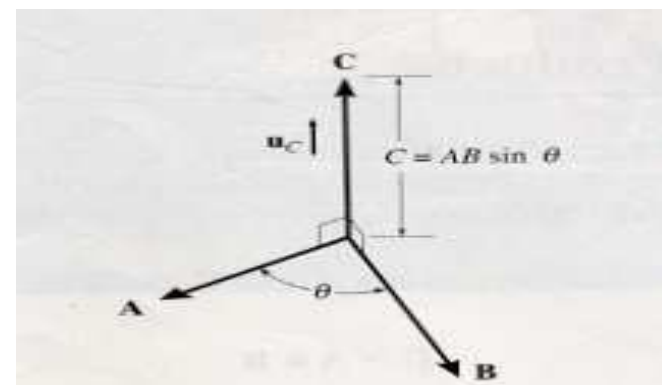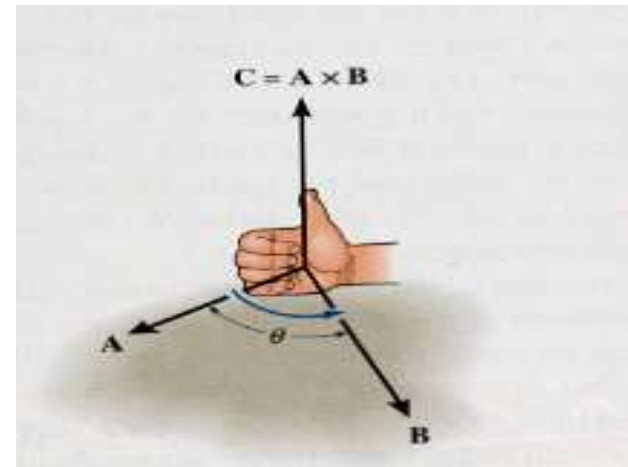# Mechanics & Materials I

Chapter 3
Force System Resultant

FAMU/FSU College of Engineering

Department of Mechanical Engineering
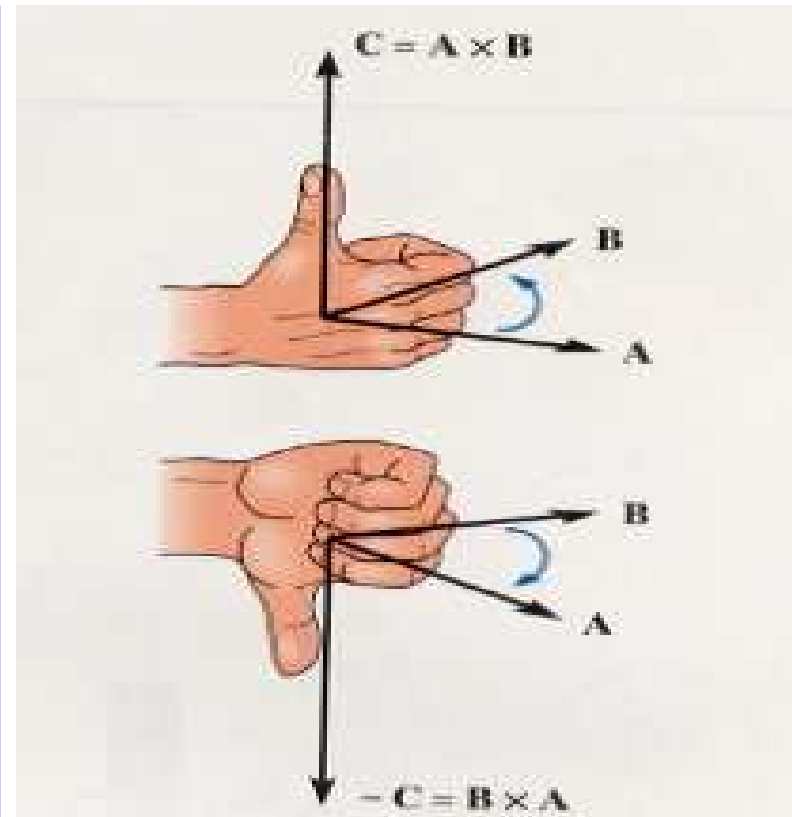
Cross Product

- Cross product of two vectors A and B:

- $A \times B = C$

- Magnitude $C = AB \sin\theta$

- Direction: C is perpendicular to both A and B

C = A × B

$C = AB \sin\theta$

Laws of Operation for Cross Product

- 1. Commutative law is not valid due to direction

- A×B≠B ×A
- A×B=-B ×A

Cartesian Formulation of Cross Product

$$\hat{A} = A_x i + A_y j + A_z k$$
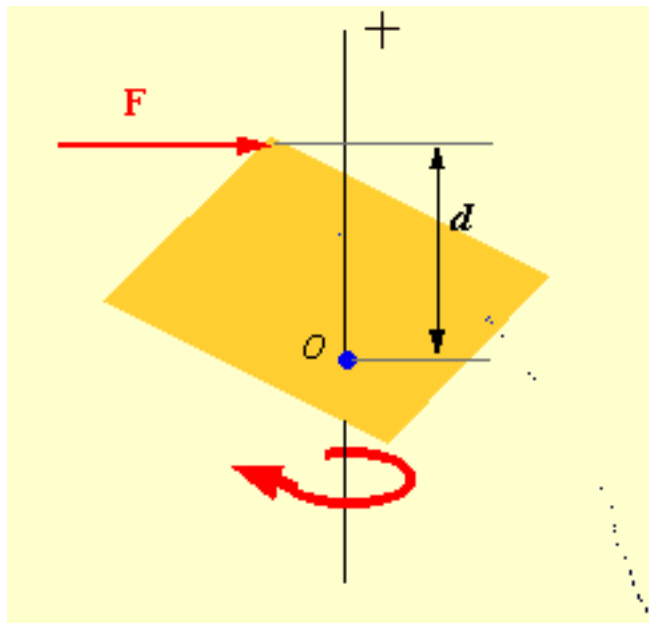
$$\hat{B} = B_x i + B_y j + B_z k$$

$$\hat{A} \times \hat{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= i (A_y B_z - A_z B_y)$$

$$- j (A_x B_z - A_z B_x)$$
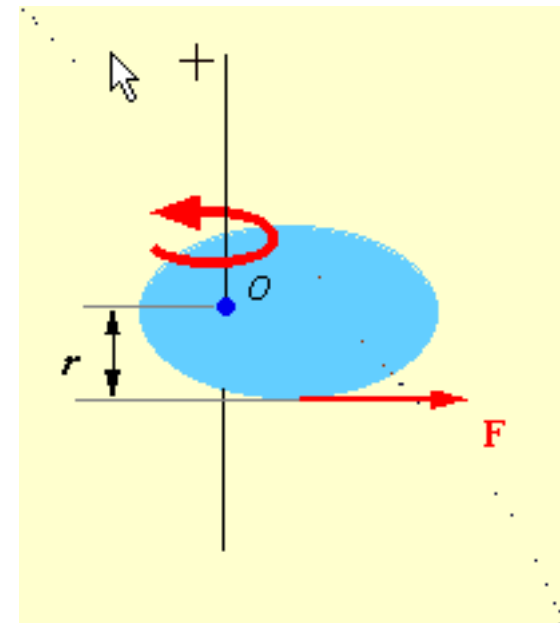
$$+ k (A_x B_y - A_y B_x)$$

