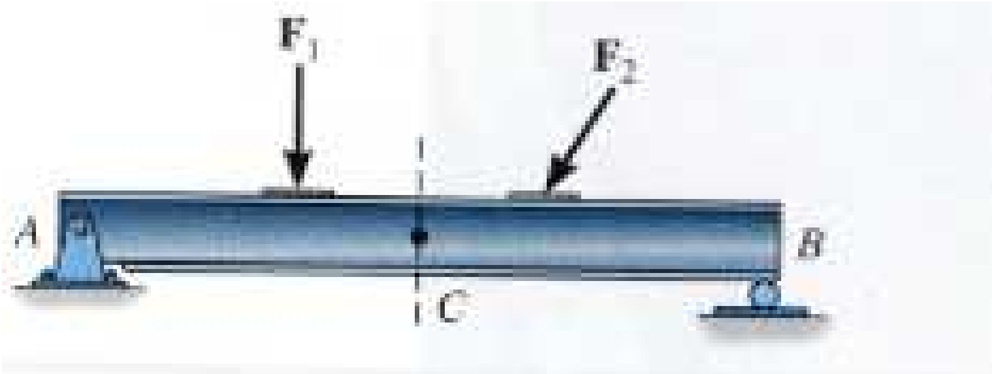
Mechanics & Materials 1

Chapter 7
Internal Loadings

FAMU-FSU College of Engineering
Department of Mechanical Engineering

Internal Loading

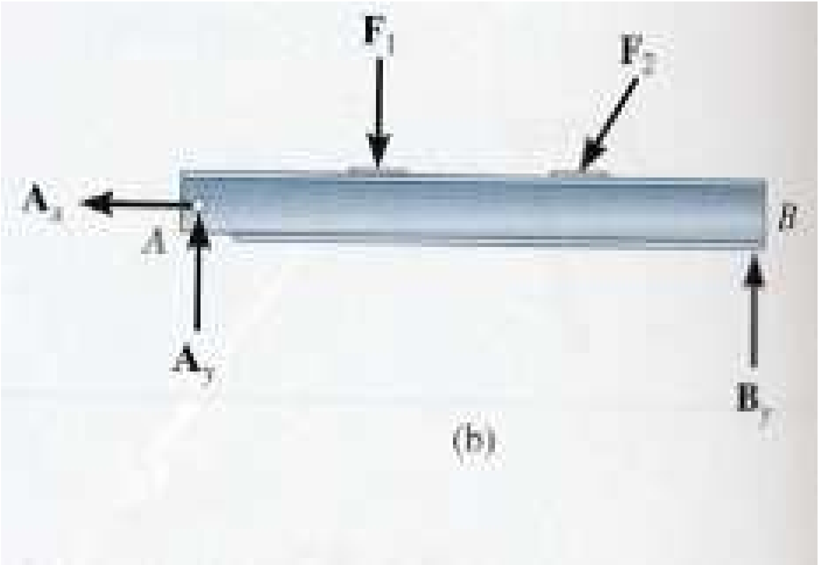
- Using the method of sections we can investigate the loading acting within a member



(a)

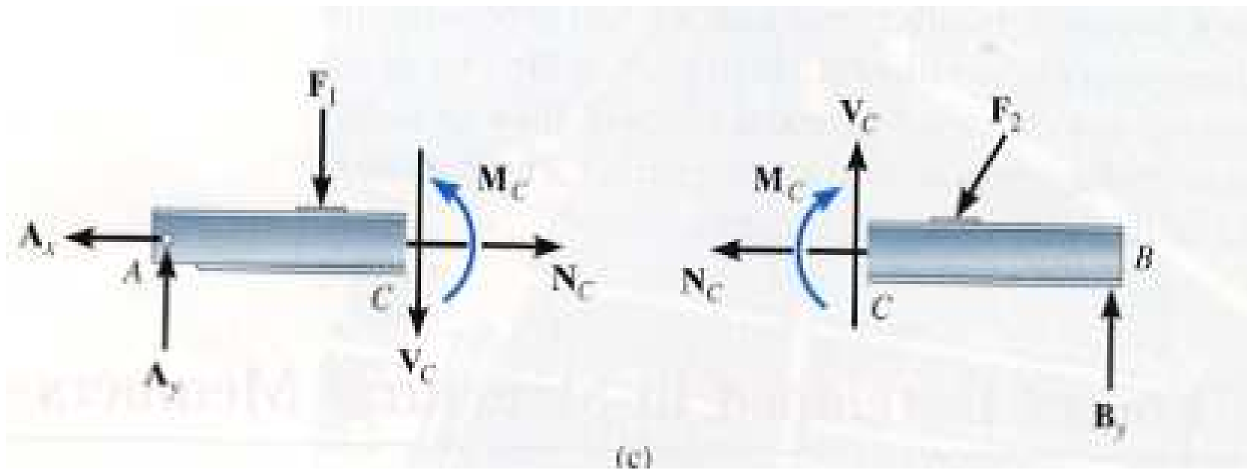
Internal Loading

- Using a free body diagram of the … beam we obtain the reactions A_x, A_y, B_x, B_y.



Internal Loading

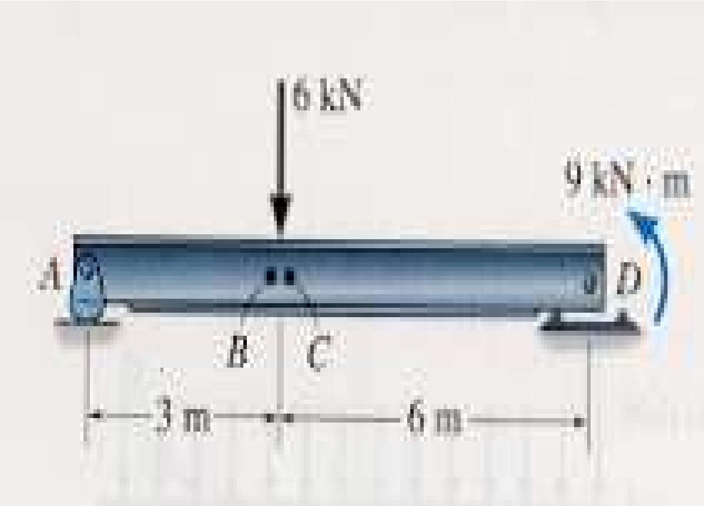
- Consider Section AC only, equilibrium of the sections requires adding three components
  - Force N along the axis of the beam is called axial loading
  - Force V perpendicular to the beam axis is called shear force
  - Moment M eliminates the moments of the forces N and V



Internal Force: Analysis Procedure

Example:

Determine the internal normal force, shear force, and the loading moment acting just to the left at point B, and just to the right at point C of the 6 kN force



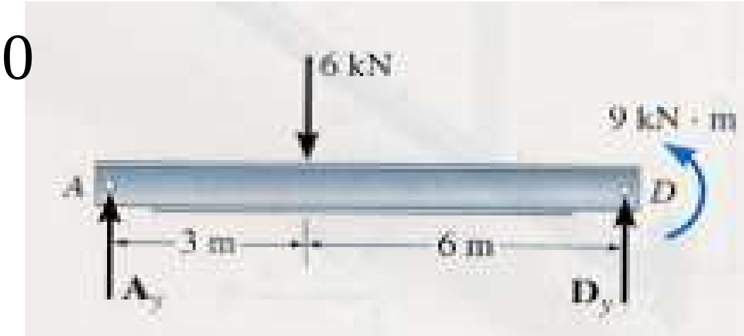
(a)

Internal Force Analysis
Step 1

- Draw a free body diagram of the entire frame and use it to determine as many of the reaction forces at the supports as you can.

$$\sum M_D = 0 \Rightarrow 9KN.m + (6KN)(6m) - A_y(9m) = 0$$

$$A_y = 5KN$$



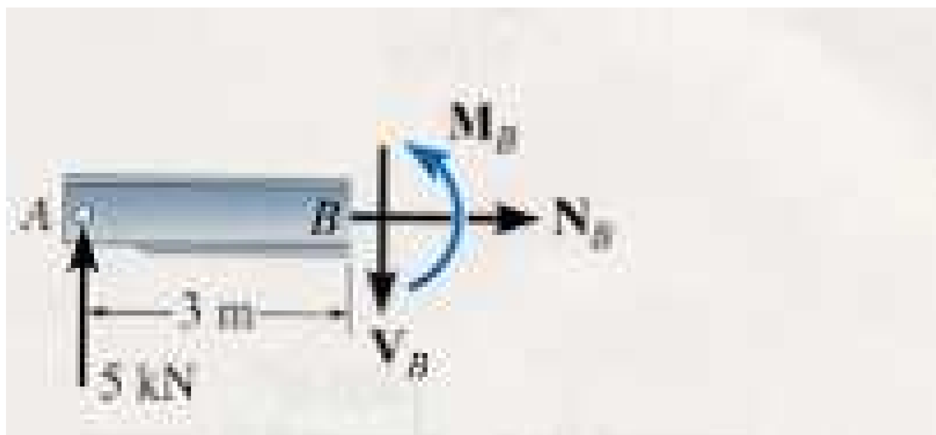
(b)

Internal Force Analysis
Step 2

- Dismember the frame and draw a free body diagram of each of its members:
  - write as many equilibrium equations as are necessary to find all the forces acting on the member on which the point of interest is located.

