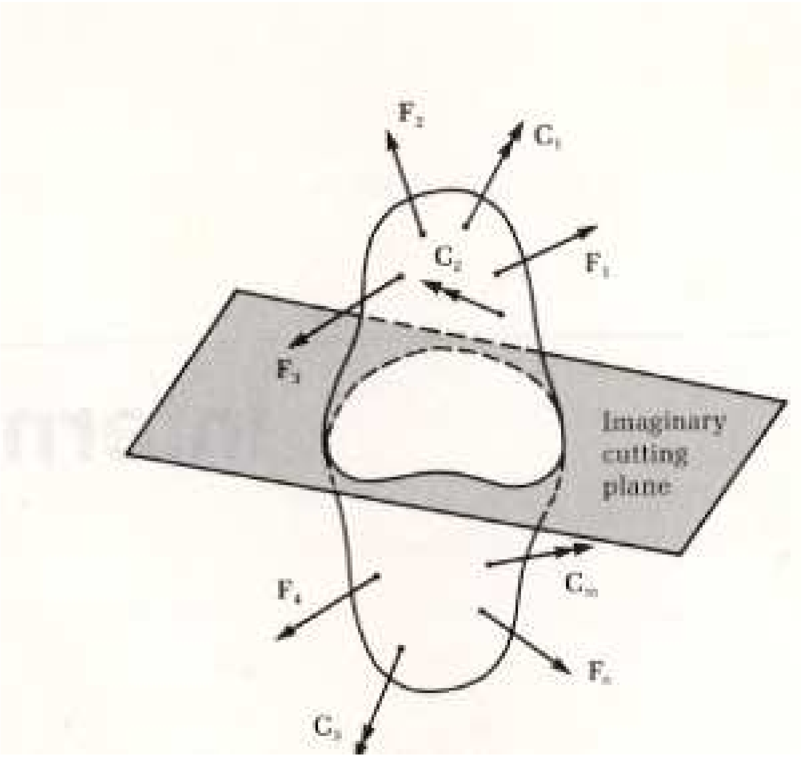
Mechanics & Materials 1

Chapter 8
Stress and Strain

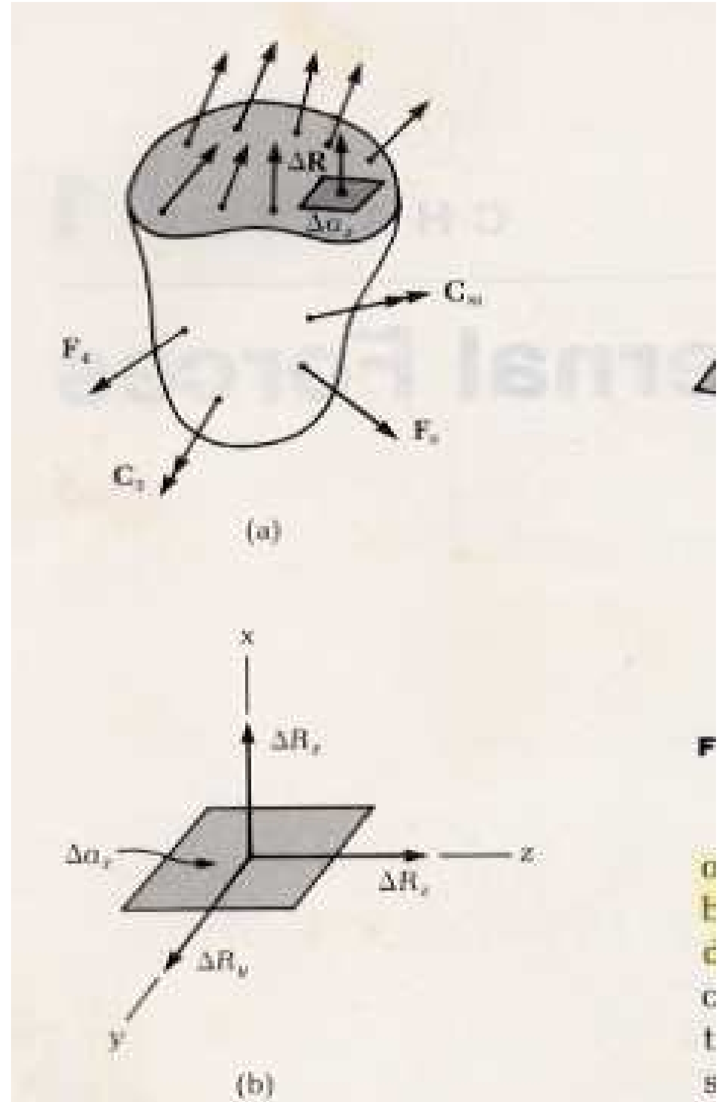
FAMU-FSU College of Engineering
Department of Mechanical Engineering

Stress

- An imaginary plane is perceived to separate the structure into two distinct portions.
- One portion is selected as the free body diagram



- This selected portion is subjected to two types of forces:
  - external forces and couples that are applied directly to it
  - distributed system of forces exerted on it by the portion that was named.



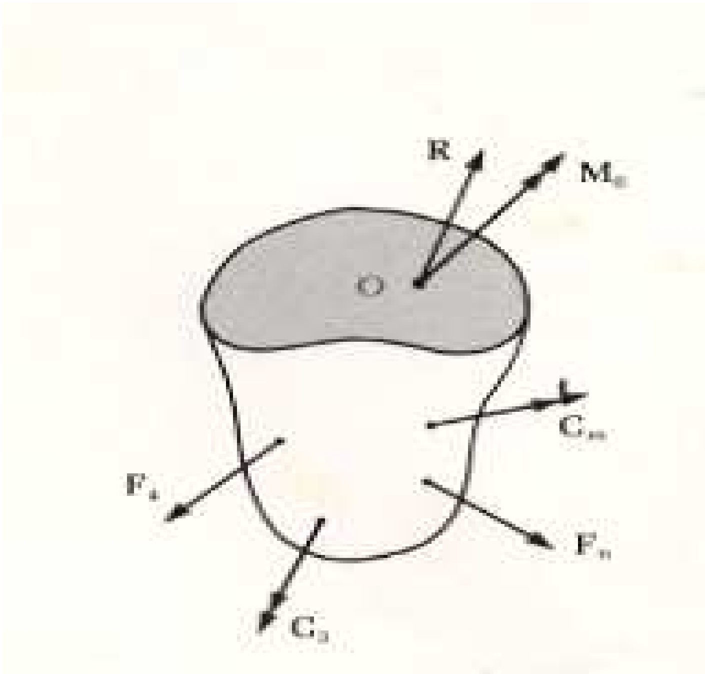
Stress

- The distributed force system that acts on the cutting plane represents the molecular forces that material particles an either side of the imaginary cutting plane exert on each other.

- The exact distribution of those molecular forces on the exposed plane of the free body diagram is unknown

Stress: Equilibrium

- Replace distributed force system by an equivalent force- couple system



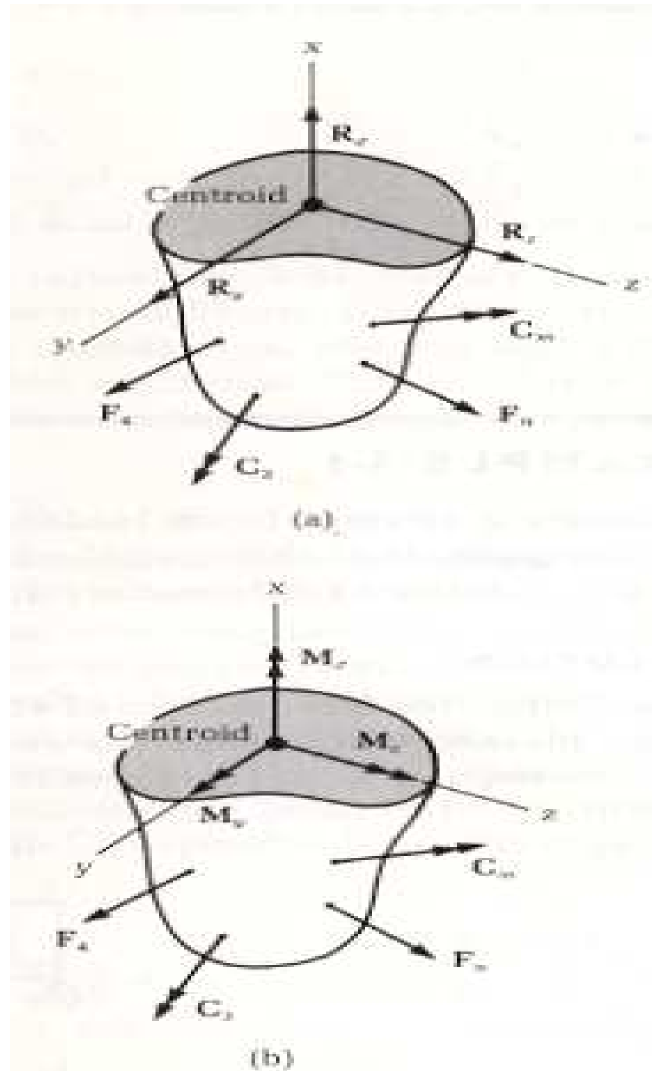
$$\overline{R} + \sum \overline{F}_i = 0$$

$$\overline{M}_0 + \sum \overline{r}_i \times \overline{F}_i + \sum \overline{G}_i = 0$$

•F_i, G_i $\rightarrow$ concentrated forces and couples applied directly to the surface