Moment Systems

The moment of a force about an axis (sometimes represented as a point in a body) can be thought of as a measure of the force's tendency to rotate the body about the axis (or point). The two bodies below are subjected to forces acting in the same direction.
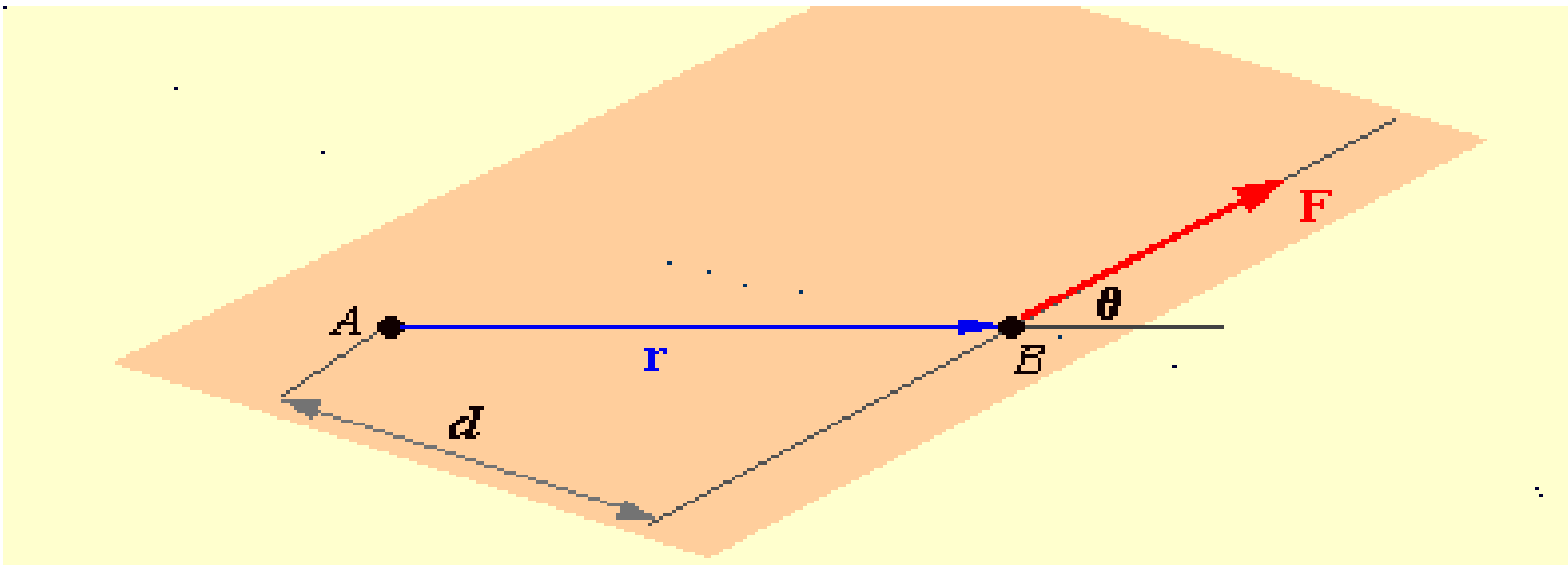
Negative Moment

Positive moment

Moment Systems

$M_A = d * F$
$D = |r| \sin\theta$
Or,

$$M_A = r \times F = |r| |F| \sin\theta \, (e)$$

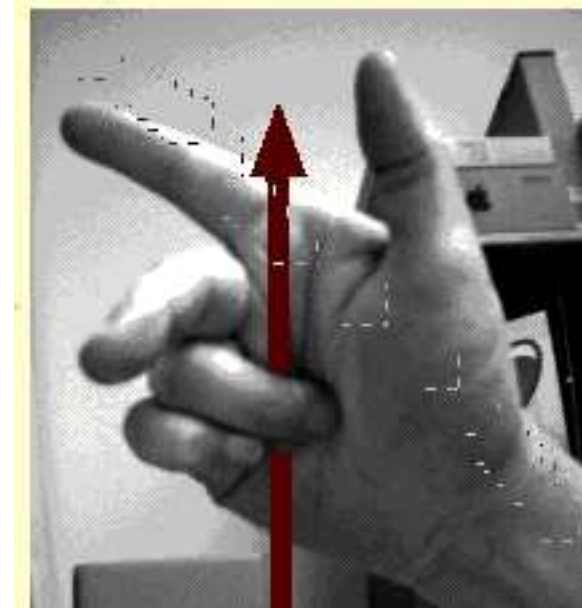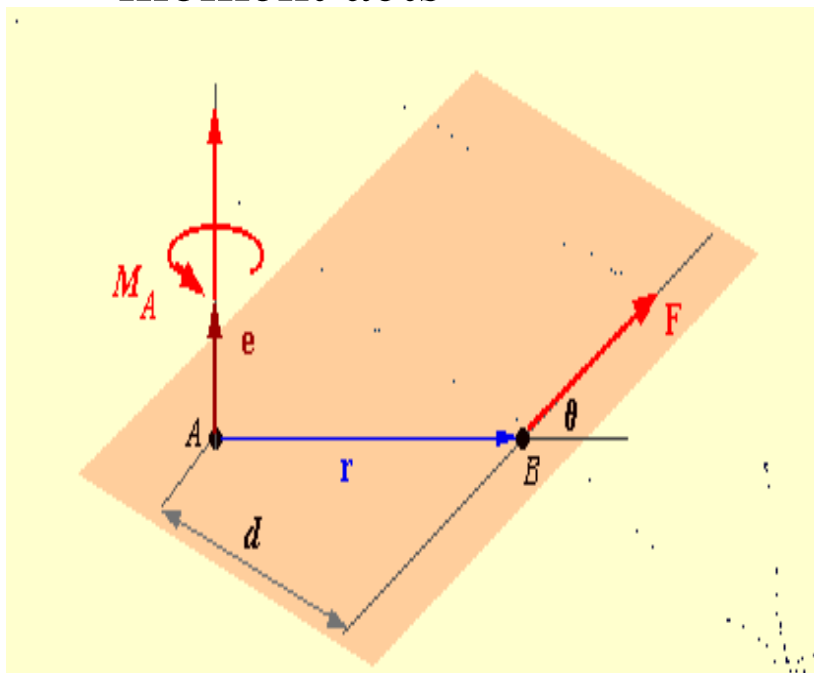Where e is the unit vector normal to both r and F. To find the direction (and sense of e) use the right hand rule.
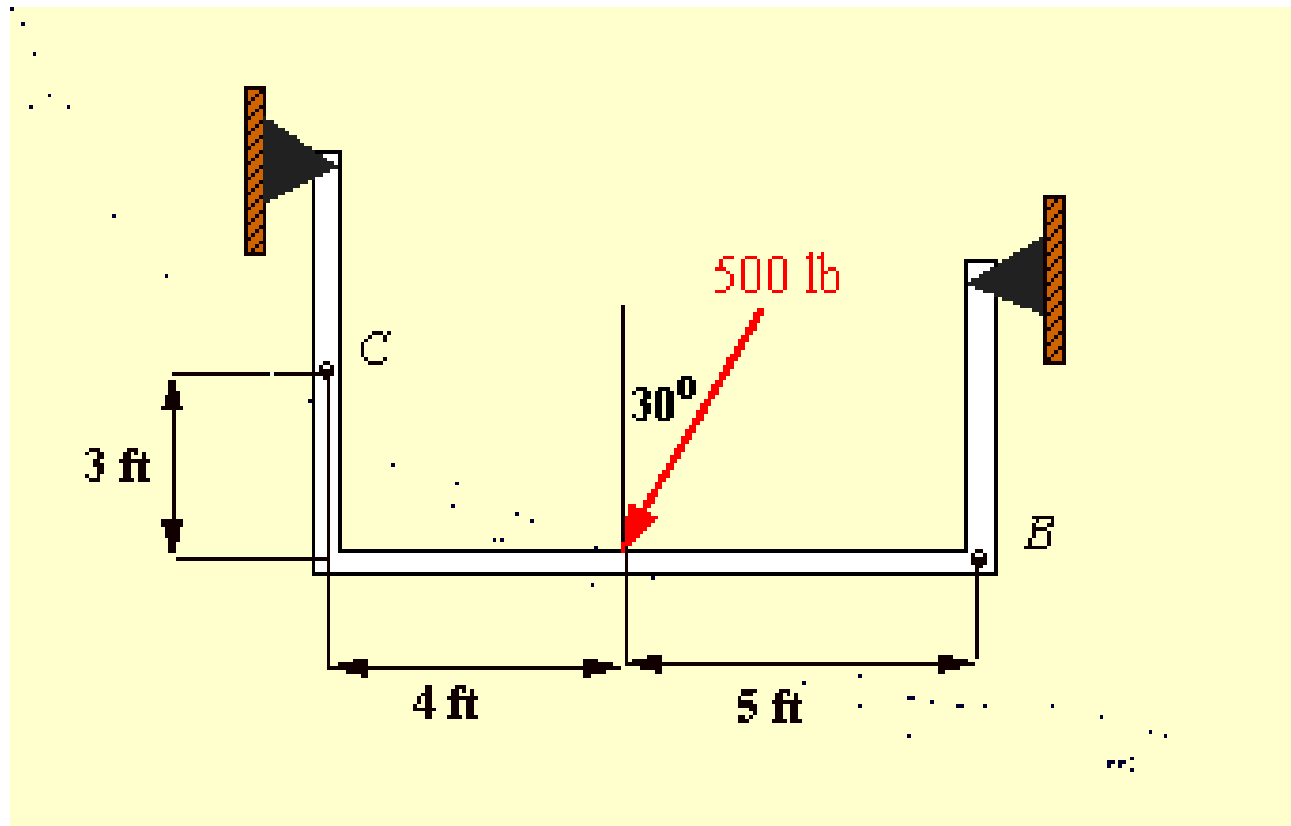
Moment Systems