Internal Force Analysis
Step 3

- Cut the member at a point of interest and draw a free body diagram of each of the two portions.



(c)

Internal Loading Analysis
Step 4

- Select one of the two free body diagrams you have drawn and use it to write three equilibrium equations for the corresponding point of the body
  - Summing Moments about the point of interest

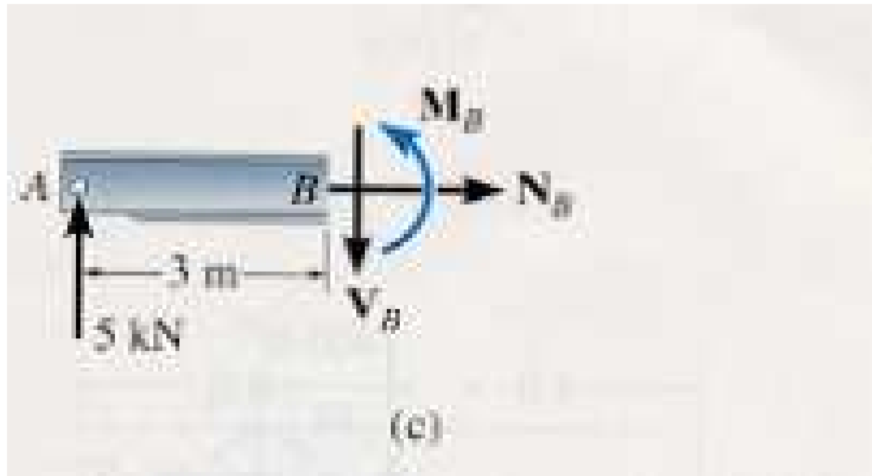
    $\sum M_p = 0 \rightarrow$ gives bending moment at point P
  - Summing Forces along the axis of the member

    $\sum F_x = 0 \rightarrow$ will produce axial force in the member

    $\sum F_y = 0 \rightarrow$ shearing force

Internal Loading Analysis
Step 4

Segment AB



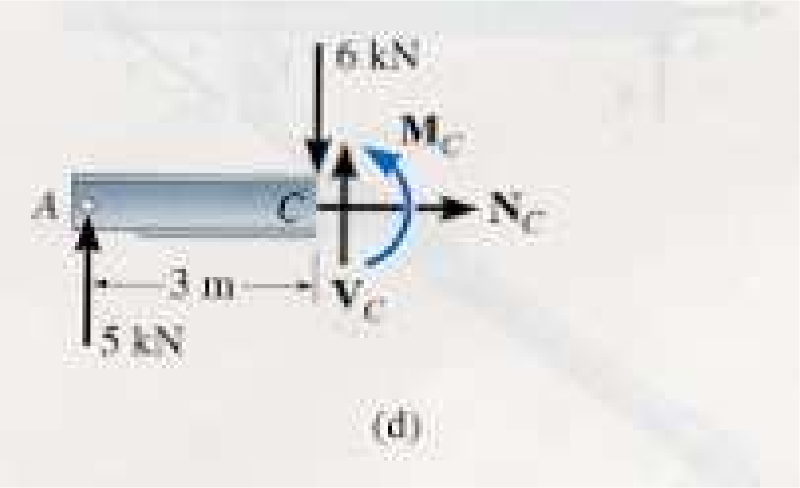
$$\xrightarrow{+} \sum F_x = 0 \qquad N_B = 0$$

$$+\uparrow \sum F_y = 0 \qquad 5kN - V_B = 0 \rightarrow V_B = 5kN$$

$$+\sum M_B = 0 \qquad (-5kN)(3m) + M_B = 0 \rightarrow M_B = 15kN \cdot m$$

Internal Loading Analysis
Step 4

Segment AC



(d)

$$\xrightarrow{+} \sum F_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$+\sum M_C = 0$$

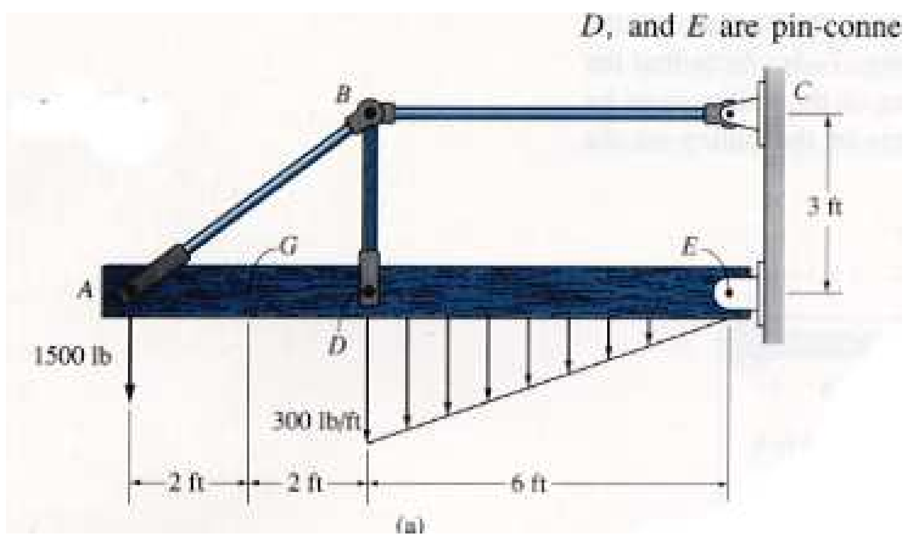
$$N_C = 0$$

$$5kN - 6kN + V_C = 0 \rightarrow V_C = 1kN$$

$$(-5kN)(3m) + M_C = 0 \rightarrow M_C = 15kN \cdot m$$

Example

- Determine the resultant internal loading acting on the cross section at G of the wooden beam AE. Assuming the joints at A, B, C, D, and E are pin-connections