Stress

- To connect the components R_x, R_y, R_z and M_x, M_y, and M_z with actual force we need the concept of stress.
- Stress - force per unit area



Stresses: Normal and Shear

- Δa_x - increment of area perpendicular to the x-axis
- ΔR - increment of force **R** acts on Δa_x
- **Note nomenclature: first subscript refers to direction of normal to plane; second subscript refers to direction of force**

$$\lim_{\Delta a_x \to 0} \frac{\Delta R_x}{\Delta a_x} = \sigma_{xx} \Rightarrow normal \quad stress$$

$$\lim_{\Delta a_x \to 0} \frac{\Delta R_y}{\Delta a_x} = \tau_{xy} \Rightarrow shear \quad stress$$

$$\lim_{\Delta a_x \to 0} \frac{\Delta R_z}{\Delta a_x} = \tau_{xz} \Rightarrow shear \quad stress$$

Stress Dimensions

Dimensions:

$$\text{stress} = \text{force per unit area}$$

Units:

$$1\,\frac{\text{lb}}{\text{in}^2} = 1\,\text{psi}; \qquad 10^3\,\frac{\text{lb}}{\text{in}^2} = 1\,\text{ksi}$$

$$1\,\frac{\text{N}}{\text{m}^2} = 1\,\text{Pa}; \qquad 10^6\,\frac{\text{lb}}{\text{in}^2} = 1\,\text{MPa}$$

Conversions:

$$1\,\text{psi} = 1\left(\frac{\text{lb}}{\text{in}^2}\right)\left(4.4482\,\frac{\text{N}}{\text{lb}}\right)\left(\frac{\text{in}}{0.0254\ \text{m}}\right)^2$$
$$= 1\left(\frac{\text{in}}{\text{in}^2}\right)$$

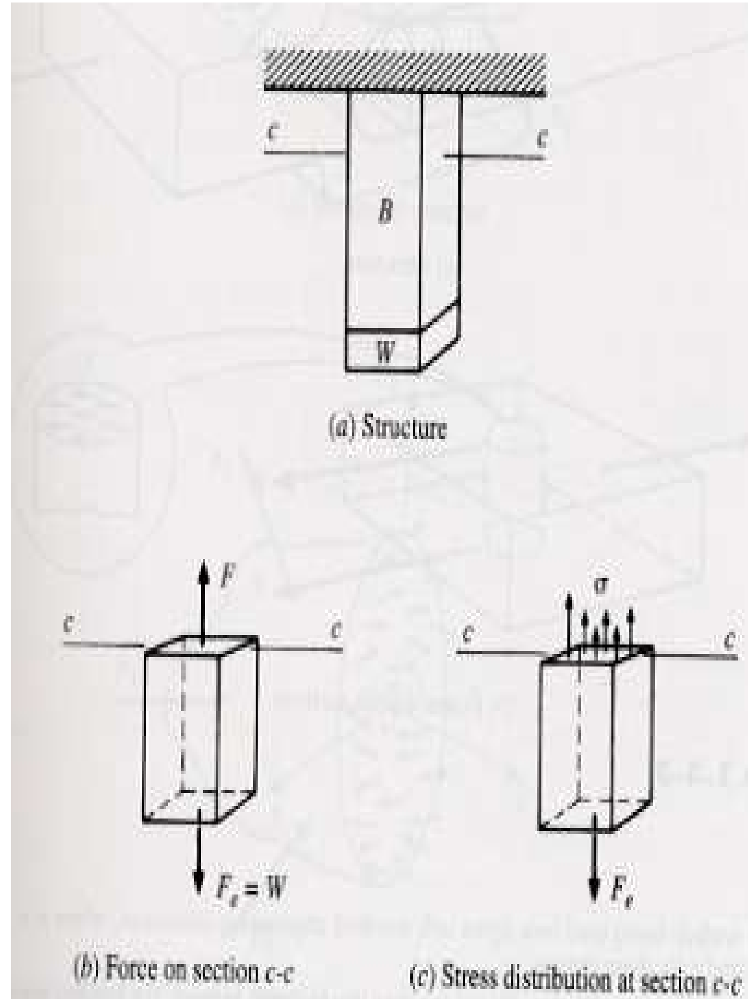
$$(1\,\text{psi} = 6890\,\text{Pa}, \qquad 1\,\text{ksi} = 6.890\,\text{MPa})$$

Normal Stress

- The stress component σ_x that is perpendicular to the imaginary plane is called normal stress
- Normal Stress either:
  - tensile: pulls the cutting plane (σ: +ve)
  - compressive: pushes the cutting plane (σ: -ve)

$$\sigma = \lim_{\Delta a \to 0} \frac{\Delta F_u}{\Delta A}$$

- F_u $\rightarrow$force normal to the area element ΔA



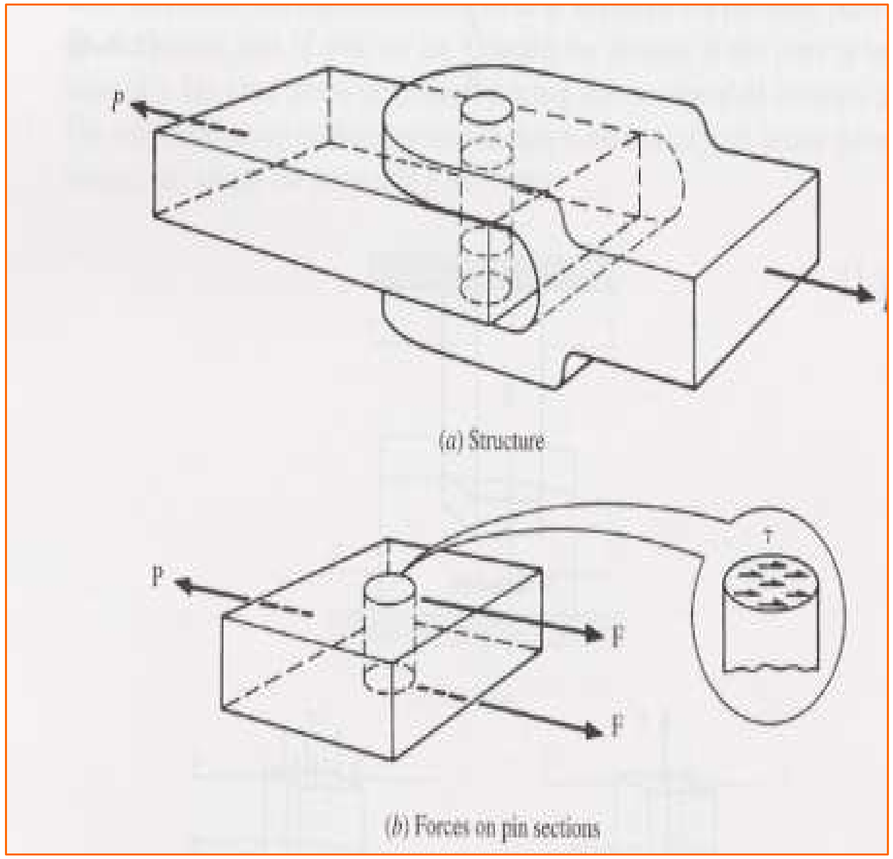
(a) Structure
(b) Force on section c-c
(c) Stress distribution at section c-c

Shear Stress

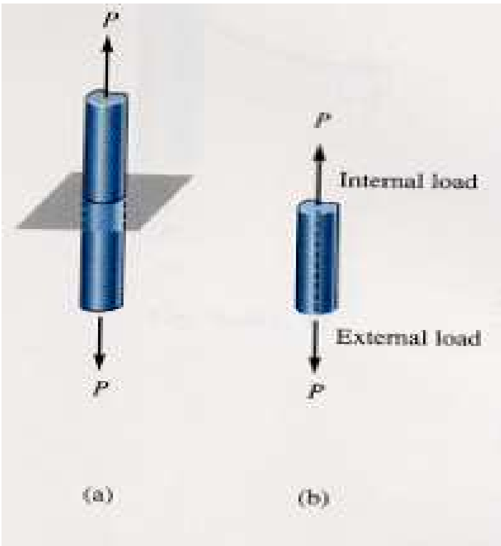
- Stress component parallel to imaginary plane

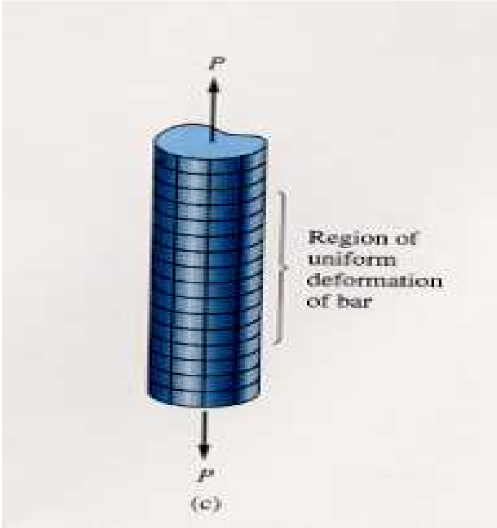
$$\tau = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