Right Hand Rule:

1. Place the base of your right hand at the moment center.
2. Curl your fingers in the direction of the force.
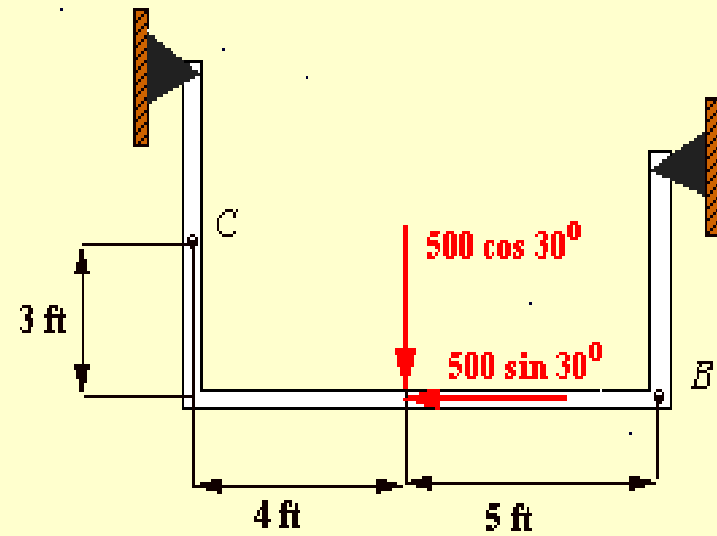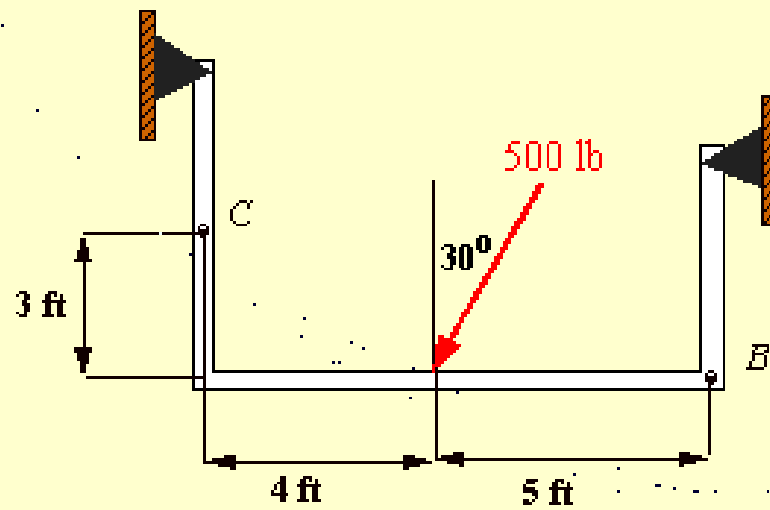3. Your thumb will point along the axis about which the moment acts

Moment Systems

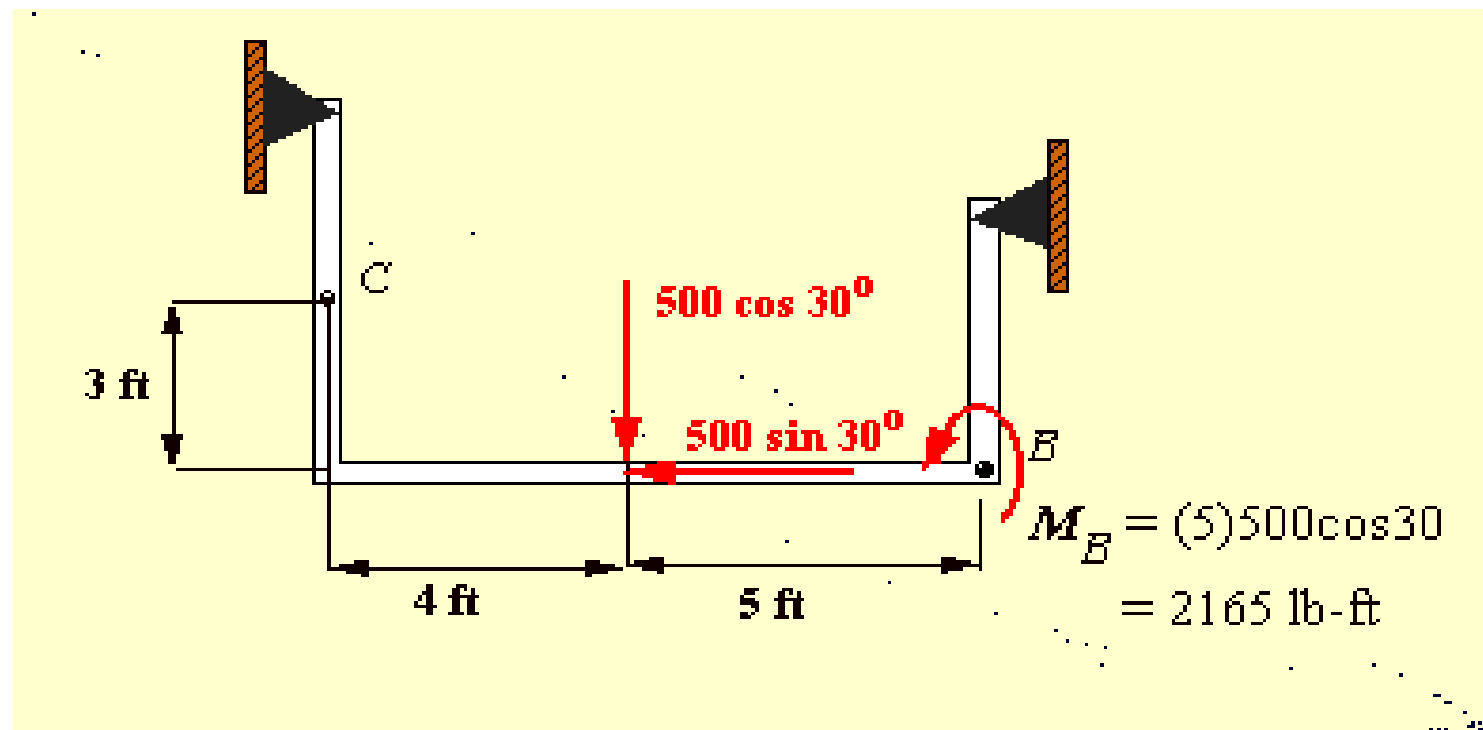Find the moment from the 500 lb force around B and C