Solution

Step 1: Support Reaction

$$+\uparrow \sum F_y = 0$$

$$-1500 + \frac{1}{2}(6\,ft)(300\,lb/\,ft)$$

$$+E_y = 0$$

$$E_y = 2400 \uparrow$$

$$+\sum M_E = 0$$

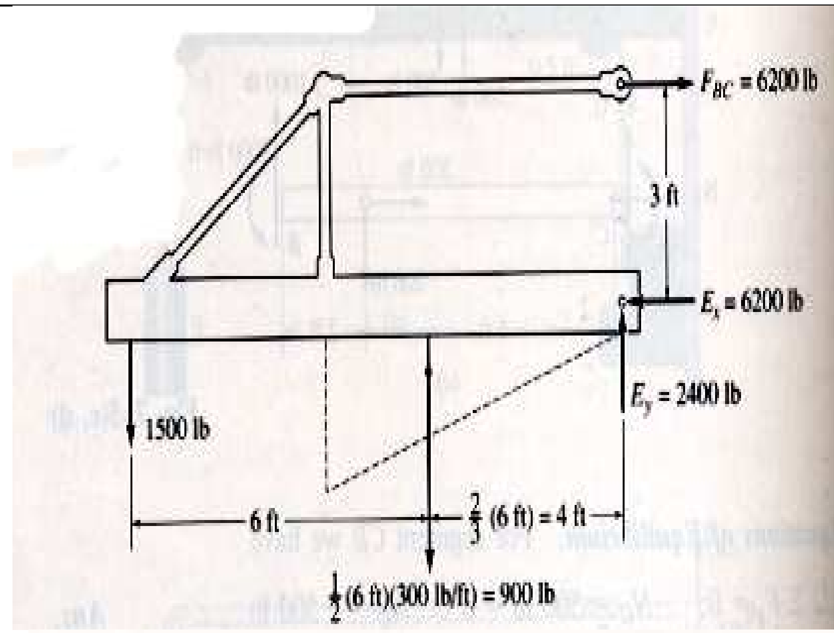
$$1500(10) + 900(4) - F_{C_x}(3) = 0$$

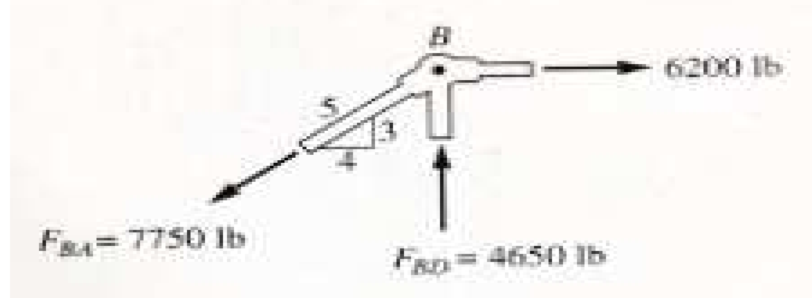
$$F_{C_x} = 6200 lb$$

$$\xrightarrow{+} \sum F_x = 0$$

$$-E_x + 6200 = 0$$

$$E_x = 6200$$

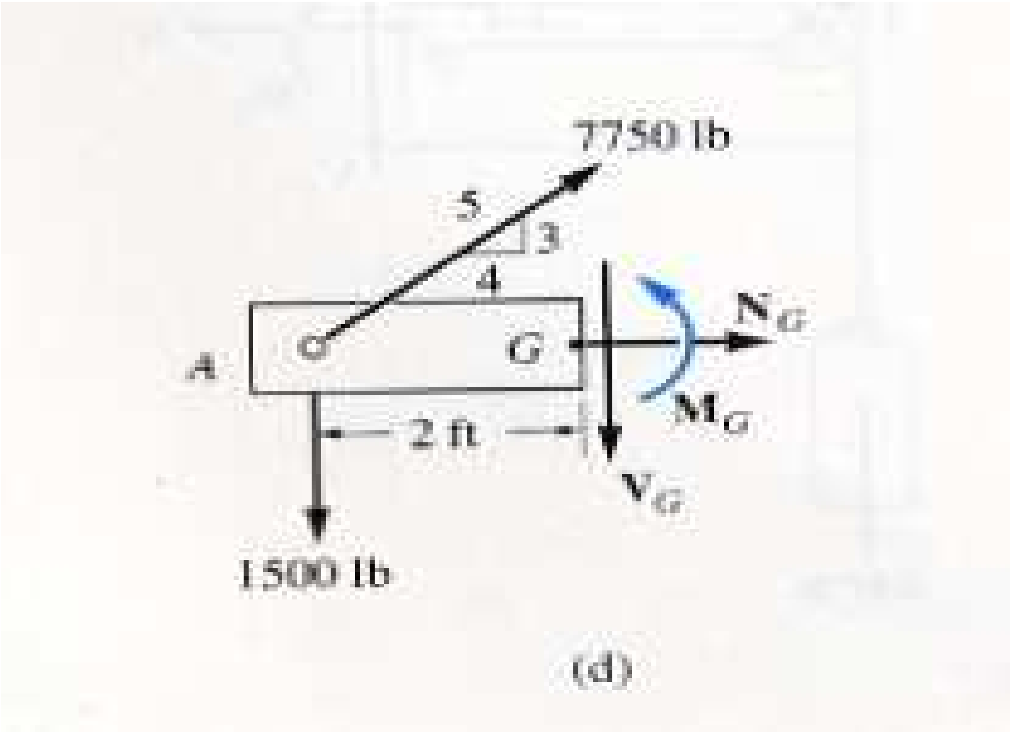




(c)

Solution

Step 2 : Free Body Diagram at point G



Solution

Step 3 : Equations of Equilibrium for segment AG

$$\xrightarrow{+} \sum F_x = 0$$

$$7750\left(\frac{4}{5}\right) + N_G = 0$$

$$N_G = -6200 lb$$

$$+\uparrow \sum F_y = 0$$

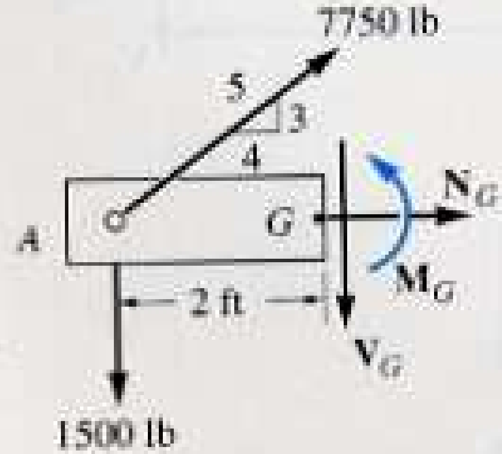
$$-1500 + 7750\left(\frac{3}{5}\right) - V_G = 0$$

$$V_G = 3150 lb$$

$$+\sum M_G = 0$$

$$M_G - 7750\left(\frac{3}{5}\right)(2) + 1500(2) = 0$$

$$M_G = 6300 lb \cdot ft$$



(d)

Question ???

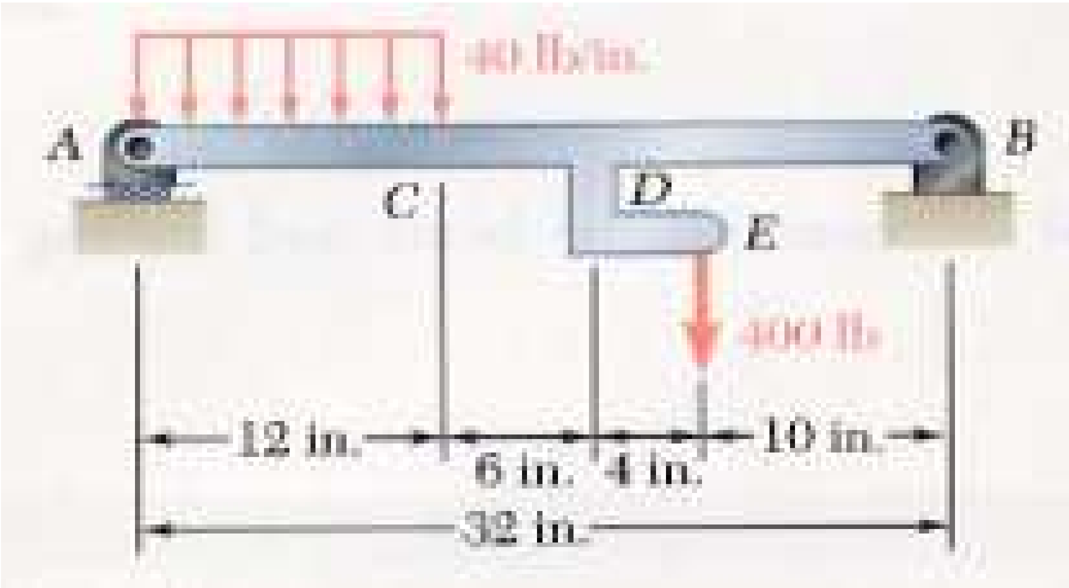
- Will we get the same results by considering segment GE rather than AG?

Determining V and M in a Beam: General Procedures

- 1. Draw a FBD of the entire beam, find reactions and supports
- 2. Cut the beam at the point of interest
- 3. Draw FBD of the section of interest including :
  - the external loads and reactions
  - the shearing force and bending couple representing the internal forces
- 4.Write the equilibrium equations for the portion of the beam you selected
- 5. Record the values and signs of V and M obtained.

Determining the Shear and Bending Moment in a Beam

- Draw the shear and bending moment diagrams for the beam shown below



Step 1

- Draw the free body diagram of the entire beam, to determine the reactions at the beam supports