- F$\rightarrow$ tangential force



Average Normal Stress in an Axially Laded Member



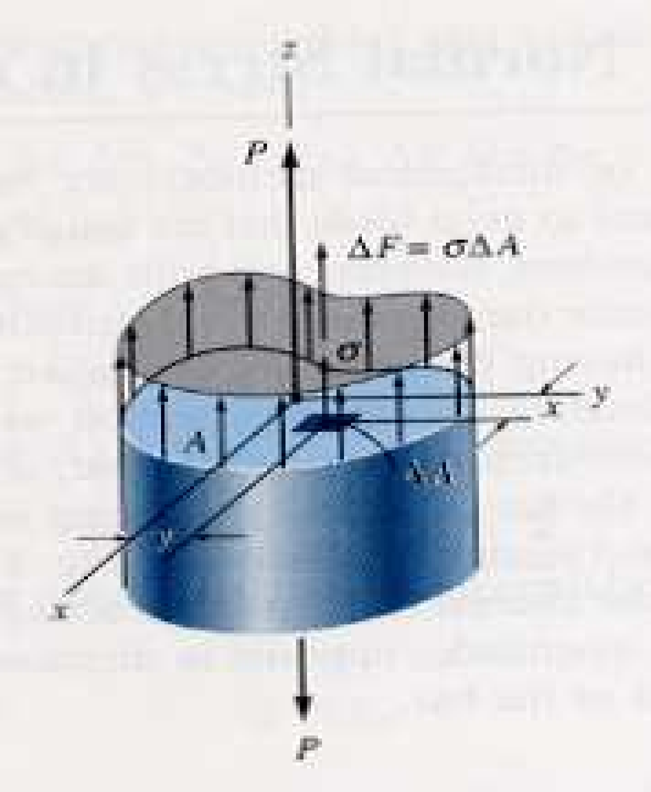


- Normal Force acts on any cutting plane producing a normal stress (either tension or compression) that is uniform over the area of the cutting plane assuming:
  1. Force acts at the centroid of the area
  2. Material deforms uniformly

Axial Loading

$$\sigma = \frac{P}{A}$$

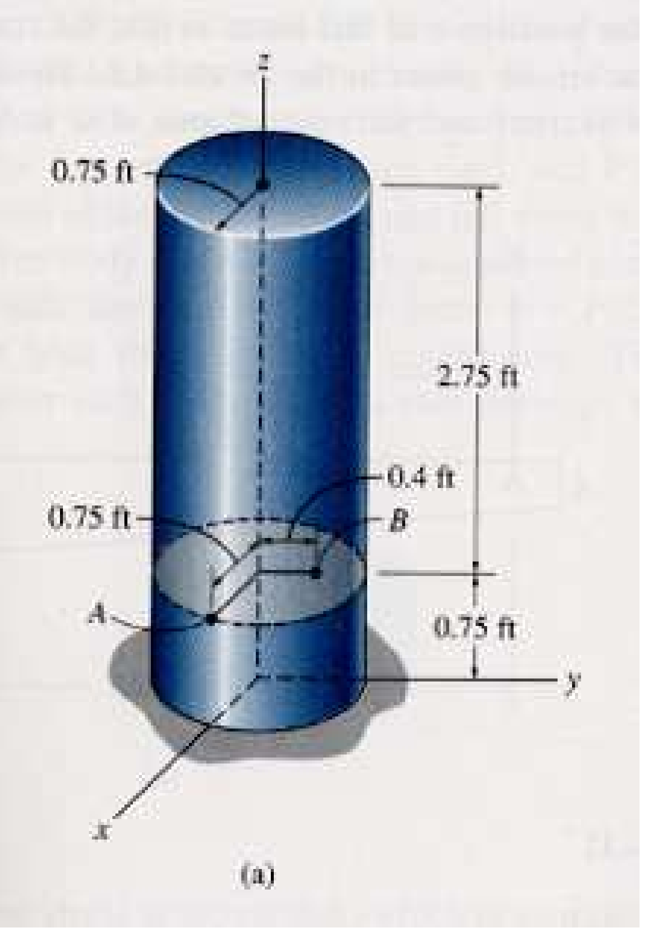
- σ- average normal stress at any point on the cross sectional area
- P- internal resultant normal force, applied through the centroid of the cross sectional area
- A- cross sectional area



(d)

Example: Normal Stress

- The casting shown is made of steel using a specific weight γ_s=490 lb/ft^3.
- Determine the average normal stress acting at points A and B



Solution

$$+\uparrow \sum F_z = 0$$

$$P - W_{st} = 0$$

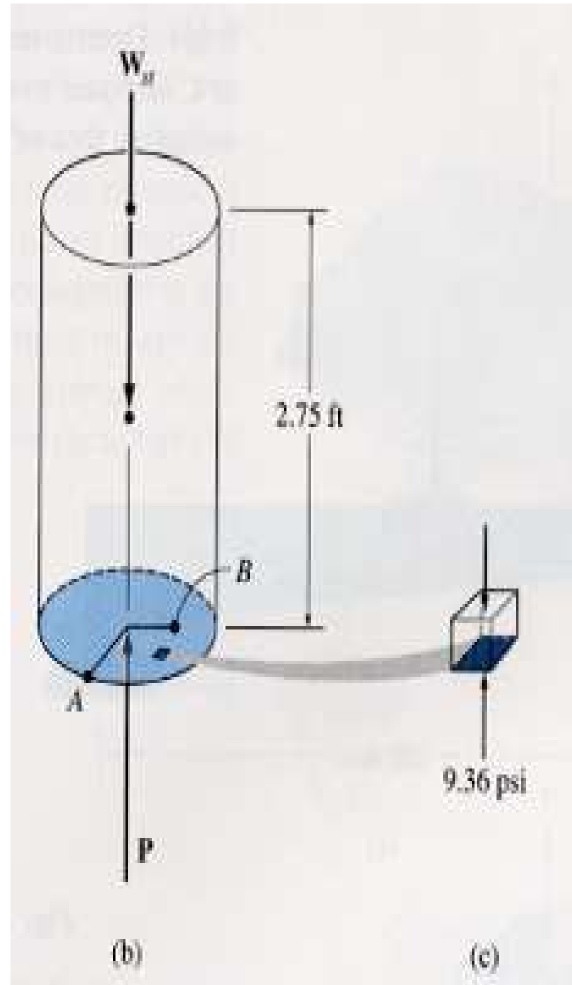
$$P - 490\,\frac{lb}{ft^3}(2.75\,ft)\pi(0.75\,ft)^2 = 0$$

$$P = 2381\text{lb}$$

$$A = \pi(0.75)^2$$

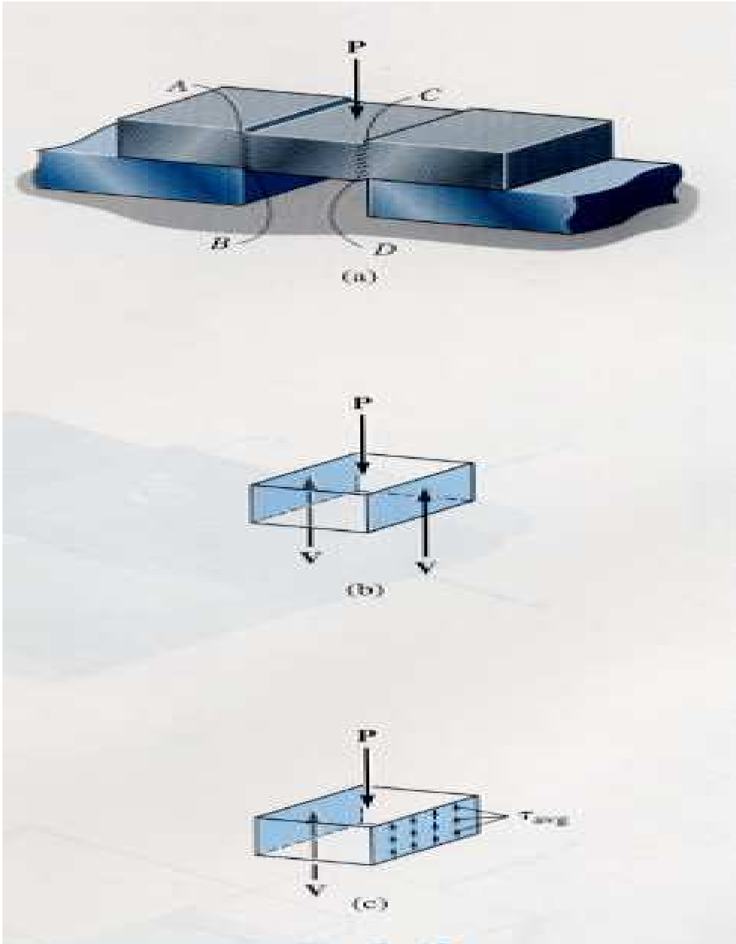
$$\sigma = \frac{P}{A} = \frac{2381}{\pi(0.75)^2} = 1347.5\frac{lb}{ft^2}$$

$$= 1347.5\frac{lb}{ft^2} * \frac{1\,ft^2}{144 in^2} = 9.36\,psi(compressive)$$



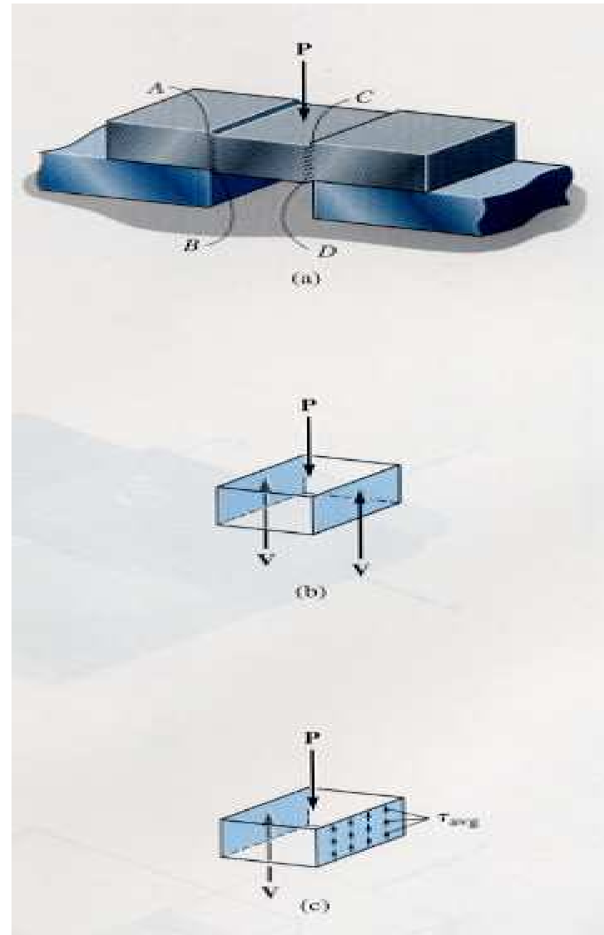
Average Shear Stress

- This situation exists in
  - rivets of riveted joints
  - pins of pin connected truss and other members



