1. Solve the Cauchy equation

\[ x^2 y'' + xy' - 4y = \ln x^2 \]

by taking \( u = \ln |x| \) as the new independent variable. To eliminate \( x \), use the chain rule of differentiation as in

\[ y' \equiv \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{dy}{du} \frac{1}{u}, \]

and once more to find \( y'' \) in terms of \( dy/du \) and \( d^2y/du^2 \). Please do not indicate \( dy/du \) also by \( y' \)!

Solution:

\[ y = -\frac{1}{2} \ln x + Ax^2 + Bx^{-2} \]

Also solve the 4 questions below*:

2. Solve the aerodynamically damped spring-mass system

\[ \ddot{y} + (\dot{y})^2 + y = 0 \]
by taking $y$ as the independent variable and $\dot{y}$ as the dependent variable. To eliminate the remaining $dt$, (in $\ddot{y} = d\dot{y}/dt$), use the chain rule of differentiation. Solution:

$$\dot{y}^2 = -y + \frac{1}{2} + C_0 e^{-2y}, \text{ hence } t = \pm \int \frac{dy}{\sqrt{-y + \frac{1}{2} + C_0 e^{-2y}}}$$

3. Solve the motion of a falling body with aerodynamic drag:

$$\ddot{x} + (\dot{x})^2 = 1.$$

Solution:

$$\dot{x} = \frac{Ce^{2t} - 1}{Ce^{2t} + 1}, \quad x = \ln |Ce^{2t} + 1| - t + D$$

4. Solve the equation for the streamfunction in a Stokes boundary layer:

$$y'' + 2xy' - 2y = 0.$$

Note that $y = x$ is one solution. Solution:

$$y = C_0 x + C_1 x \int \frac{e^{-x^2}}{x^2} \, dx$$

**Also:** Make exam 3 of 1998. Give yourself 50 minutes. Include your solutions with homework set I and grade yourself using the solutions on the web after you get it back.