$$+\sum M_A = 0$$

$$B_y(32) - 480(6) - 700(22) = 0$$

$$B_y = 365lb \uparrow$$

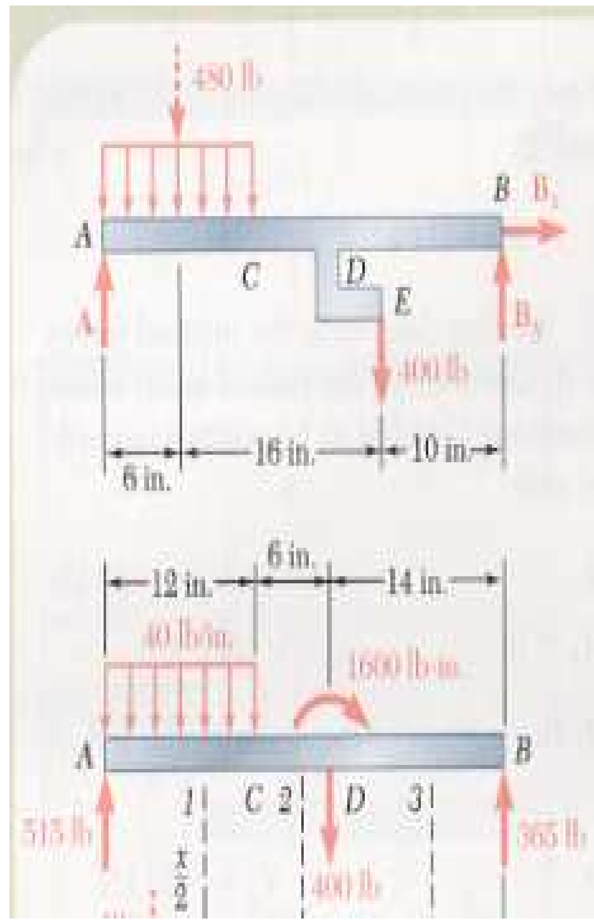
$$+\sum M_B = 0$$

$$480(26) + 400(10) - A(32) = 0$$

$$A = 515lb \uparrow$$

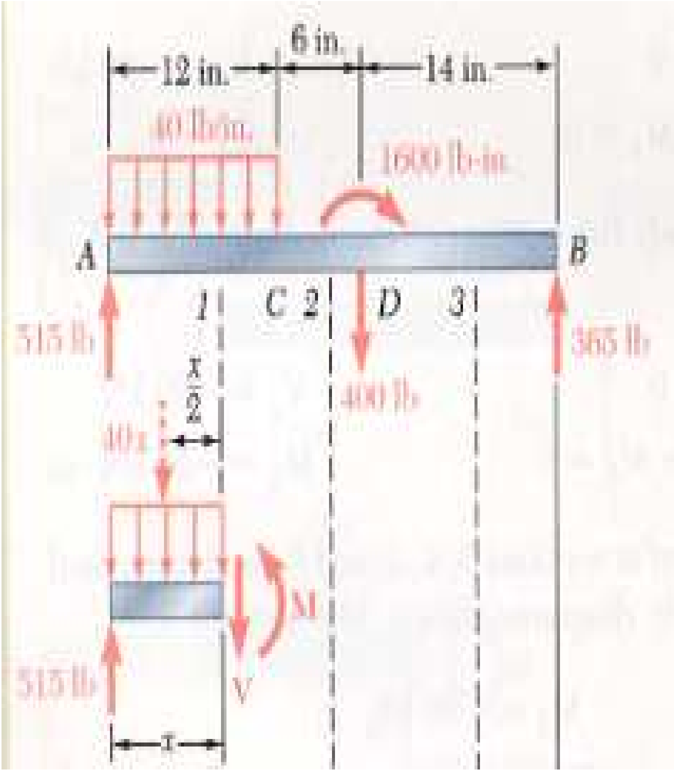
$$\xrightarrow{+} \sum F_x = 0$$

$$B_x = 0$$



Step 2

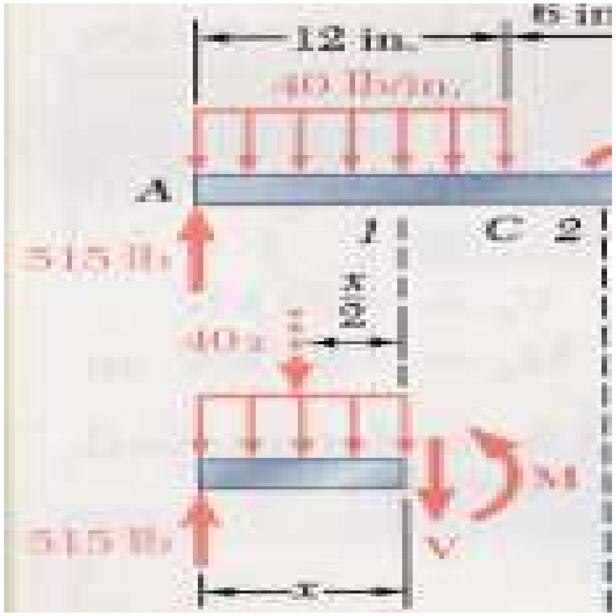
- Cut the beam at the point C (point of interest



Step 3

- Draw free body diagram of the portion of the beam you have selected showing:
  - The loads and reactions exerted on that portion of the beam, replace distributed loads by equivalent concentrated loads
  - The shearing force and the bending moment representing the internal force at the point of interest