Average Shear Stress

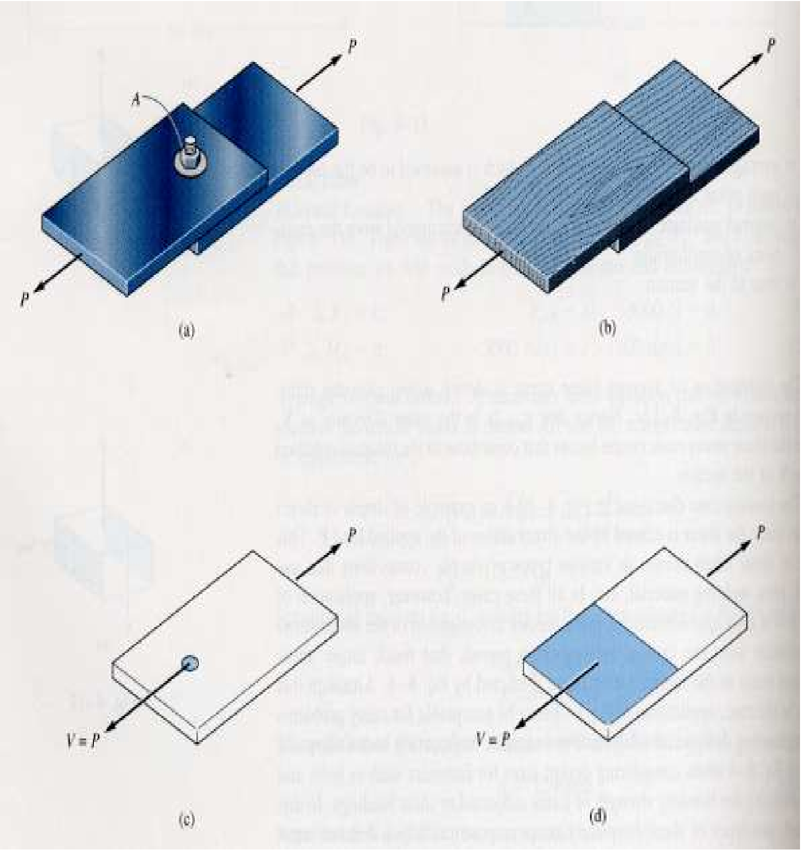
$$\tau_{avg} = \frac{V}{A}$$



- $\tau_{avg.}$=average shear stress at the section
- V= internal resultant shear force at the section
- A= area of the section

Single Shear

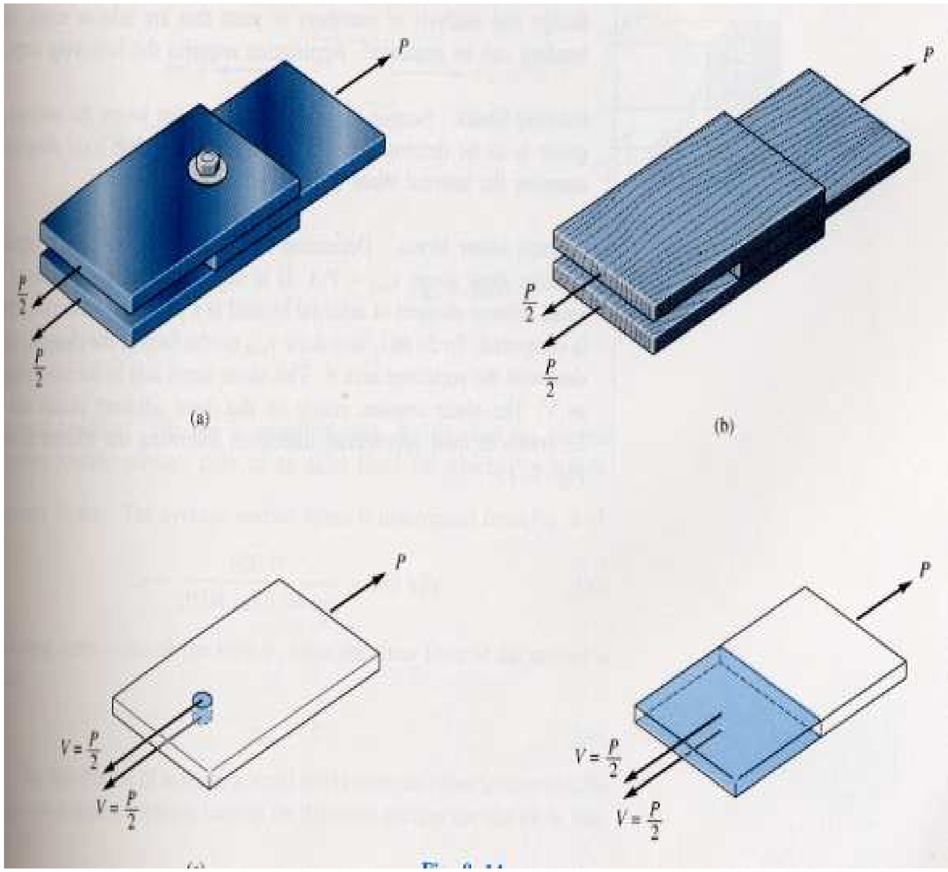
- single shear connections:
  - Lap Joints
- bonding surfaces between the members are subjected to single shear force V=P



Double Shear

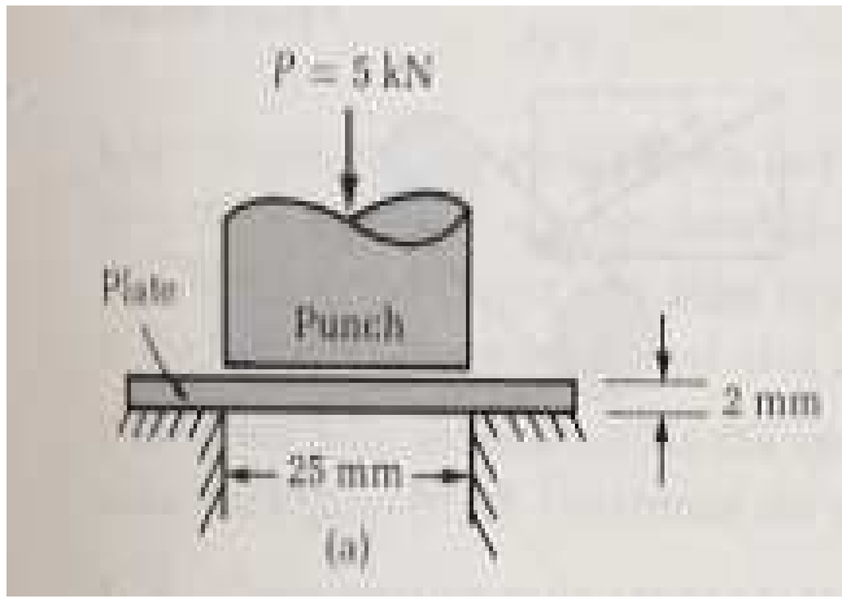
- Double lap joints

- Double shear
- V=p/2



Example

- A 25mm diameter hole is to be punched in an aluminum plate 0.2mm thick by a punching machine.
- Calculate the stress in the plate when the punching force is 5kN.



