# Introduction

Multiple integrals:

- **Areas (cost, ...):**
  \[ dA = dx \, dy \quad dA = \rho \, d\rho \, d\theta \]

- **Volumes (weight, ...):**
  \[ dV = dx \, dy \, dz \quad dV = \rho \, d\rho \, d\theta \, dz \quad dV = r^2 \sin \phi \, dr \, d\phi \, d\theta \]

- **Centroids (center of gravity, center of pressure, ...)**
  \[ \bar{x} = \frac{\int x \, dA}{\int dA} \quad \bar{x} = \frac{\int x \, dV}{\int dV} \]

- **Moments of inertia (solid body dynamics, center of pressure, ...)**
  \[ I_x = \int y^2 \, dA \quad I_0 = \int x^2 + y^2 \, dA \]
  \[ I_x = \int y^2 + z^2 \, dV \quad I_{xy} = -\int xy \, dV \]

- ...

**Notes:**

- Draw the region to be integrated over.
- When integrating, say \( \int \int \int f(a, b, c) \, da \, db \, dc \), you have to decide whether you want to do \( a, b \), or \( c \) first.
- Usually, you do the coordinate with the easiest limits of integration first.
- If you decide to do, say, \( b \) first, \( (\int_{b_1}^{b_2} f(a, b, c) \, db \) first), the limits of integration \( b_1 \) and \( b_2 \) must be identified from the graph at *arbitrary* \( a \) and \( c \), and are normally functions of \( a \) and \( c \): \( b_1 = b_1(a, c) \), \( b_2 = b_2(a, c) \).
- After integrating over, say, \( b \), the remaining double integral should no longer depend on \( b \) in any way. Nor does the region of integration: redraw it without the \( b \) coordinate. Then integrate over the next easiest coordinate in the same way.