Shear and Bending Moment from A to C



$$+\uparrow \sum F_y = 0$$

$$515 - 40(X) - V = 0$$

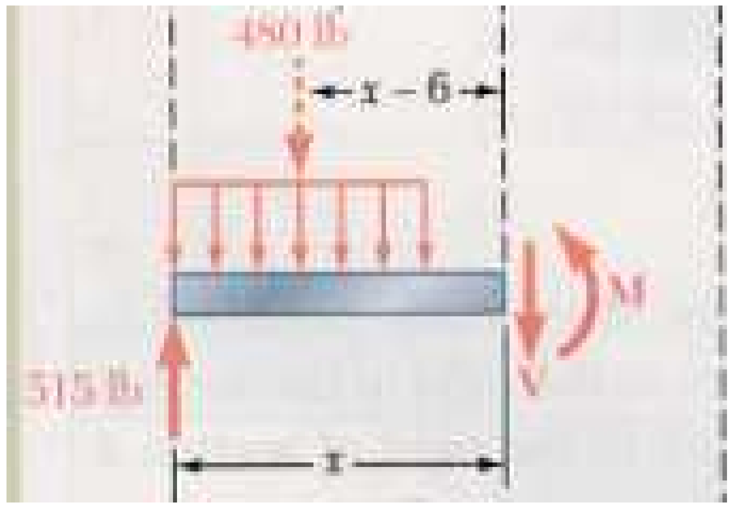
$$V = 515 - 40Xlb$$

$$+\sum M_1 = 0$$

$$-515X - 40X\left(\frac{1}{2}X\right) + M = 0$$

$$M = 515 - 20X^2 lb \cdot in$$

Shear and Bending Moment from C to D



$$+\uparrow\sum F_y = 0$$

$$+\sum M_2 = 0$$

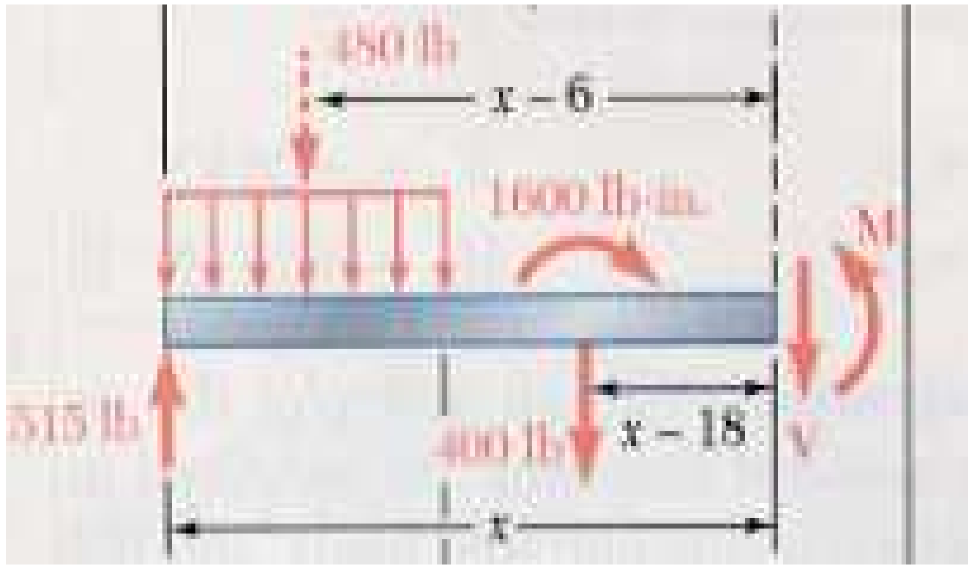
$$515 - 480 - V = 0$$

$$V = 35lb$$

$$-515X - 480(X-6) + M = 0$$

$$M = (2880 + 35X)lb \cdot in$$

Shear and Bending Moment from D to B



$$+\uparrow\sum F_y = 0$$

$$+\sum M_3 = 0$$

$$515 - 480 - 400V = 0$$

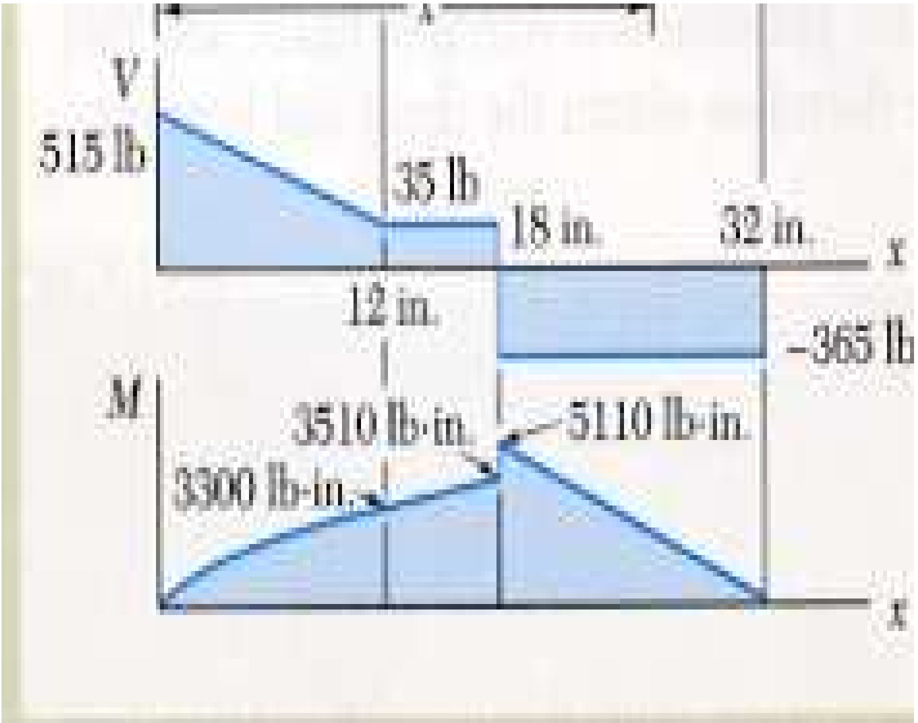
$$V = 365lb$$

$$-515X - 480(X-6) - 1600 + 400(x-18) + M = 0$$

$$M = 11 \cdot 680 - 365Xlb \cdot in$$

Step 5

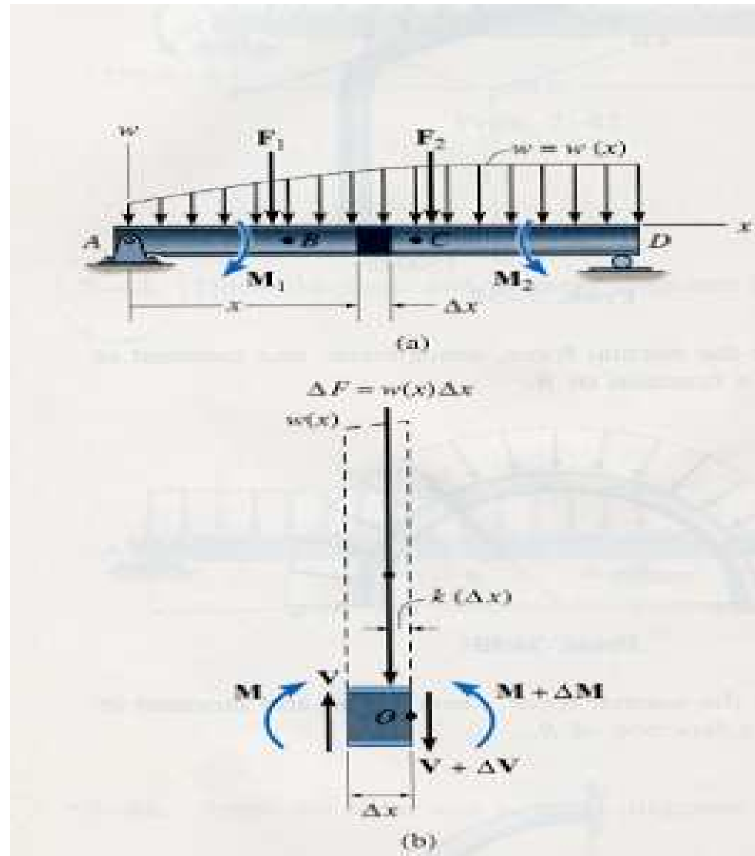
- Draw the shear and moment diagram



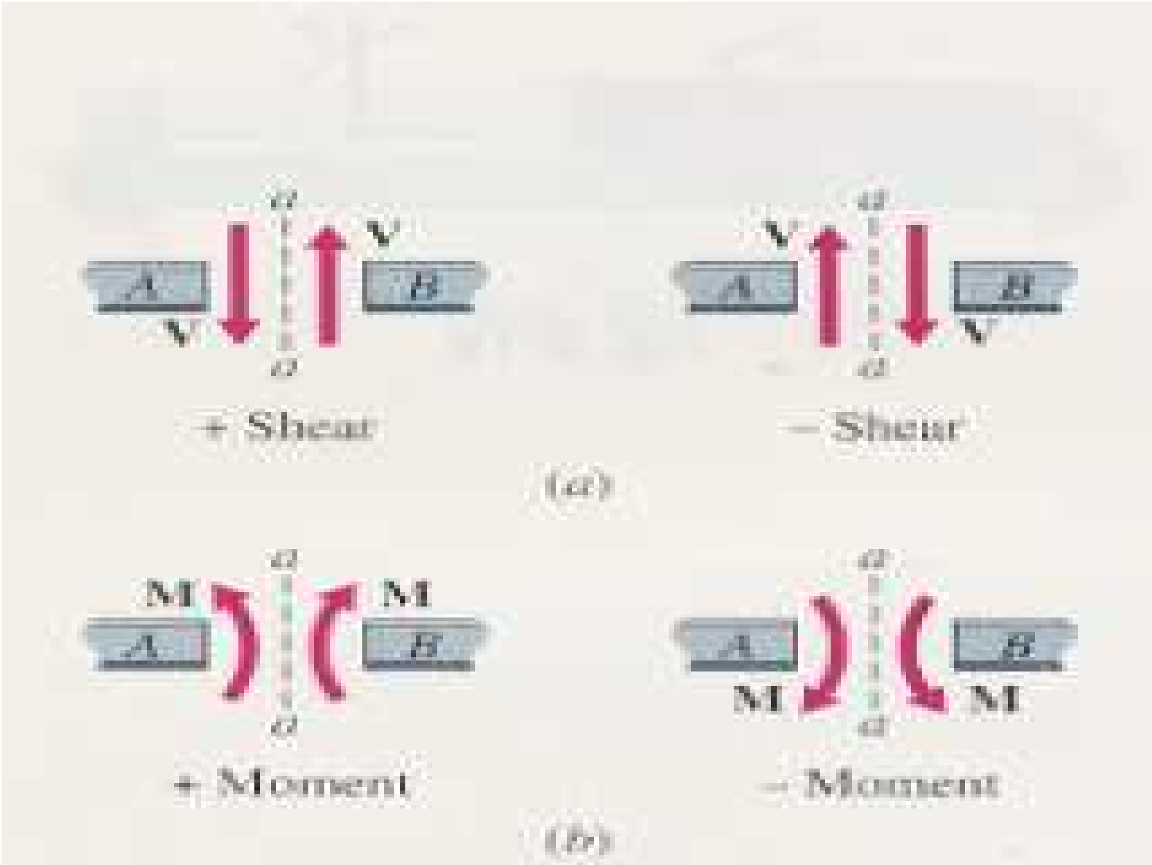
Relationship Between Distributed Load , Shear , and Moment

- W = w(x) is a distributed load
- Distributed load considered positive when acts downward, and negative when acts upward.

- Positive sense of shear force and bending moments are given



Shear Force and Bending Moment Sign Convention



w, V, M Relations

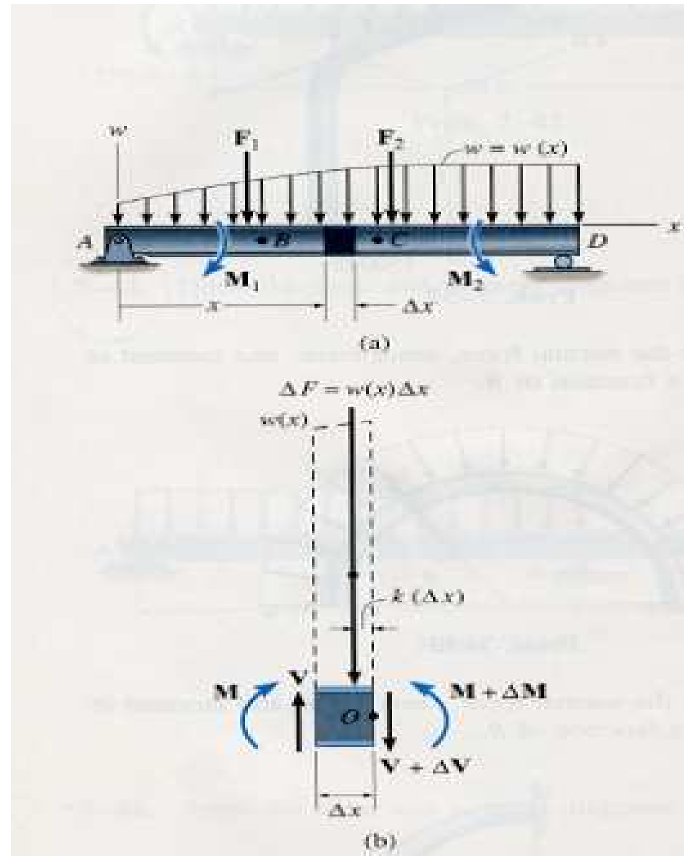
- Consider the equilibrium of the element cut from the beam:

$$+\uparrow \Sigma F_y = 0; \qquad V - w(x)\,\Delta x - (V + \Delta V) = 0$$

$$\Delta V = -w(x)\,\Delta x$$

$$\circlearrowleft\!+\Sigma M_O = 0; \quad -V\,\Delta x - M + w(x)\,\Delta x[k(\Delta x)] + (M + \Delta M) = 0$$

$$\Delta M = V\,\Delta x - w(x)k(\Delta x)^2$$



Relation Between V&w

- Using the limits theory, dividing by Δx and taking the limit as $\Delta x \longrightarrow 0$

$$\frac{dV}{dx} = -w(x)$$

Slope of shear diagram = Negative of distributed load intensity

Relation between M &V

$$\frac{dM}{dx} = V$$

$$\text{Slope of moment diagram} = \text{Shear}$$

Area Method to Find V(Shear Force)

