Objectives

- Introduce the thermodynamic property entropy (S) using the Clausius inequality
- Recognize the fact that the entropy is always increasing for an isolated system (or a system plus its surroundings) based on the increase of entropy principle
- Analysis of entropy change of a thermodynamic process (how to use thermodynamic table, ideal gas relation)
- Property diagrams involving entropy (T-s and h-s diagrams)
- Entropy balance: entropy change = entropy transfer + entropy change
Entropy

• Entropy: a thermodynamic property, can be used as a measure of disorder. The more disorganized a system the higher the entropy.
• Defined using Clausius inequality \( \int \frac{\delta Q}{T} \leq 0 \)
• This inequality is valid for all cycles, reversible and irreversible.
• Consider a reversible Carnot cycle

\[
\int \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}, \quad \text{from Carnot efficiency } \eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}, \quad \frac{Q_L}{Q_H} = \frac{T_L}{T_H}
\]

Therefore, \( \int \frac{\delta Q}{T} = 0 \) for a reversible Carnot cycle \( \int \left( \frac{\delta Q}{T} \right)_{rev} = 0 \)

• Define a thermodynamic property entropy \((S)\), such that

\[
dS = \left. \frac{\delta Q}{T} \right|_{rev}, \quad \text{for any reversible process } \int_{1}^{2} dS = \int_{1}^{2} \left. \frac{\delta Q}{T} \right|_{rev} = S_2 - S_1
\]

The change of entropy can be defined based on a reversible process
Entropic-2

- Since entropy is a thermodynamic property, it has fixed values at a fixed thermodynamic states.

\[ \oint \frac{\delta Q}{T} = \int_1^2 \left( \frac{\delta Q}{T} \right)_\text{rev} + \int_1^2 \left( \frac{\delta Q}{T} \right) \leq 0 \]

From entropy definition

\[ dS = \left( \frac{\delta Q}{T} \right)_\text{rev} \]

\[ \oint dS = 0 = \oint \left( \frac{\delta Q}{T} \right)_\text{rev} = \int_1^2 \left( \frac{\delta Q}{T} \right)_\text{rev} + \int_1^2 \left( \frac{\delta Q}{T} \right) \]

Therefore,

\[ \int_1^2 \left( \frac{\delta Q}{T} \right)_\text{rev} \leq \int_1^2 \left( \frac{\delta Q}{T} \right) = \int_1^2 dS = S_2 - S_1 = \Delta S \]

\[ \Delta S = S_2 - S_1 \geq \int_1^2 \left( \frac{\delta Q}{T} \right)_\text{rev} \]

This is valid for all processes

\[ ds \geq \frac{\delta Q}{T} \]

- The entropy change during an irreversible process is greater than the integral of \( \delta Q/T \) during the process. If the process is reversible, then the entropy change is equal to the integral of \( \delta Q/T \). For the same entropy change, the heat transfer for a reversible process is less than that of an irreversible. Why?
Entropy Increase Principle

\[ \Delta S = S_2 - S_1 \geq \int_1^2 \left( \frac{\delta Q}{T} \right), \] define entropy generation \( S_{\text{gen}} \)

\[ \Delta S_{\text{system}} = S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right) + S_{\text{gen}} \geq \int_1^2 \left( \frac{\delta Q}{T} \right) \]

where \( S_{\text{gen}} \geq 0 \). If the system is isolated and "no" heat transfer

The entropy will still increase or stay the same but never decrease

\[ \Delta S_{\text{system}} = S_{\text{gen}} \geq 0, \] entropy increase principle

• A process can take place only in the direction that complies with the increase of entropy principle, that is, \( S_{\text{gen}} \geq 0 \).

• Entropy is non-conservative since it is always increasing. The entropy of the universe is continuously increasing, in other words, it is more disorganized and is approaching chaotic.

• The entropy generation is due to the existence of irreversibilities. Therefore, the higher the entropy generation the higher the irreversibilities and, accordingly, the lower the efficiency of a device since a reversible system is the most efficient system.
Entropy Generation Example

Example: Show that the heat cannot transfer from the low-temperature sink to the high-temperature source based on the increase of entropy principle.

\[ \Delta S(\text{source}) = \frac{2000}{800} = 2.5 \text{ (kJ/K)} \]
\[ \Delta S(\text{sink}) = -\frac{2000}{500} = -4 \text{ (kJ/K)} \]
\[ S_{\text{gen}} = \Delta S(\text{source}) + \Delta S(\text{sink}) = -1.5 \text{ (kJ/K)} < 0 \]

It is impossible based on the entropy increase principle \( S_{\text{gen}} \geq 0 \), therefore, the heat cannot transfer from low-temp. to high-temp. without external work input.

- If the process is reversed, 2000 kJ of heat is transferred from the source to the sink, \( S_{\text{gen}} = 1.5 \text{ (kJ/K)} > 0 \), and the process can occur according to the second law.

- If the sink temperature is increased to 700 K, how about the entropy generation?
  \[ \Delta S(\text{source}) = -\frac{2000}{800} = -2.5 \text{ (kJ/K)} \]
  \[ \Delta S(\text{sink}) = \frac{2000}{700} = 2.86 \text{ (kJ/K)} \]
  \[ S_{\text{gen}} = \Delta S(\text{source}) + \Delta S(\text{sink}) = 0.36 \text{ (kJ/K)} < 1.5 \text{ (kJ/K)} \]

Entropy generation is less than when the sink temperature is 500 K, less irreversibility. Heat transfer between objects having large temperature difference generates higher degree of irreversibilities.