1.59(b)

1 1.59(b), §1 Asked

**Given:** The hyperboloid of one sheet

\[ x^2 + 3y^2 - 5z^2 = 160 \]

and the point P with position vector (3,-2,1) on that hyperboloid.

**Asked:** A normal vector \( \vec{N} \) to the surface at P and the tangent plane at P.

2 1.59(b), §2 Solution

\[ x^2 + 3y^2 - 5z^2 = 160 \quad P = (3, -2, 1) \]

Correct problem:

\[ x^2 + 3y^2 - 5z^2 = 16 \quad P = (3, -2, 1) \]

Bring equation of surface in **standard form** (zero right hand side):

\[ x^2 + 3y^2 - 5z^2 - 16 \equiv F(x, y, z) = 0 \]

A normal vector to a surface in standard form is given by the gradient of \( F \):

\[
\nabla F \equiv \begin{pmatrix}
\frac{\partial F}{\partial x} \\
\frac{\partial F}{\partial y} \\
\frac{\partial F}{\partial z}
\end{pmatrix} = \begin{pmatrix}
2x \\
6y \\
-10z
\end{pmatrix}
\]

At P, \((x, y, z) = (3, -2, 1)\), so:

\[
\vec{N} = \nabla F \bigg|_P = \begin{pmatrix}
6 \\
-12 \\
-10
\end{pmatrix}
\]

Tangent plane:

\[ \vec{N} \cdot \vec{r} = \vec{N} \cdot \vec{r}_P \]
or

\[ 6x - 12y - 10z = 6 \cdot 3 - 12 \cdot (-2) - 10 \cdot 1 = 32 \]

Can divide by 2 to simplify:

\[ 3x - 6y - 5z = 16 \]