- *Change in shear between points B and C is equal to the negative of the area under the distributed - loading curve between these points*

$$\Delta V_{BC} = -\int w(x)\, dx$$

Change in shear = Negative of area under loading curve

Area Method to Find M (Bending Moment)

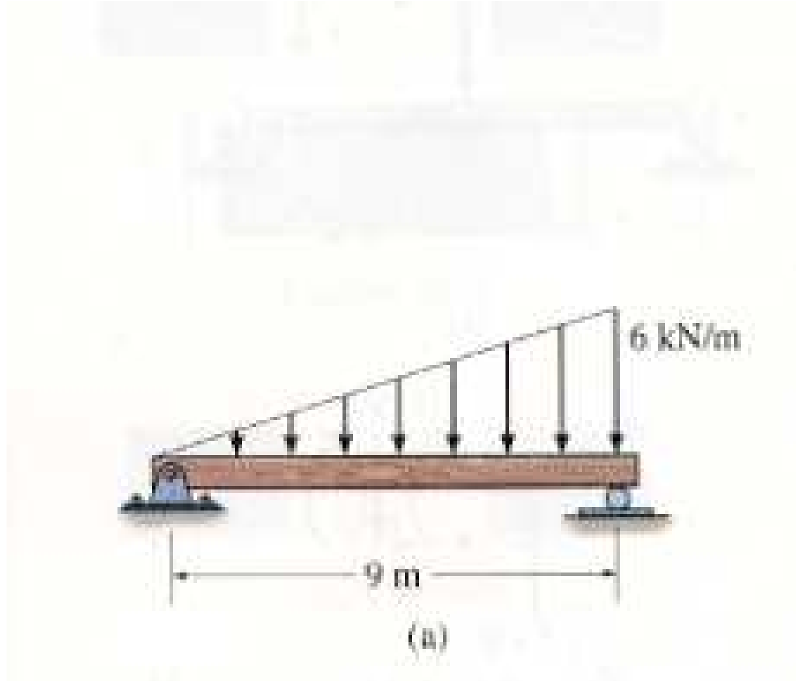
- *The change in moment between points B and C is equal to the area under the shear diagram within region BC*

$$\Delta M_{BC} = \int V\,dx$$

Change in moment = Area under shear diagram

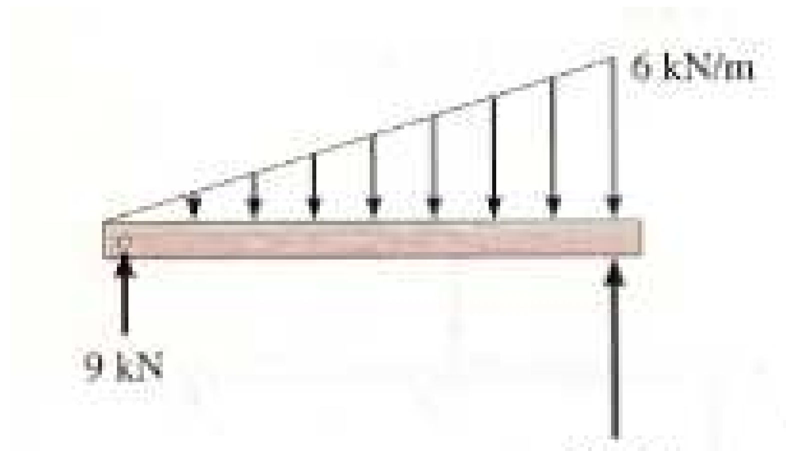
Example

- Draw the shear and bending moment diagrams for the beam shown .

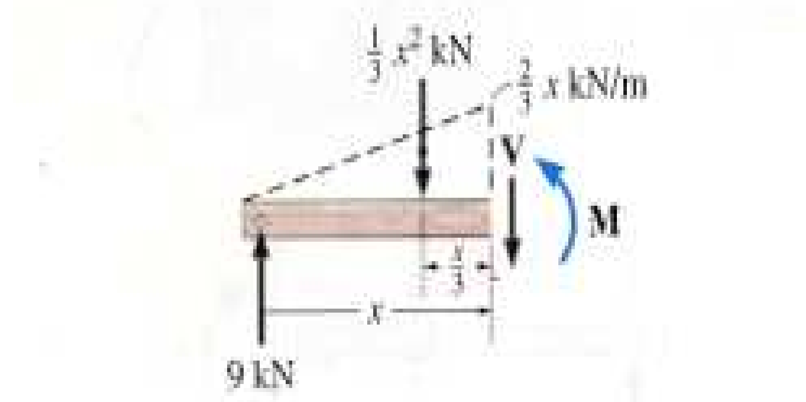


Solution

- Step 1: FBD and reactions



- Step2: Cut a portion of the beam
- Step3: Draw a FBD of that section



(b)

Solution, Cont...

- Step4: write the equilibrium equations for that section

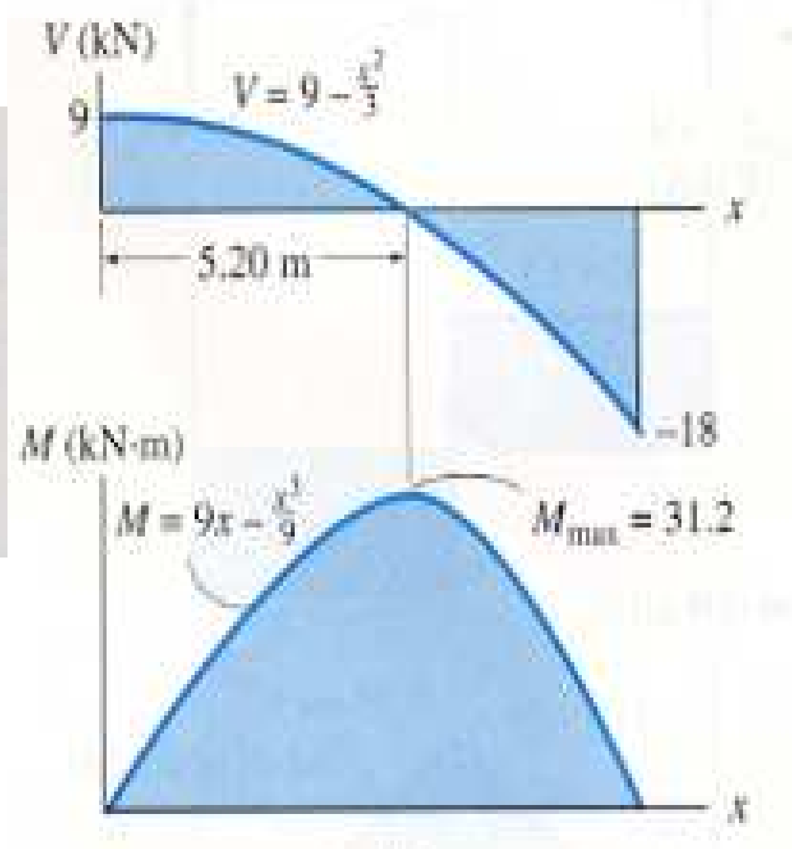
$$+\uparrow \Sigma F_y = 0; \qquad 9 - \frac{1}{3}x^2 - V = 0$$

$$V = \left(9 - \frac{x^2}{3}\right) \text{kN}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{3}x^2\left(\frac{x}{3}\right) - 9x = 0$$