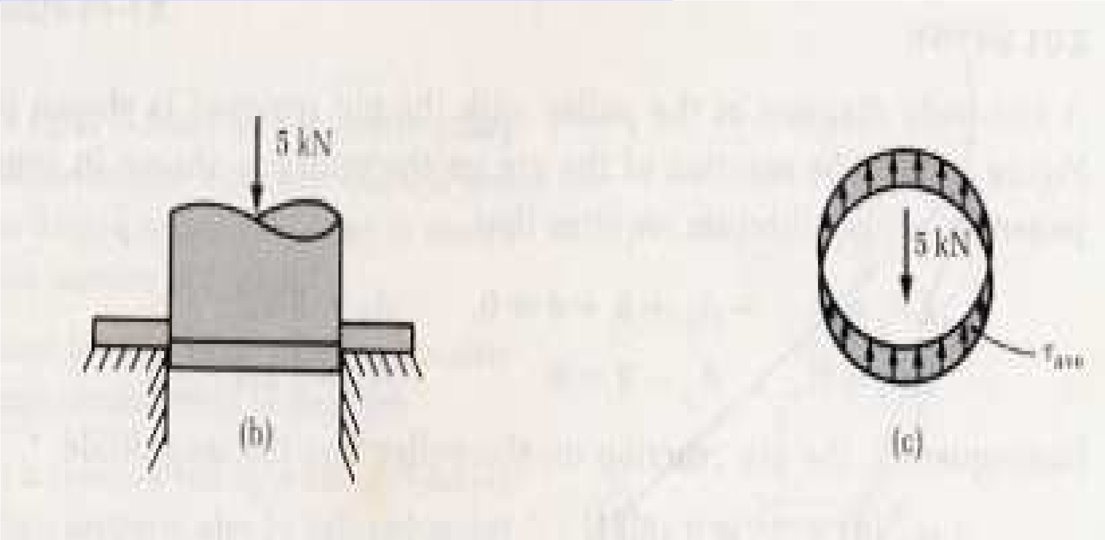
Solution

$$A = \pi * d * t$$

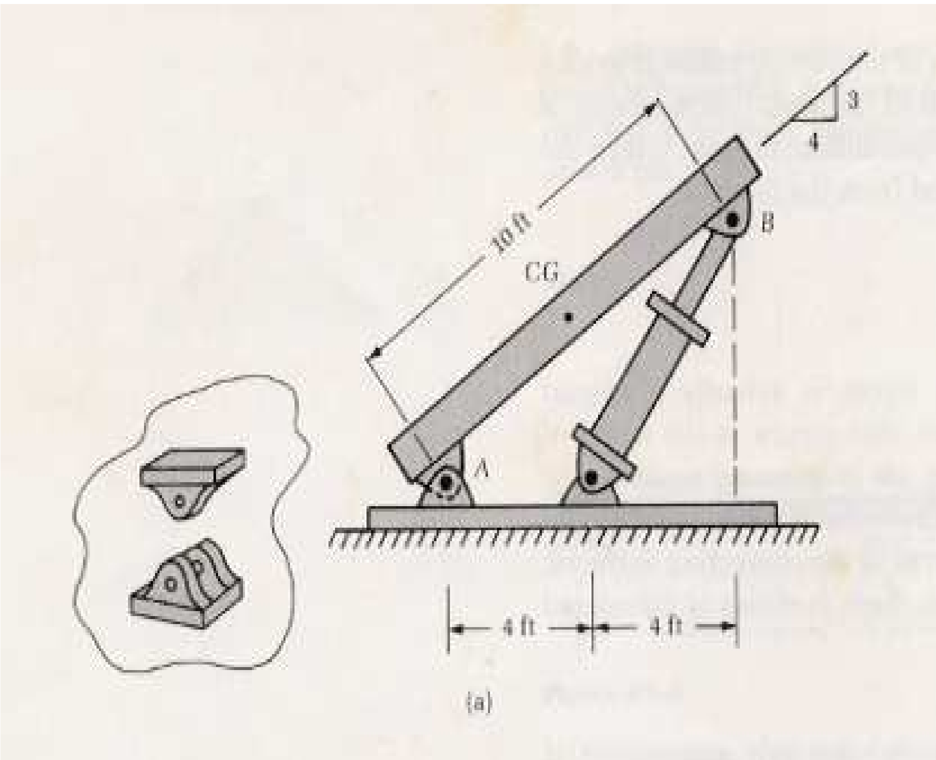
$$= \pi (0.025)(0.002) = 1.57 \times 10^{-4} m^2$$

$$\tau_{av} = \frac{5000}{1.57 \times 10^{-4}} = 31.9 \times 10^{6} \frac{N}{m^2}$$

$$= 31.9 MPa$$



Example



- Calculate the average normal stress in a 0.75in diameter plunger of the hydraulic cylinder
- The mechanism that the hydraulic cylinder supports weighs 6000lb.
- Calculate the average shearing stress in the 0.5in diameter pin at A

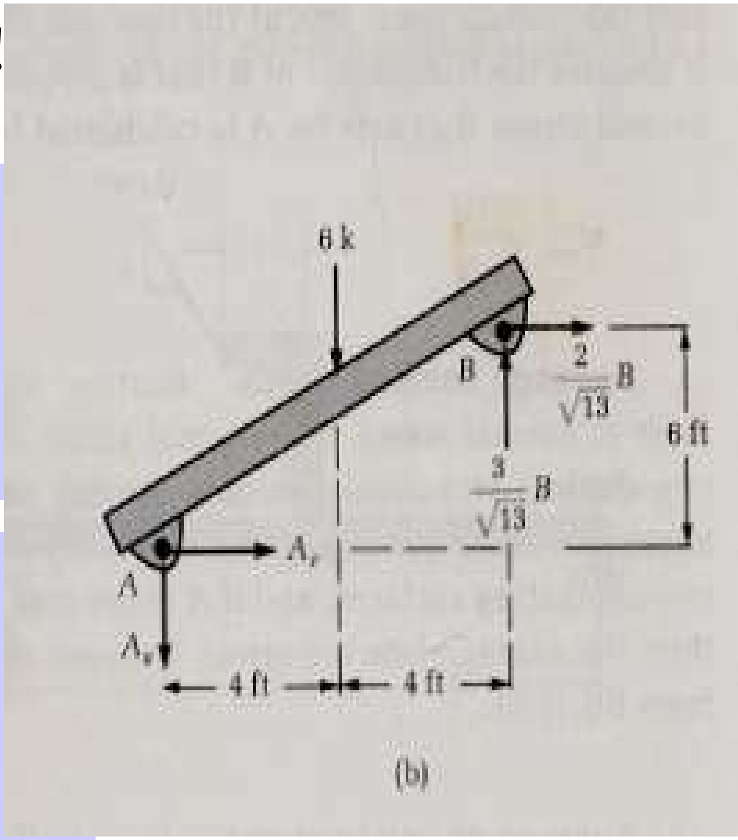
Solution

- First step: FREE BODY DIAGRAM!
- From FBD

$$\sum M_A = 0$$

$$8\left(\frac{3}{\sqrt{13}}B\right) - 6\left(\frac{2}{\sqrt{13}}B\right) - 4(6000) = 0$$

$$B = 2000\sqrt{13}\ lb$$



$$\xrightarrow{+} \sum F_x = 0$$

$$A_x + \frac{2}{\sqrt{13}}B = 0$$

$$A_x = -4000\ lb$$

$$+\uparrow \sum F_y = 0$$

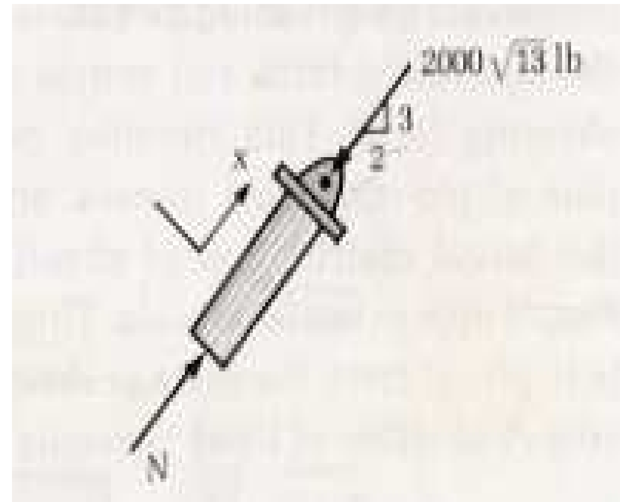
$$A_y - 6000 + \frac{3}{\sqrt{13}}B$$

$$A_y = 0$$

•Equilibrium of the forces parallel to axes of the plunger

$$N - 2000\sqrt{13} = 0$$

$$N = 2000\sqrt{13}lb$$



•Average normal stress in the plunger

$$\sigma_{av} = \frac{2000\sqrt{13}}{\pi\left(0.75/2\right)^2} = 16.135\,psi$$

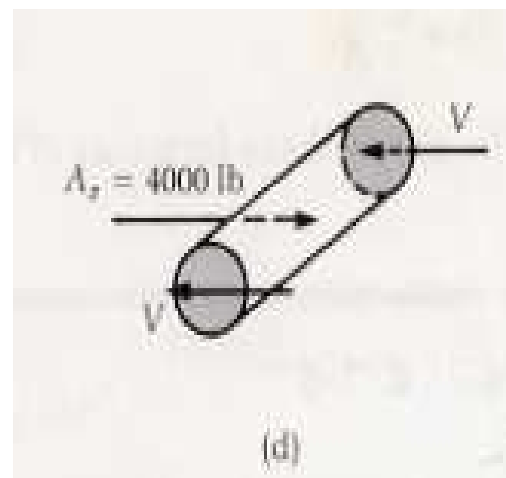
•Free Body Diagram of the pin

$$2V - A_x = 0$$

$$2V - 4000 = 0$$

$$V = 2000lb$$

$$\tau_{av} = \frac{2000}{\pi\left(0.5/2\right)^2} = 10{,}200\,psi$$



Allowable Stress

- Factor of safety

  ratio of a maximum load that can be carried by the member until it fails divided by an allowable load

