1. Small-scale perturbations in a supersonic flow satisfy the following PDE for the perturbation pressure $p(x, y, z)$:

$$(u^2 - a^2)p_{xx} + (v^2 - a^2)p_{yy} + (w^2 - a^2)p_{zz} + 2uvp_{xy} + 2vwp_{yz} + 2wp_{zx} = 0$$

where $a$ is the speed of sound, which is to be assumed constant, and $(u, v, w)$ is the supersonic flow velocity; take it as $(u, v, w) = (2a, a, a)$. Derive the simplified canonical equation by rotating the coordinate system using class PDE transformation procedures. Give the simplified equation and the new coordinate system. Explain why the results you obtained could have been expected physically without doing the math.

2. Find the unsteady temperature distribution $u(x, t)$ in a bar extending from $x = 0$ to $x = \infty$, if the initial temperature at time $t = 0$ is zero, and the end at $x = 0$ is held at 5 degrees centigrade for $0 < t < 1$ and at zero degrees for $1 < t$. The heat conduction coefficient is $9 \, \text{m}^2/\text{s}$. Be sure to simplify your answer and write it in terms of what is given only.

3. Find the unsteady temperature distribution $u(x, t)$ in a bar of length 3 m if the end at $x = 0$ m is held at 15 degrees Centigrade, while the other end at $x = 3$ m is insulated (i.e. it has a homogeneous Neumann boundary condition.) Assume the initial temperature to be of the form $u(x, 0) = f(x)$ where $f(x) = 20C$ in the range $0m < x < 1m$ and $f(x) = 15C$ for $1m < x < 3m$. The heat conduction coefficient is $9\, \text{m}^2/\text{s}$

4. Answer the preassigned PDE problem. Your solution may be brought into the exam.