Ideal Gas Model

• For many gases, the ideal gas assumption is valid and the P-v-T relationship can be simplified by using the ideal gas equation of state: \( P_v = RT \) (Compressibility factor \( Z = 1 \))
• For an ideal gas, it can be shown that the specific internal energy \( u \) is a function of temperature only, \( u = u(T) \)
• Accordingly, specific enthalpy is a function of temperature only, since \( h(T) = u(T) + P_v = u(T) + RT \)
• Physically, a gas can be considered ideal only when the intermolecular forces are weak. This usually happens in low pressure and high temperature ranges.

\[
c_v = \frac{du}{dT}, \quad c_p = \frac{dh}{dT} = \frac{d(u + P_v)}{dT} = \frac{d(u + RT)}{dT} = c_v + R
\]

\[
c_p = c_v + R
\]
Ideal Gas Model (cont.)

- Specific ratio $k = c_p/c_v$, $c_p = c_v + R$

$$c_p(T) = \frac{kR}{k-1}, \quad c_v(T) = \frac{R}{k-1}$$

$$du = c_v(T)\,dT, \text{ integrate from 1 to 2}$$

$$u(T_2) - u(T_1) = \int_1^2 c_v(T)\,dT, \text{ similarly}$$

$$h(T_2) - h(T_1) = \int_1^2 c_p(T)\,dT$$

- In some cases, the temperature dependence of the specific heat can be written in polynomial form, see Table A-2, p. 845
- Otherwise, ideal gas tables are also available. They are easier to use compared to the thermodynamic tables since temperature is the only parameter.
Example: Two tanks filled with air are connected by a valve as shown. If the valve is opened and the gases are allowed to mix while receiving heat from the surrounding. The final temperature is 227°C. Determine (a) the final pressure of the mixture, (b) the amount of heat transfer during the mixing process. Assume ideal gas model is valid.

Air, 1 kg, 127°C 0.05 MPa
Air, 2 kg 247°C 0.02 MPa

Assume: little change of KE & PE
Both the initial and the final states are in equilibrium

From table A-1, $T_c=132.5$ K, $P_c=3.77$ Mpa
$T_r=3$, $P_r=0.0133$ for tank 1
From Figure A-13, compressibility factor $Z=1$, good ideal gas assumption
\[ P_f = \frac{mRT_f}{V} \], where \( V = V_1 + V_2 \) is the total volume of both tanks.

\( m = m_1 + m_2 \) is the total mass of both tanks.

\[ V_1 = \frac{m_1RT_1}{P_1}, \quad V_2 = \frac{m_2RT_2}{P_2} \]

\[ P_f = \frac{(m_1 + m_2)RT_f}{\frac{m_1RT_1}{P_1} + \frac{m_2RT_2}{P_2}} = \frac{(1 + 2)(273 + 227)}{(1)(400) + (2)(520)} = 0.025(MPa) \]

(b) The heat transfer can be found from the energy balance

\[ E_f - E_i = U_f - U_i = Q - W \]

\[ Q = U_f - U_i = (m_1 + m_2)u(T_f) - [m_1u(T_1) + m_2u(T_2)] \]
\[ Q = m_1[u(T_f) - u(T_1)] + m_2[u(T_f) - u(T_2)] \]

\[ = m_1 C_{v,avg,f-1}(T_f - T_1) + m_2 C_{v,avg,f-2}(T_f - T_2) \] (see eq 3-46, p.112)

where \( C_{v,avg,f-1} = 1/2(0.726 + 0.742) = 0.734(\text{kJ/kgK}) \)

can be found using averaged \( C_v \) and table A-2 (p.844)

\( C_{v,avg,f-2} \approx 0.742 (\text{kJ/kg K}) \) since \( T_f \approx T_2 \)

\[ Q = (1)(0.734)(500 - 400) + (2)(0.742)(500 - 520) = 43.72(\text{kJ}) \]