- $$\text{F.S.} = \frac{P_{fail}}{P_{allow}}$$

  $$= \frac{\sigma_{fail}}{\sigma_{allow}} \rightarrow$$ Failure in the normal stress

  $$= \frac{\tau_{fail}}{\tau_{allow}} \rightarrow$$ Failure in shear stress

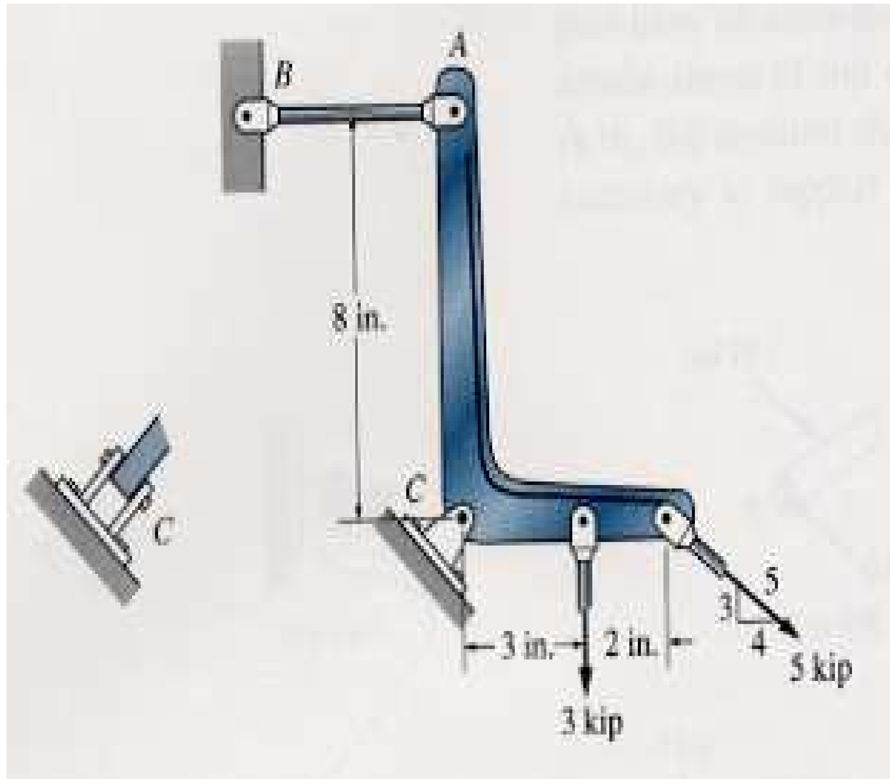
- In Geometry

  $$A = \frac{P}{\sigma_{allow}} \rightarrow$$ Subjected to normal stress

  $$A = \frac{V}{\tau_{allow}} \rightarrow$$ Subjected to shear force

Example

- The control arm is subjected to the loading shown in the figure. . Determine to the nearest ¼ in. the required diameter of the steel pin at C if the allowable shear stress for the steel is τ_{allow}=8 ksi. Note that the pin is subjected to double shear.



Solution

$$\sum M_c = 0$$

$$F_{AB}(8) - 3(3) - 5\left(\frac{3}{5}\right)5 = 0$$

$$F_{AB} = 3kip$$

$$\xrightarrow{+} \sum F_x = 0$$

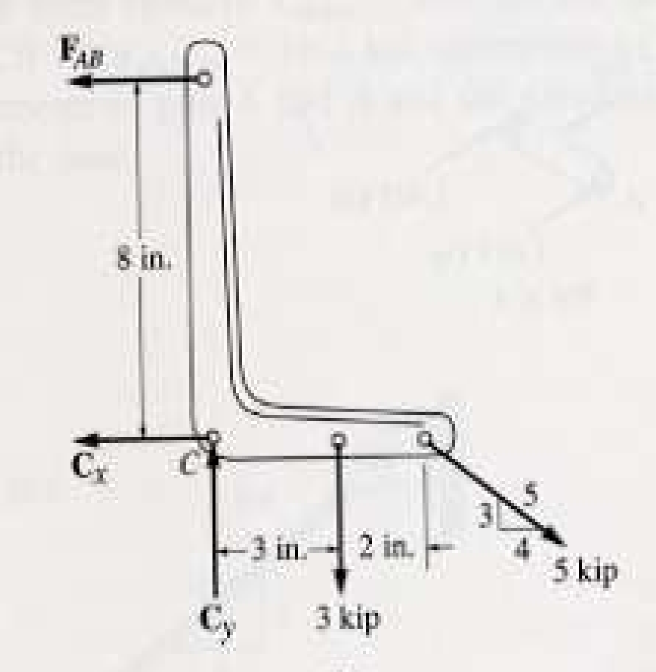
$$-3 - C_x + 5\left(\frac{4}{5}\right) = 0$$

$$C_x = 1kip$$

$$+\uparrow \sum F_y = 0$$

$$C_y - 3 - 5\left(\frac{3}{5}\right) = 0$$

$$C_y = 6kip$$



(b)

- Resultant force C

$$F_c = \sqrt{(1)^2 + (6)^2} = 6.082kip$$

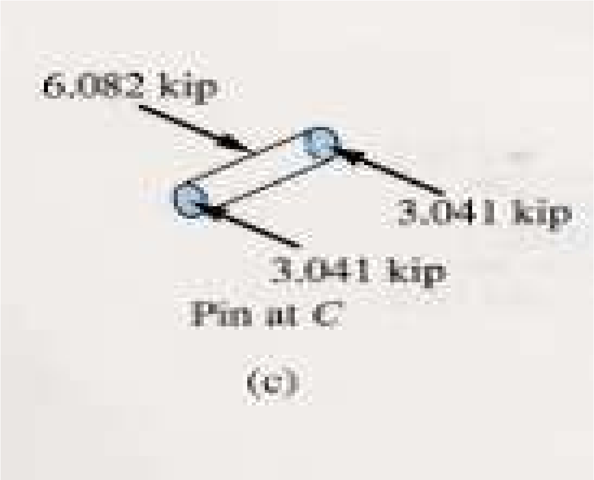
•Since the pin is subjected to double shear

$$2V - F_c = 0$$

$$2V - 6.082 = 0$$

$$V = 3.041\,kip$$

$$A = \frac{V}{\tau_{all}} = \frac{3.041\,kip}{8\,kip/in^2} = 0.3802 in^2$$



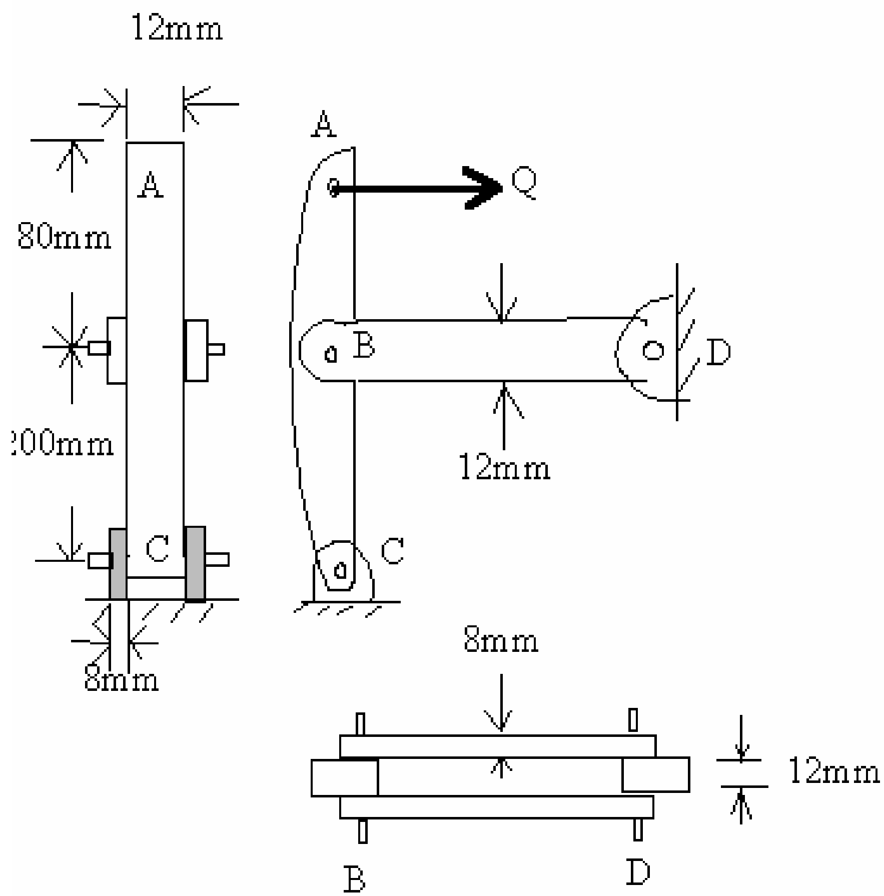
$$\pi\left(\frac{d}{2}\right)^2 = 0.3802 in^2$$

$$d = 0.696 in$$

•Use a pin having a diameter of d=0.75 in

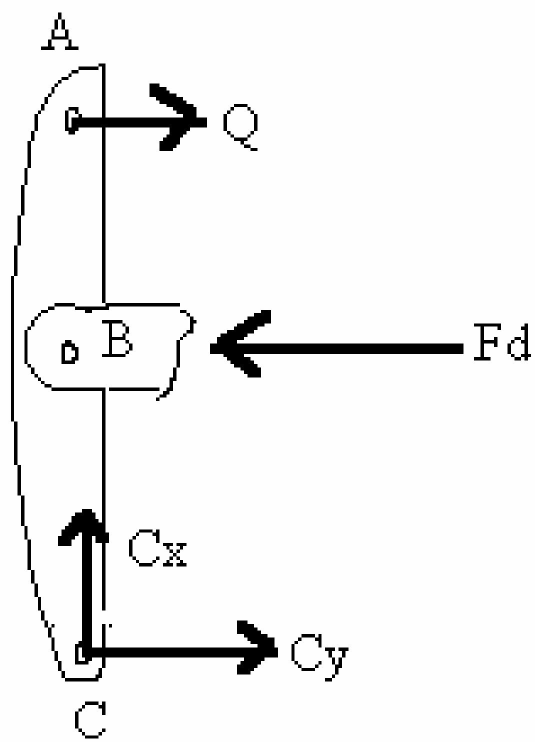
Practice !!

