Vectors:

- Statics and Dynamics
- Physics
- Geometry
- Computer solutions
- ...

Properties:

- The vector above as a list of numbers:
  \[ \mathbf{r} = \mathbf{x} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \text{ or } \mathbf{r}^T = (4, 2) \]

If the list is written vertically, the vector is called a column vector; if it is written horizontally, it is a row vector. An \(n\)-dimensional row vector is equivalent to a \(1 \times n\) size matrix; an \(n\)-dimensional column vector to a \(n \times 1\) matrix,

- Components of the vector above: \(r_1 = r_x = x_1 = x = 4, \ r_2 = r_y = x_2 = y = 2\).
- Addition of vectors:

  \[ (4, 2) + (1, 3) = (4 + 1, 2 + 3) = (5, 5). \]
• Multiplication of a vector by a scalar:

\[ 1.5(4, 2) = (1.5 \cdot 4, 1.5 \cdot 2) = (6, 3) \]

• Length or norm of a vector:

- Definition:

\[ ||\vec{a}|| = |\vec{a}| = a = \sqrt{a_x^2 + a_y^2 + a_z^2 + \ldots} \]

- Unit vectors: Unit vectors have length one.

- Distance: The distance between two points \( \vec{r}_1 \) and \( \vec{r}_2 \) is by definition \( ||\vec{r}_2 - \vec{r}_1|| \):

• Dot (scalar) product:

- Definition:

\[ \vec{a} \cdot \vec{b} = a_xb_x + a_yb_y + a_zb_z + \ldots = ||\vec{a}|| \cdot ||\vec{b}|| \cos \theta \]
- Orthogonality: If the dot product is zero, the vectors are by definition orthogonal to each other.
- Length: $||\vec{a}|| = \sqrt{\vec{a} \cdot \vec{a}}$.

- Projection:

\[
\begin{align*}
\text{The magnitude of the (orthogonal) component (or coordinate) of } \vec{a} \text{ in the direction of } \vec{b} & \text{ is:} \\
a_b &= a \cos(\varphi) = \vec{a} \cdot \hat{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||} \\
\text{The projection of } \vec{a} \text{ onto } \vec{b} & \text{ is} \\
\text{proj}(\vec{a}, \vec{b}) &= \vec{a}_b = a_b \hat{b} = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||} \hat{b}
\end{align*}
\]