Moment Systems

Moment Systems

- The moment around point B

Moment Systems

- The moment around point C

$M_C = (3)500\sin30$
$= -750$ lb-ft

500 cos 30⁰

3 ft

500 sin 30⁰

$M_B = (5)500\cos30$
$= 2165$ lb-ft

4 ft   5 ft

$M_c = -2482$ lb-ft

500 cos 30⁰

3 ft

500 sin 30⁰

$M_B = (5)500\cos30$
$= 2165$ lb-ft

4 ft   5 ft
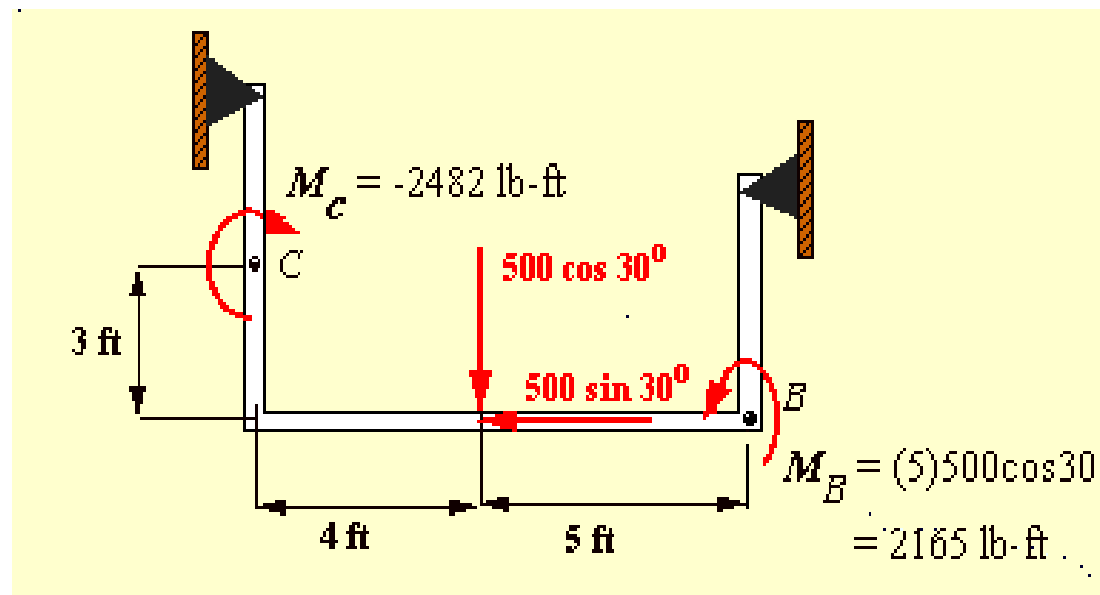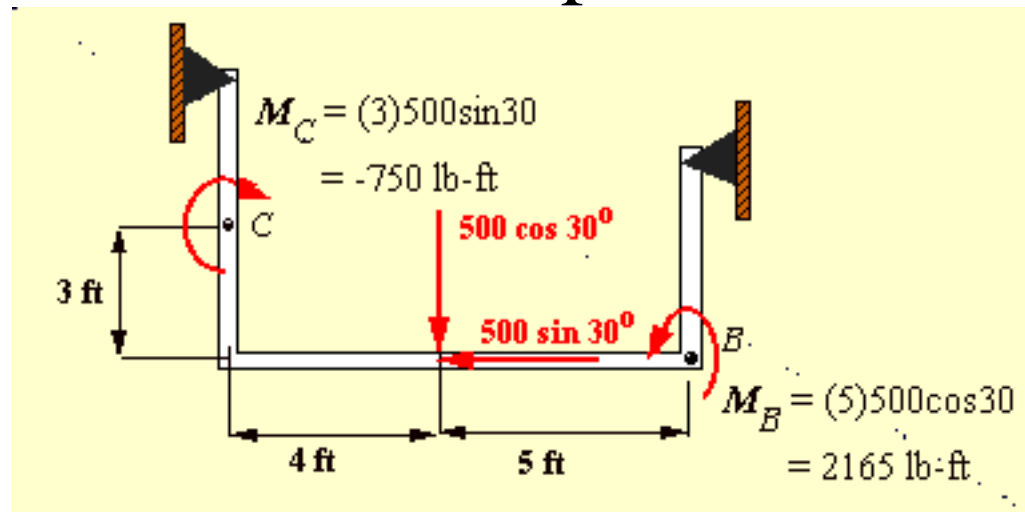
Moment Systems