- An 8-mm-diameter pin is used at C, while pins of 12-mm diameter are used at both B and D.
- The ultimate shearing stress is 100MPa at all connections and the ultimate normal stress in the links BD is 250MPa.
- Determine the load Q for which the factor of safety is 3.0.
- Must check pins B,C, D, and link BD!



Step by Step Solution

- Note that member BD is a single-force-member. C is a pin joint, so it has two reaction forces.
- C_x and C_y in the free-body-diagram are reversed. C_x should be horizontal and C_y should be vertical.
- Fd= F_{BD}



Cont.

The safety factor is first considered: Allowable shear stress = (ultimate shear stress)/F.S.=100MPa/3.0=33.3MPa.

Allowable normal stress=UTS/F.S.=250MPa/3.0=83.3MPa.

We look at the pin connectors at B and D, and the link BD, first. Then we look at pin C.

The pins for link BD is in double shear, thus allowable shear stress= 33.3MPa=F_{BD}/(2A), where F_{BD} is the allowable force in the link. A is the cross-section area of the pins $(\pi/4)*(0.012m)^2$.

This gives $F_{BD}=(33.3MPa)(2)\ (\pi/4)*(0.012m)^2 = 7.54$ kN

Cont.

The double link BD is in normal stress, allowable normal stress=83.3MPa=F_{BD}/(2A) where A is the cross-section area of the link BD = (0.008m)x(0.02m). So the allowable force in BD (F_{BD}) = 26.7kN.

Comparing Fd from 3 and 4, we should take the **smaller** of the two forces, so the allowable force F_{BD} = 7.54kN.

Using the free-body diagram, summing moment about C,

(Q * 0.38m-F_{BD} * 0.2m= 0), and using F_{BD}=7.54kN, we find the allowable force Q=3.97kN, based upon the limits on B,D, and BD.

Cont.

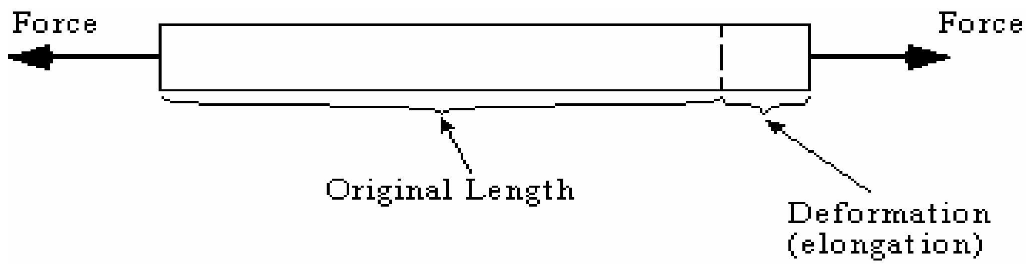
Now we can look at pin C. Pin C is in double shear, so the allowable shear stress 33.3MPa=C/(2A) where A is $(\pi/4)(0.008\text{m})^2$; so maximum allowable C=3.35kN. (note that C_y=0, so C= C_x).

Sum moments about B:

$(Q * 0.18\text{m} - C_x * 0.2\text{m} = 0)$,

using C_x=3.35kN, we find Q=3.72kN which is smaller than Q (3.97kN) from considering link BD and pins B,D. So the answer is Q=3.72kN.

Deformation



- Whenever a force is applied to a body, its shape and size will change. These changed are referred as deformations.

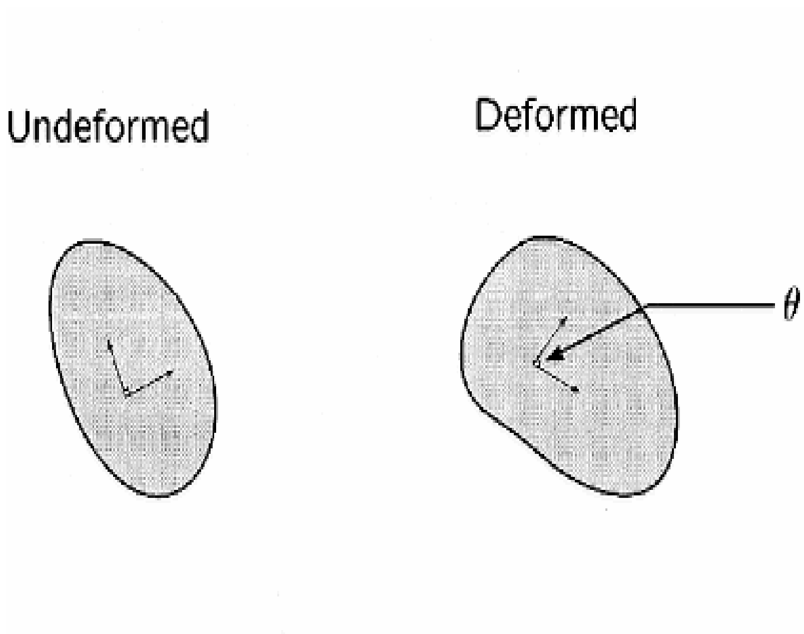
- These deformations can be thought of being either positive (elongation) or negative (contraction) in sign.

- It is however hard to make a relative comparison between bodies of different size and length as their individual deformations will be different. This requires the development of the concept of STRAIN, which relates the bodies deformation to its initial length.

Strain

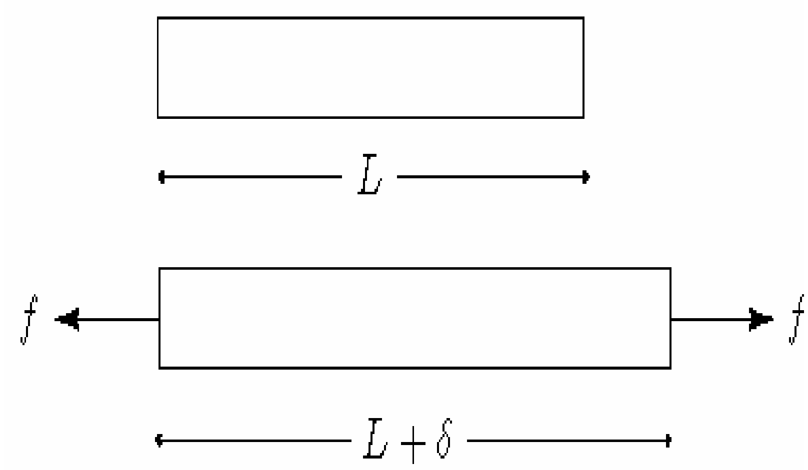
- The elongation (+ve) or contraction (-ve) of a structure or body per unit length is termed Strain.

- It can thus be equated as the change in length of the body over its original length, and is given the symbol ε (epsilon)



Normal Strain-ε

- Normal Strain = elongation per unit length.
- ε>o tensile strain(+)
- ε<0 compression strain (-)
- **Normal strain causes a change in volume**
- **Strain is dimensionless !**



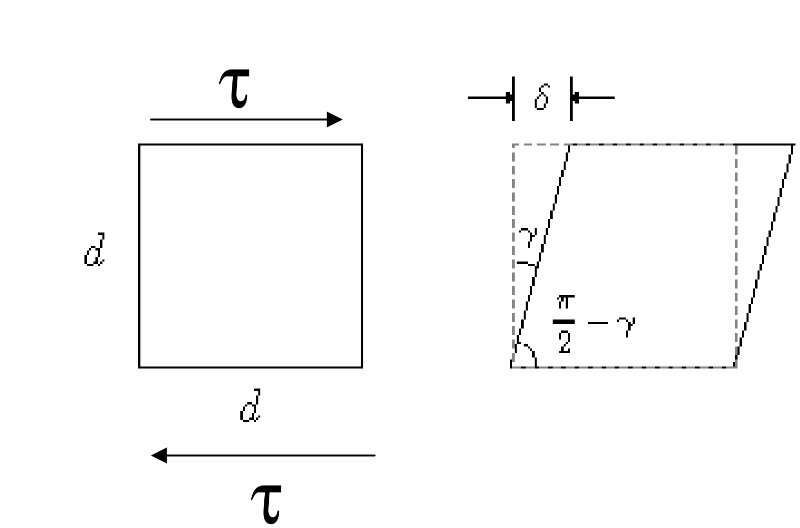
Engineering definition of "normal strain:"

$$\varepsilon = \frac{L_{\text{final}} - L_{\text{initial}}}{L_{\text{initial}}} = \frac{\text{change in length}}{\text{initial length}}$$

