Homogeneous Equations

Homogeneous equation:

\[
\frac{dy}{dx} = f\left(\frac{y}{x}\right)
\]

Solution: define a new unknown

\[
u = \frac{y}{x},
\]

so replace \( y \) by \( xu \). The equation for \( u(x) \) will be separable.

Note: Do not confuse this use of the word “homogeneous” for first order equations with the use of the same word for “homogeneous” for linear ODEs!

Note: To check exactness, you can replace \( x \) by \( tx \) and \( y \) by \( ty \) and see whether the \( ts \) cancel.