- 3D form

$$M_o = r \times F = (r_x i + r_y j + r_z k) \times (F_x i + F_y j + F_z k) = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$
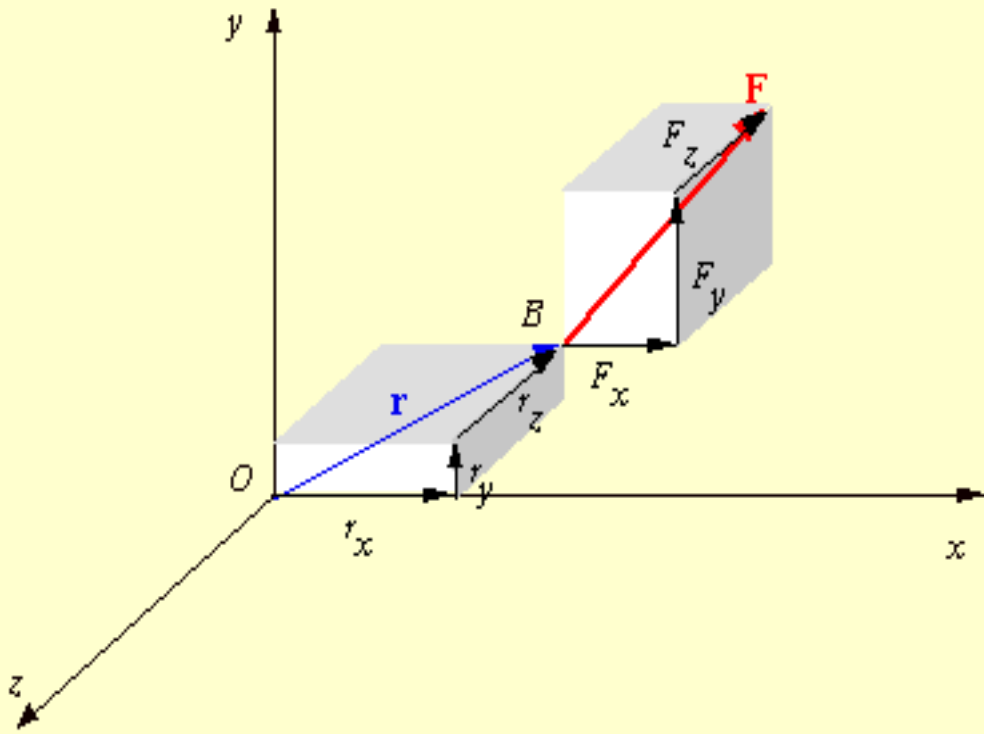
$$= (r_y F_z - r_z F_y) i + (r_z F_x - r_x F_z) j + (r_x F_y - r_y F_x) k$$
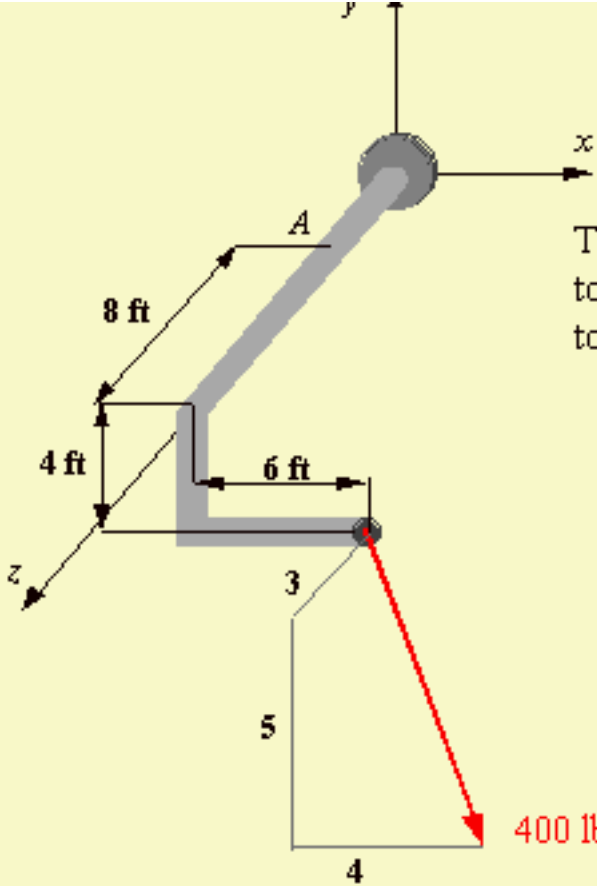
Moment Systems

- 3D form

$$M_o = (r_y F_z - r_z F_y)\mathbf{i} + (r_z F_x - r_x F_z)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$
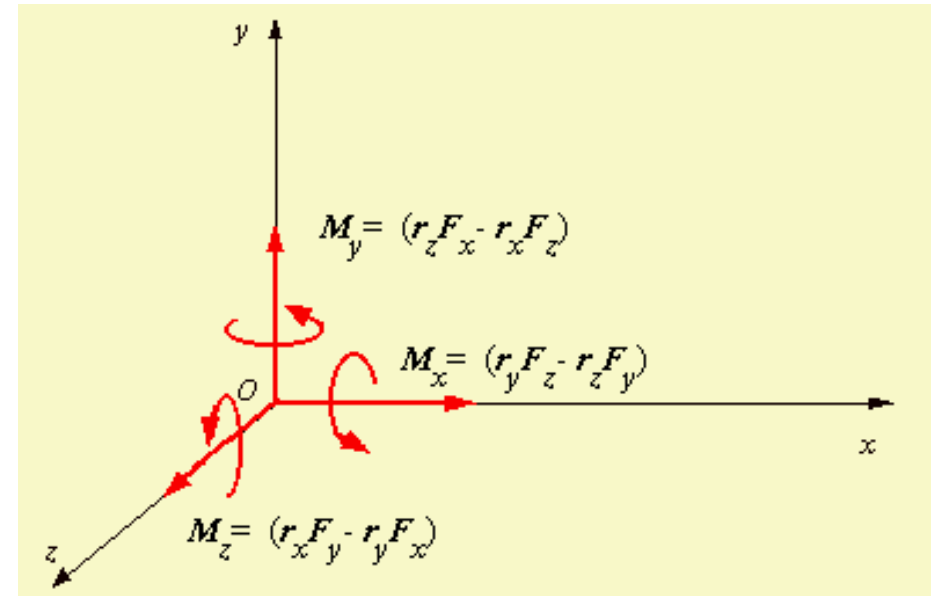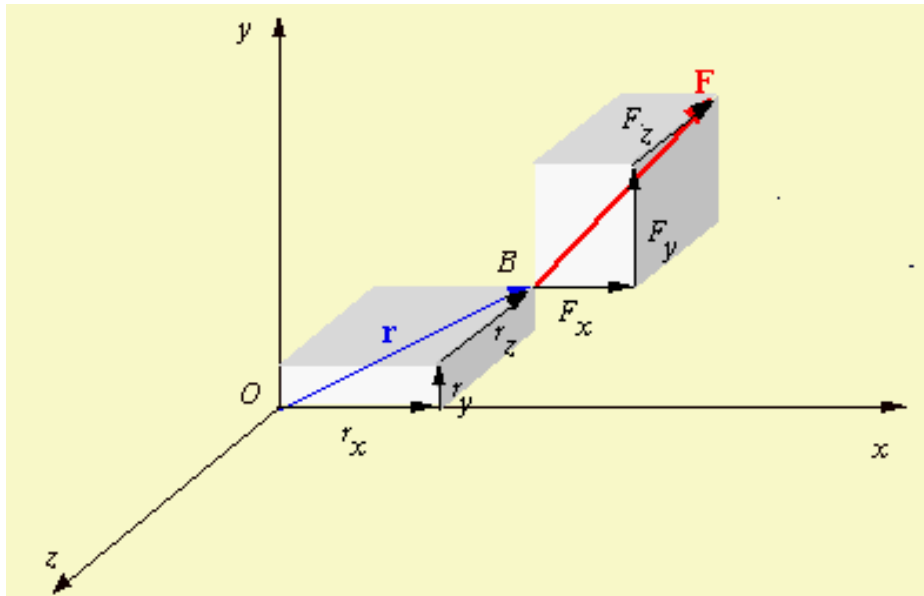
Moment Systems

- Example 3D

$$M_A = r \times F = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The force vector can be defined easily, since its location relative to the point of application is known. The procedure may be easier to follow if a box is constructed around the force.