$$M = \left(9x - \frac{x^3}{9}\right) \text{kN} \cdot \text{m}$$

- Step5: use the resulting equations to draw the V and M diagrams



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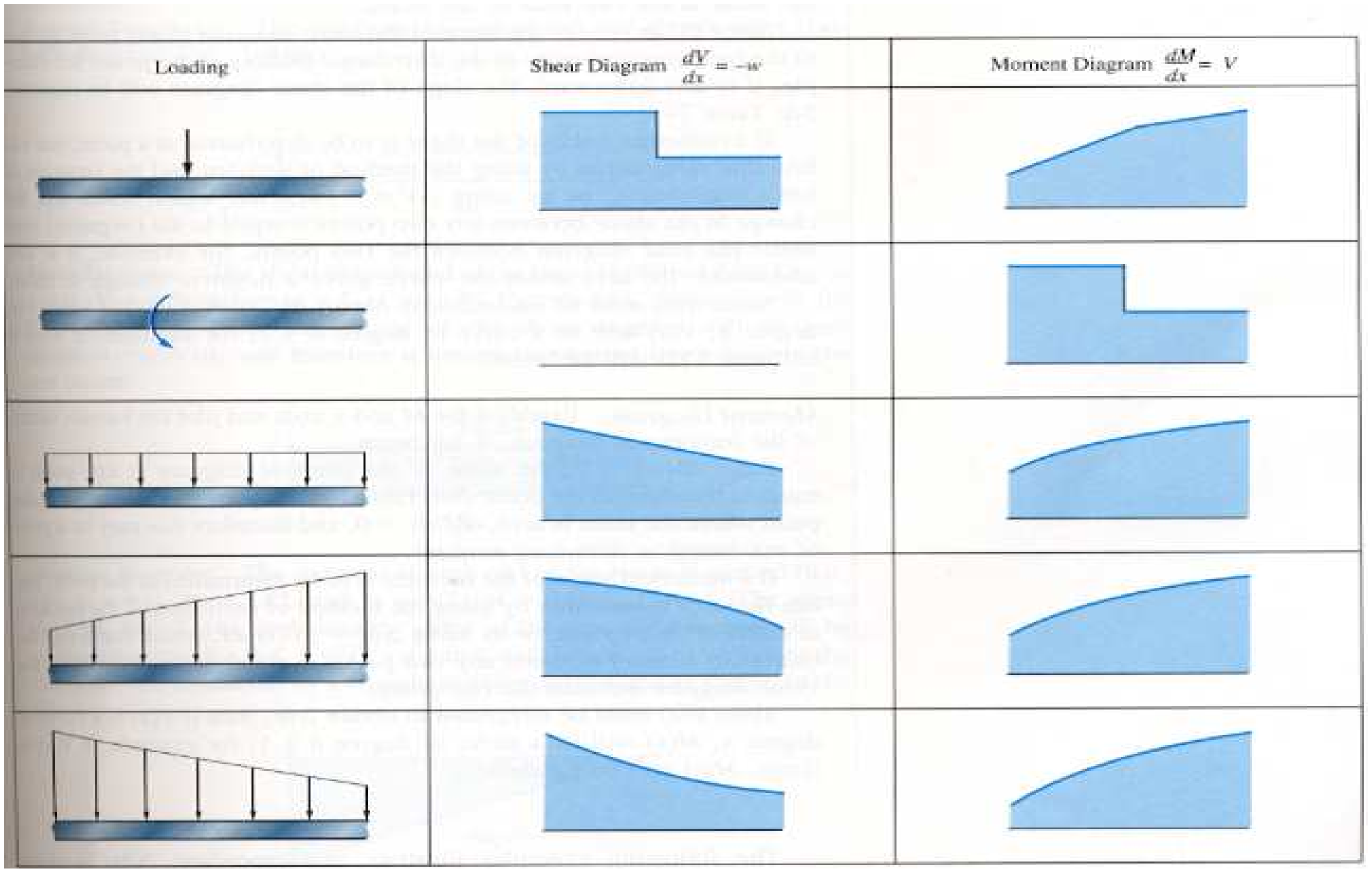
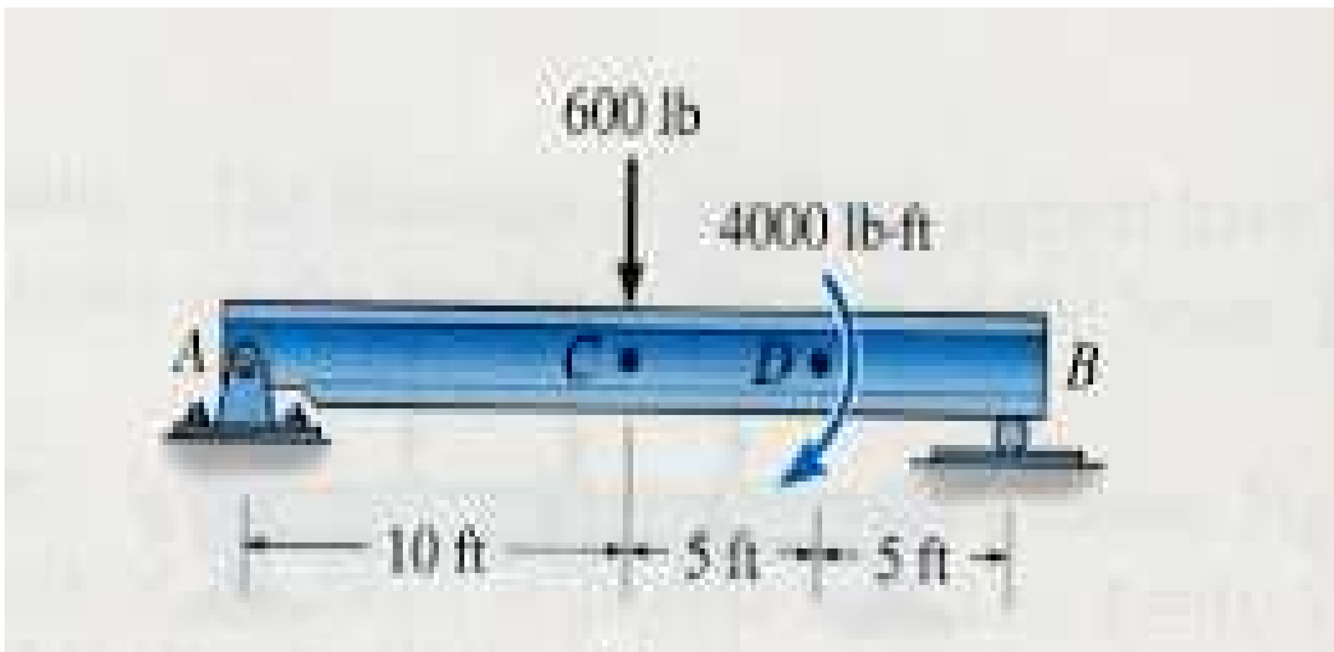
Drawing V and M Diagrams Using the Area Method



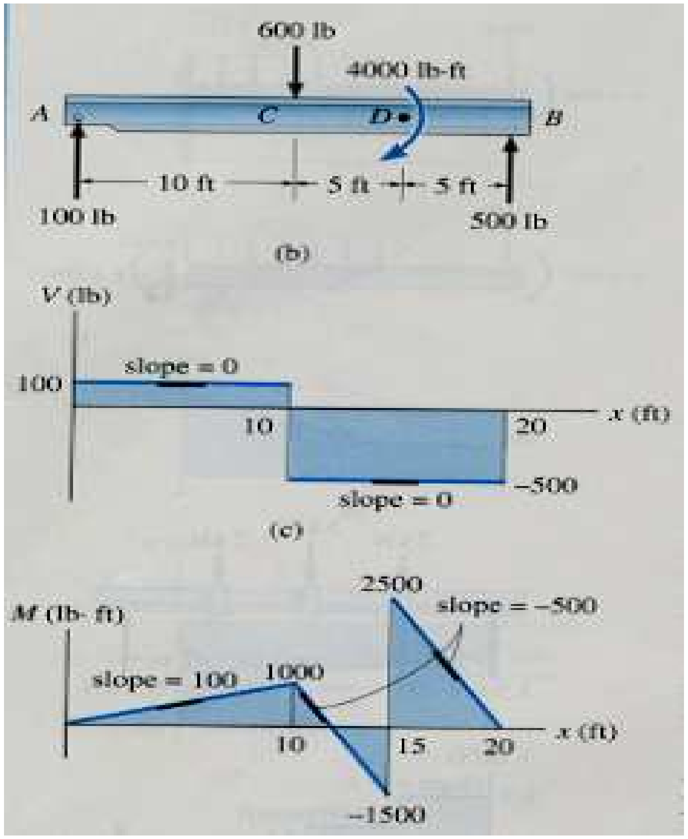
Example : Area Method

- Using the area method draw the shear and moment diagrams of the beam shown below



