac{T_H}{T_C}
\]

\[
Q_H = Q_{\text{in}}
\]

\[
Q_C = Q_{\text{out}}
\]

\[
W_{\text{cycle}} = Q_{\text{in}} - Q_{\text{out}}
\]

To obtain the maximum performance measure we replace the heat ratios by the corresponding absolute temperature ratios in the previously defined performance measures.

**Power cycles**

\[
\eta_{\text{max}} = 1 - \frac{T_C}{T_H}
\]
Since a refrigerator/heat pump is a Carnot cycle in “reverse,” the same maximum performance formulation also applies. Thus we may write:

**Refrigeration cycles**

\[
\beta_{\text{max}} = \frac{T_C}{T_H - T_C}
\]

**Heat Pump cycles**

\[
\gamma_{\text{max}} = \frac{T_H}{T_H - T_C}
\]