A

8 ft

4 ft

6 ft

3

5

4

400 lb

Moment Systems:Example 3D

For the structure shown, determine the moment of the 400-lb force about point A. Since a three-dimensional moment is required, the first step is to express both the force and position vectors in terms of their **i**, **j**, and **k** components. Then the cross product can be formed and the determinant expanded, resulting in the moment about A:
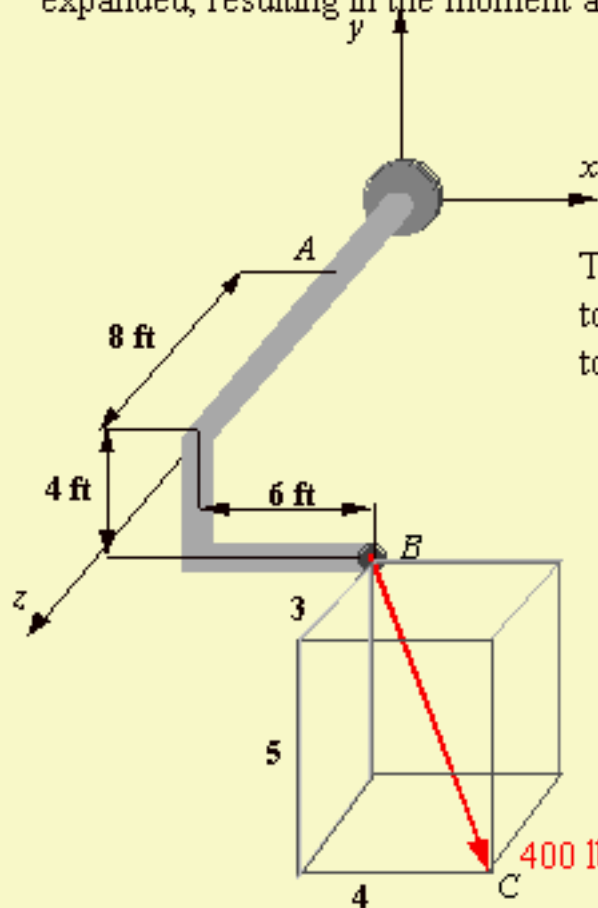
$$M_A = r \times F = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The force vector can be defined easily, since its location relative to the point of application is known. The procedure may be easier to follow if a box is constructed around the force.

The distance from B to C is

$$(3^2 + 5^2 + 4^2)^{1/2} = 7.07$$

The vector defining the direction of the force is 4**i** - 5**j** + 3**k**.

Combining this with the 400 lb magnitude results in a force vector of

$$F = 400 \frac{4i - 5j + 3k}{7.07}$$

$$= 226.3 \, i - 282.9 \, j + 169.7 \, k$$

$$F = 226.3i - 282.9j + 169.7K$$
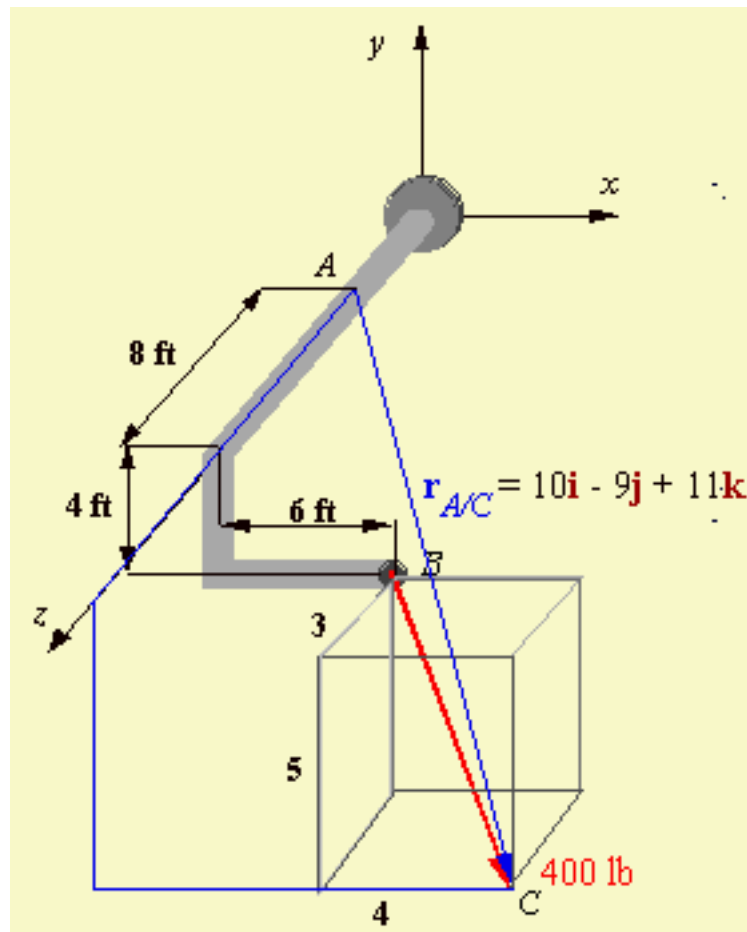
8 ft

4 ft

6 ft

A

B

3

5

4

C

400 lb

x

y

z

$r_{A/B} = 6i - 4j + 8k$

$$M_A = r \times F = \begin{vmatrix} i & j & k \\ 6 & -4 & 8 \\ 226.3 & -282.9 & 169.7 \end{vmatrix}$$

Expanding the determinant results in

$$M_A = 1584.4i + 792.2j - 791.6k \quad \text{lb-ft}$$

Moment Systems :Example 3D

$$M_A = r \times F = \begin{vmatrix} i & j & k \\ 6 & -4 & 8 \\ 226.3 & -282.9 & 169.7 \end{vmatrix}$$