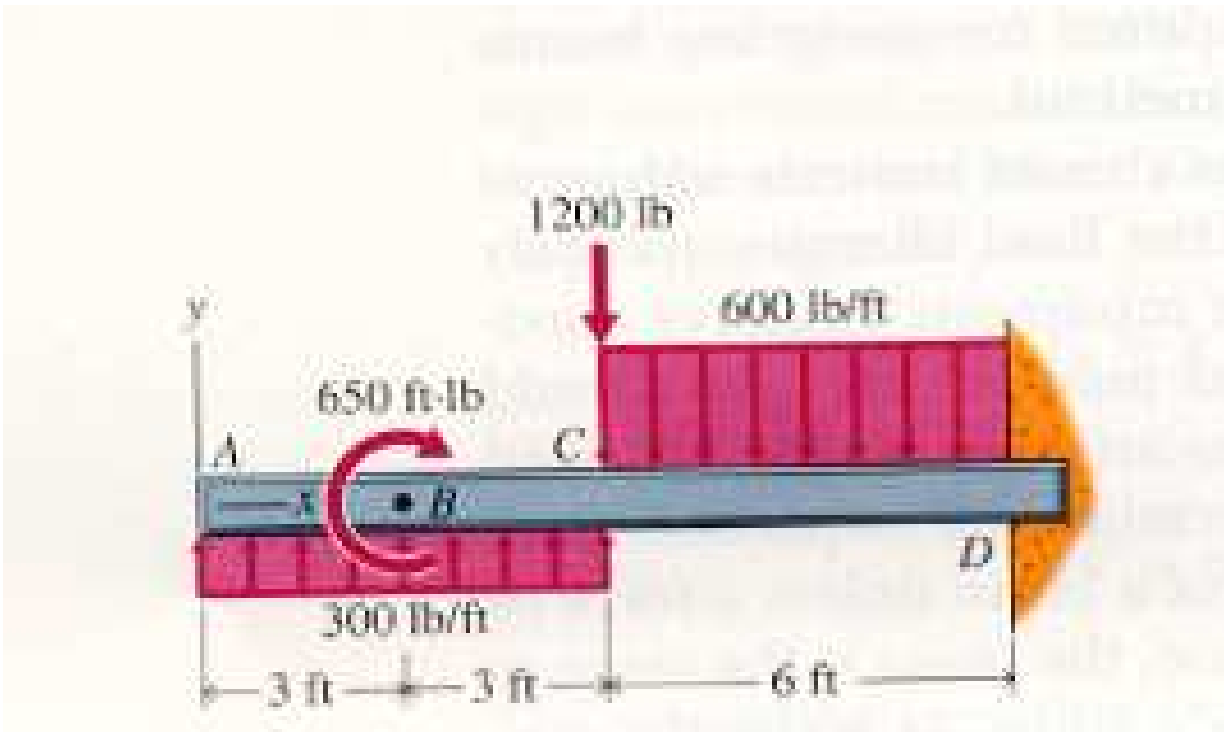
Solution

- **Note that when having a concentrated couple affecting the beam at certain point, that will not affect the shear diagram, however, the bending moment diagram will show a discontinuity at that point rising or falling by an amount equivalent to the magnitude of that couple**

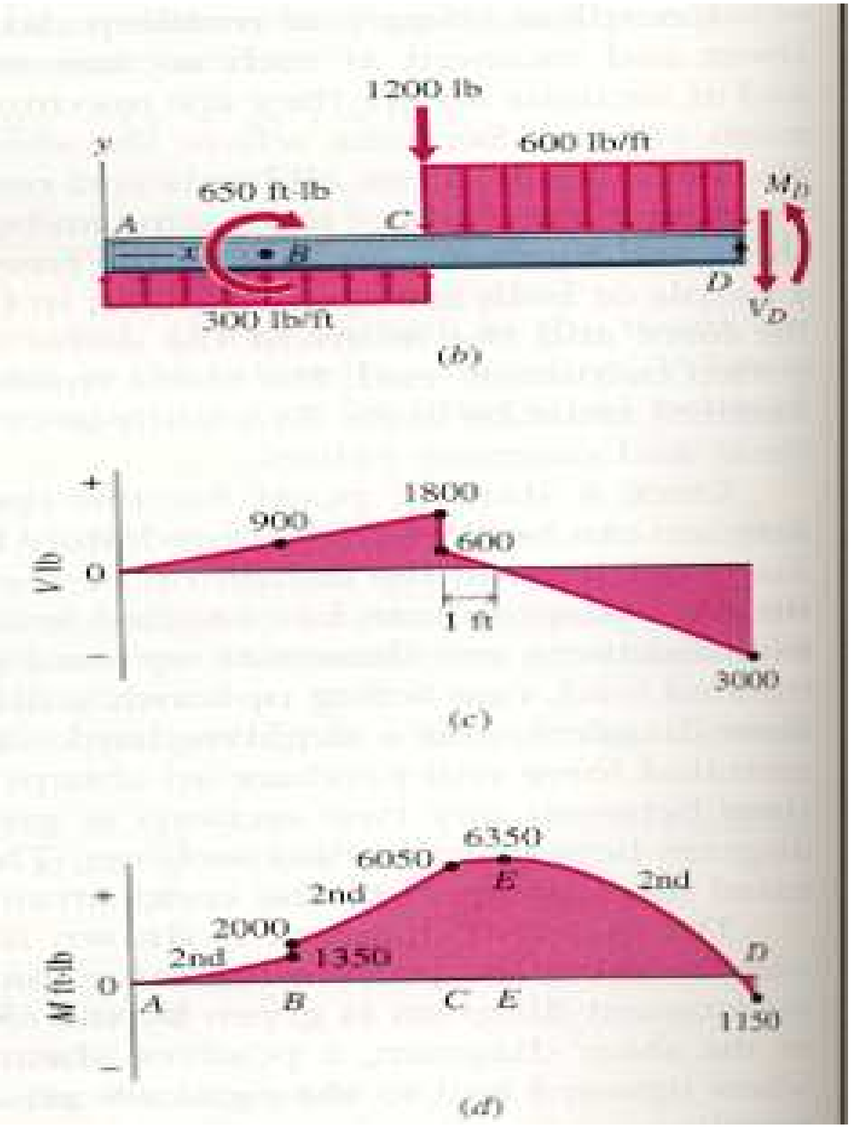


Practice !

- Draw a complete shear and bending moment diagrams for the beam shown below

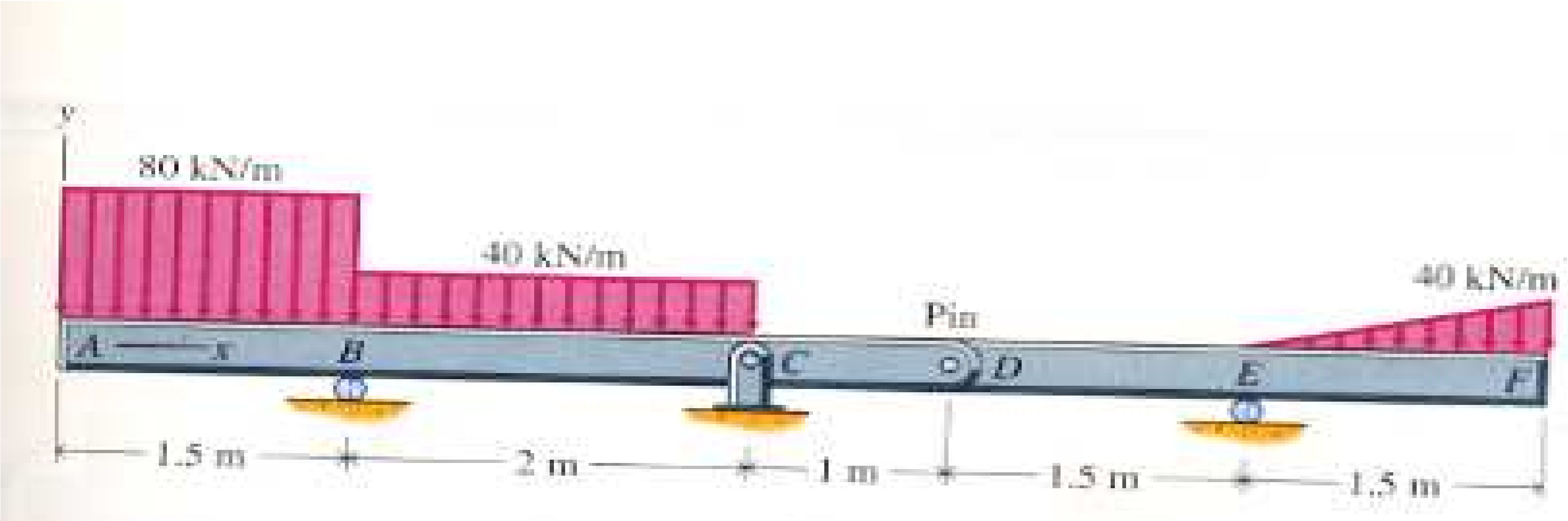


Answer to Practice



Practice

- Draw the shear and bending moment diagrams for the following beam



(a)

Answer to Practice