1.48(a)

1  1.48(a), §1 Asked

Given: The vectors

\[ \vec{v} = (2, 5) \quad \vec{u}_1 = (1, 2) \quad \vec{u}_2 = (3, 5) \]

Asked: Write \( \vec{v} \) as a linear combination \( a \vec{u}_1 + b \vec{u}_2 \), i.e., find \( a \) and \( b \) so that \( \vec{v} = a \vec{u}_1 + b \vec{u}_2 \)

2  1.48(a), §2 Solution

Write \( \vec{v} \) as a linear combination \( a \vec{u}_1 + b \vec{u}_2 \), i.e., find \( a \) and \( b \) so that \( \vec{v} = a \vec{u}_1 + b \vec{u}_2 \)

\[
\begin{pmatrix} 2 \\ 5 \end{pmatrix} = a \begin{pmatrix} 1 \\ 2 \end{pmatrix} + b \begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 1a + 3b \\ 2a + 5b \end{pmatrix}
\]

\[
\begin{align*}
a + 3b &= 2 \quad (1) \\
2a + 5b &= 5 \quad (2)
\end{align*}
\]

Eliminate \( a \) from equation (2) by subtracting 2 times (1):

\[
\begin{align*}
a + 3b &= 2 \quad (1) \\
0 - b &= 1 \quad (2') = (2) - 2(1)
\end{align*}
\]

Solve from the bottom up, \( (2') \) giving that \( b = -1 \) and then (1) giving that \( a = 5 \).