Expanding the determinant results in

$$M_A = 1584.4i + 792.2j - 791.6k \quad lb\text{-}ft$$

792.2 lb-ft

791.6 lb-ft

1584.4 lb-ft

8 ft

4 ft
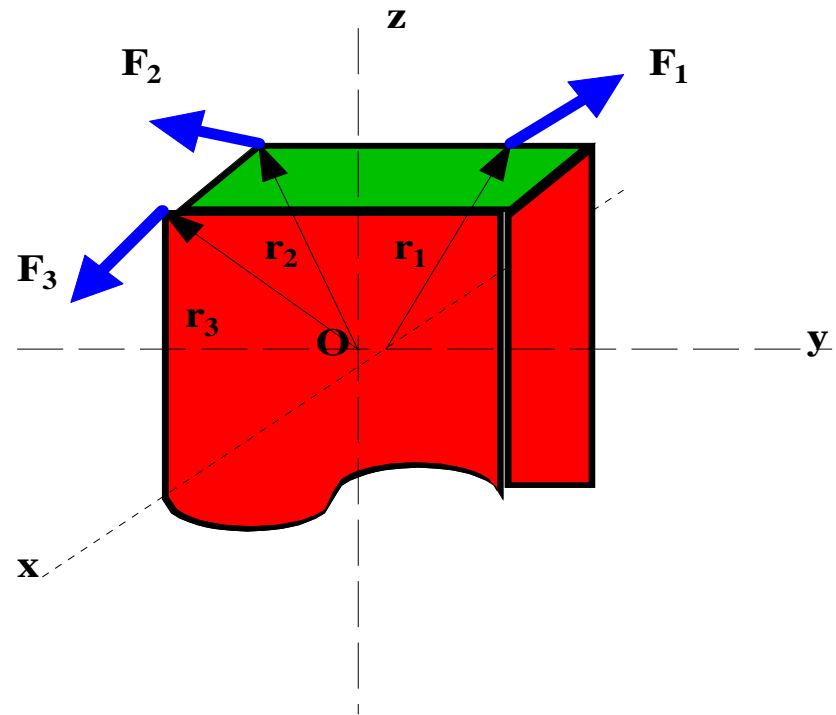
6 ft

$r_{A/C} = 10i - 9j + 11k$

3

5

400 lb

4

The moment can now be translated
To point A
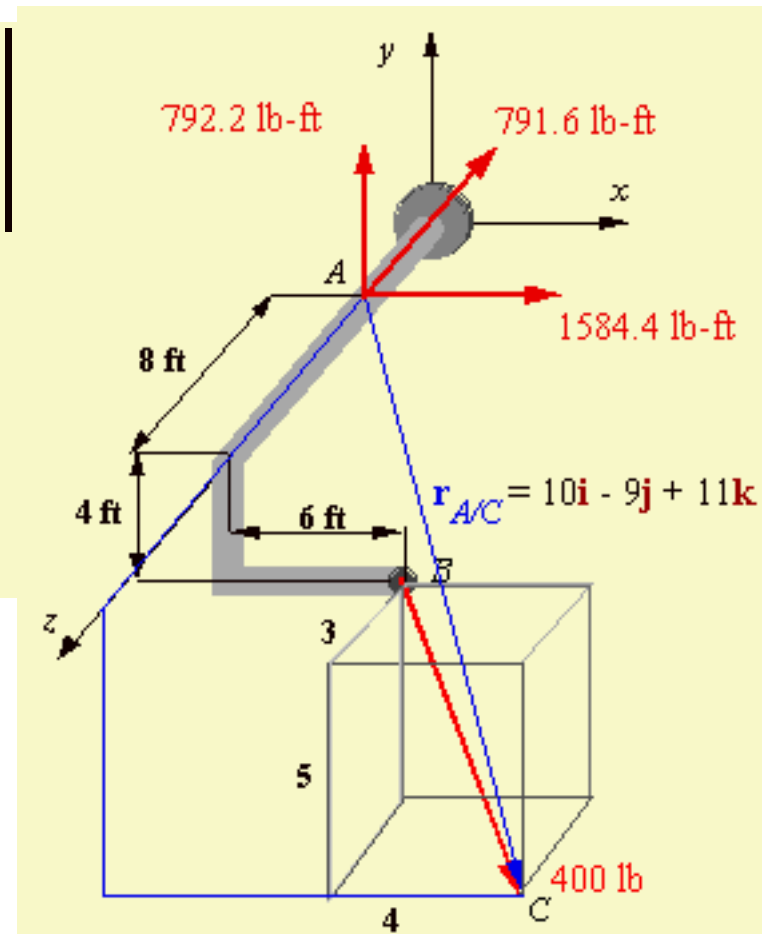
Resultant Moment of a System of Forces

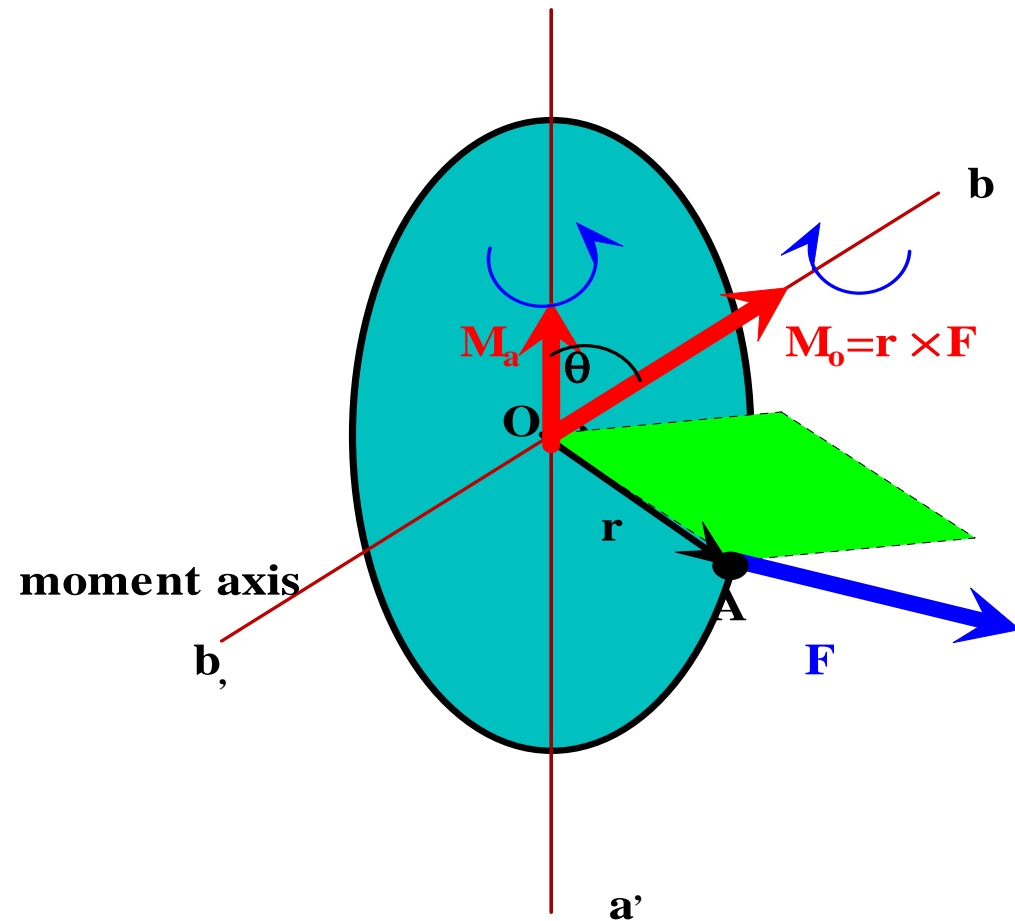- The resultant moment of a system of forces about point O can be determined by vector addition of all the moments corresponding to the different forces

$$\Sigma M_{Ro} = \Sigma \bar{r} \times \bar{F}$$

$$= \bar{r}_1 \times \bar{F}_1 + \bar{r}_2 \times \bar{F}_2 + \bar{r}_3 \times \bar{F}_3$$

Moment of a Force About Specified Axis

- The effect of F in tending to rotate the body about aa' is given by Ma

Moment of a Force about Specified axis, Cont..

$$M_o = \bar{r} \times \bar{F}$$

r : distance between O and A

$$M_a = \bar{M}_o \cos \theta$$
$$= \bar{M}_o . \bar{u}_a$$

u_a : unit vector defines the direction of aa$'$

Moment of a Force about Specified axis, Cont

$$M_a = \overline{M}_o \overline{u}_a$$

$$= \overline{u}_a \cdot (\overline{r} \times \overline{F})$$

$$= (u_{ax} i + u_{ay} j + u_{az} k) \cdot \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Moment of a Force about Specified axis, Cont

$$M_a = \bar{M}_o \bar{u}_a$$

$$= \bar{u}_a \cdot (\bar{r} \times \bar{F})$$

$$= (u_{ax}i + u_{ay}j + u_{az}k) \cdot \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \begin{vmatrix} u_{ax} & u_{ay} & u_{az} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

- **u**$_a$: unit vector defines the direction of of aa' axis

- **r**: position vector from any point O on aa', to any point A on the line of action of the force F

Couples

- A couple is a moment which results when two force vectors of equal magnitude and opposite direction are separated by a distance.