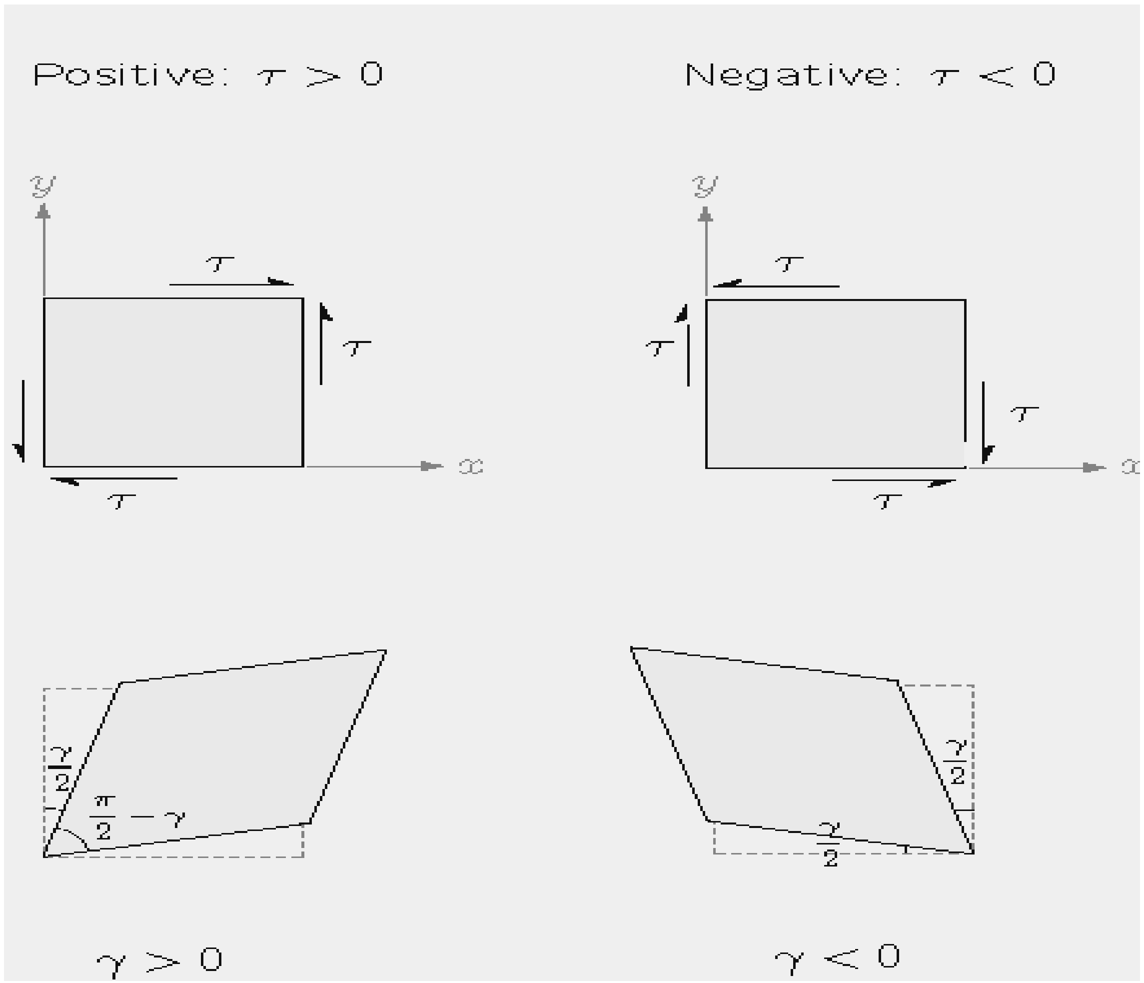
$$\varepsilon = \frac{(L+\delta) - L}{L} = \frac{\delta}{L}$$

Shear Strain- γ

- Shear stress will result in a shear strain
- Shear strain: change in angle between two segments that were perpendicular to one another
- Shear strain causes change in shape.
- $\tan \gamma = (\delta/d)$
  - For sufficiently small γ, $\tan \gamma \sim \gamma$
  - $\gamma = (\delta/d)$

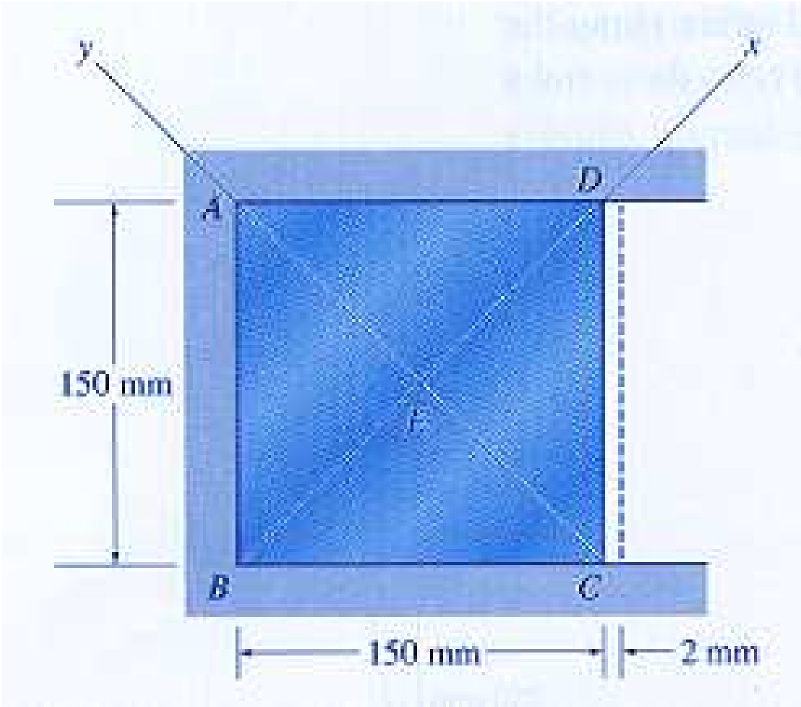


Sign Convention for Shear Strain



Example

- The plate shown in the figure is held in the rigid horizontal guides at its top and bottom AD and BC. If its right side CD is given a uniform horizontal displacement of 2 mm, determine the average normal strain along the diagonal AC, and the shear strain at E relative to the x-y axes.



Solution

After deformation AC becomes AC'

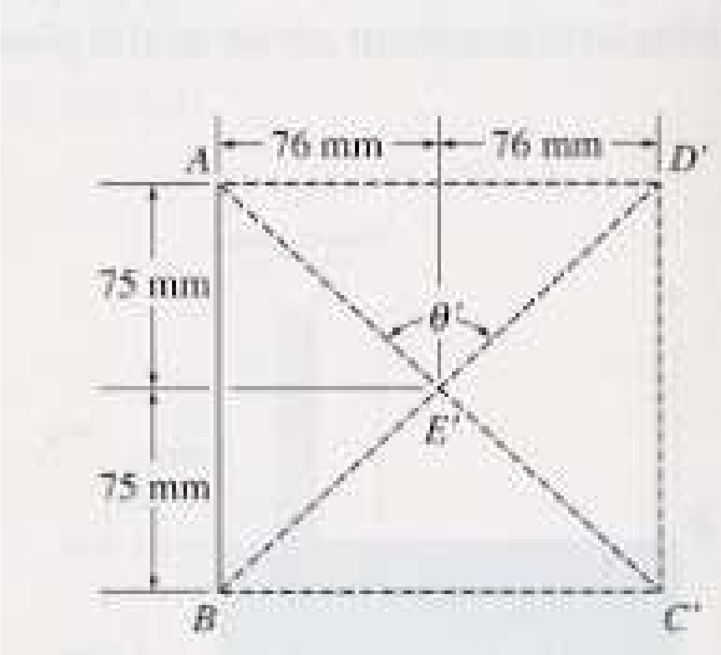
$$\mathrm{AC} = \sqrt{(0.15)^2 + (0.15)^2} = 0.21213mm$$

$$AC' = \sqrt{(0.15)^2 + (0.152)^2} = 0.21355mm$$

Average normal strain along the diagonal

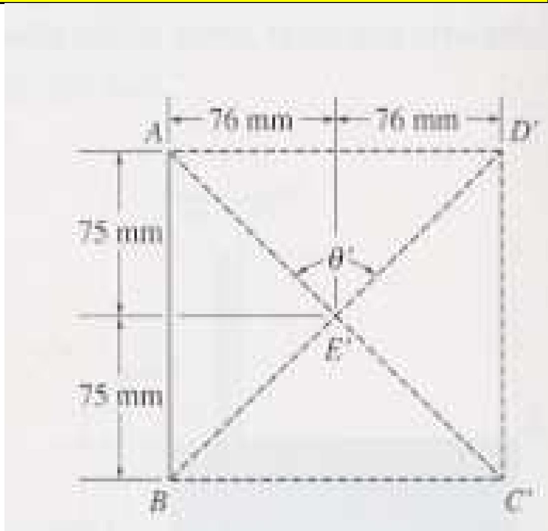
$$\varepsilon_{\mathrm{AC}} = \frac{AC' - AC}{AC} = \frac{0.21355mm - 0.21213mm}{0.21213mm}$$

$$\varepsilon_{\mathrm{AC}} = 0.00669 = 0.669\%$$



Solution Cont.

- To find the shear strain at E relative to the x and y axis. Need to find θ’ that specifies the angle between the two axes after deformation



$$\tan\left(\frac{\theta'}{2}\right) = \frac{76\ mm}{75\ mm} \Rightarrow \theta' = 90.759^0$$

$$\theta' = 90.759^0 = \frac{\pi}{180^0} 90.759^0 = 1.58404\ \ rad$$

$$\gamma_{xy} = \frac{\pi}{2} - \theta' = \frac{\pi}{2} - 1.58404\ \ rad\ \ = -0.0132\ \ rad$$