The point of application of each force vector can be located by a position vector
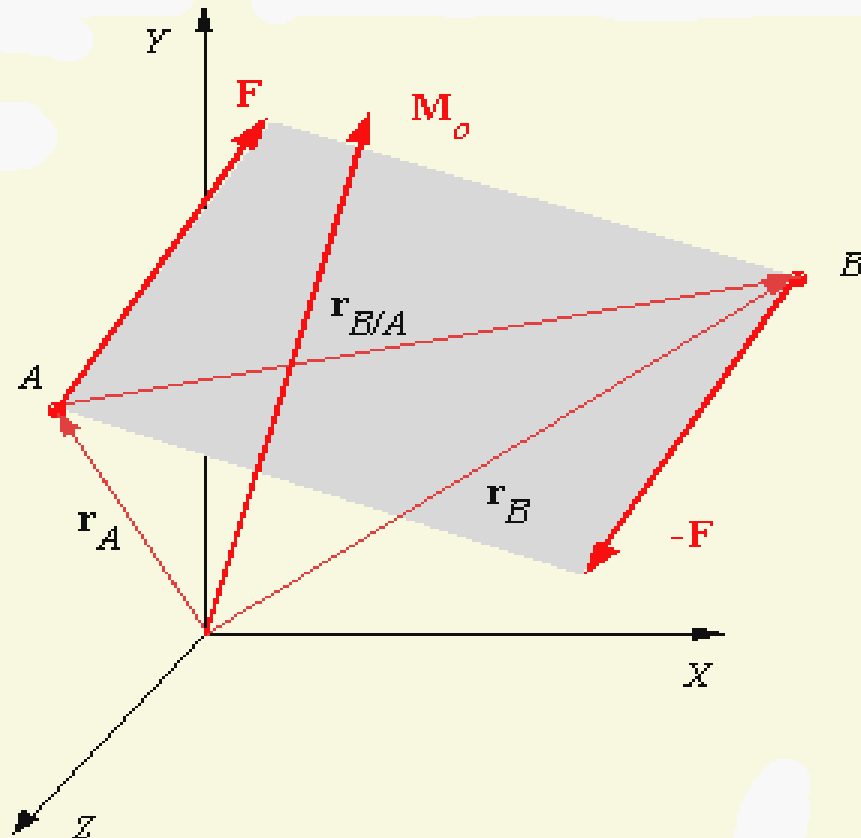
Couples

Y

F   M$_o$

r$_{B/A}$

B

A
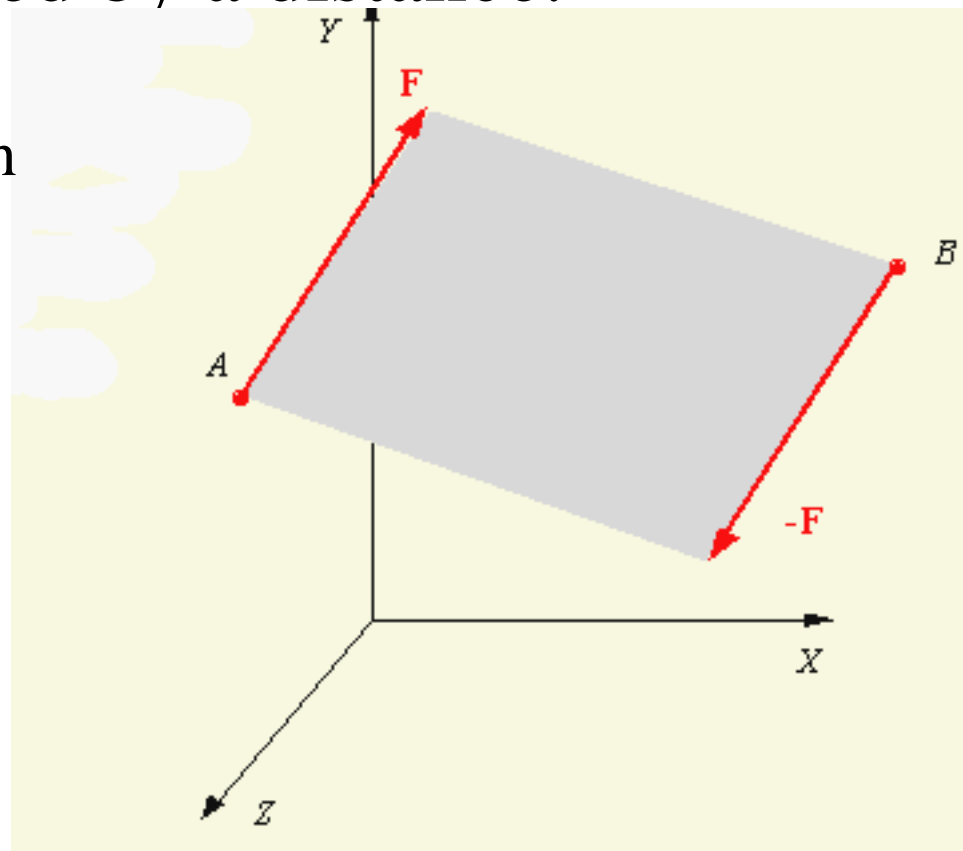
Taking moments of each force about the origin
results in a moment about that point.

r$_A$

r$_B$

-F

$$M_o = r_A \times F + r_B \times (-F) = (r_A - r_B) \times F$$

This equation can be rewritten in terms of the
distance between the two vectors.

X

Z

$$r_A + r_{B/A} = r_B \quad \text{and}$$

$$r_A - r_B = -r_{B/A} = r_{A/B} \quad \text{therefore} \quad M = r_{A/B} \times F.$$

Couples
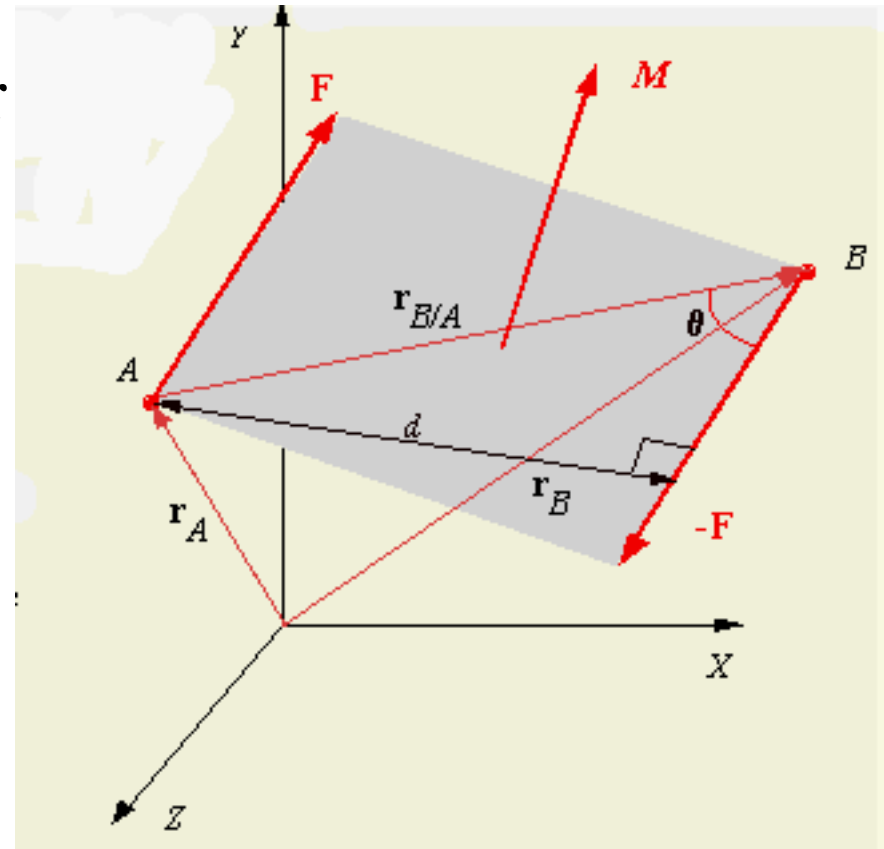
The resultant Moment vector must be
Perpendicular to the plane containing
The forces and $r_{A/B}$

The magnitude of the moment:

$$M = r_{B/A}(F) \, Sin\theta = Fd$$

Couples

The moment vector formed by a couple is a free vector (it can be applied at any point). The sense of the moment is in Accordance with the right hand rule. A pair of equal and opposite forces on a plane, separated by a distance, will produce a moment perpendicular to the plane, with a magnitude equal to the product of the force and distance.

$M = d_1 F_1$ (perpendicular to the plane)
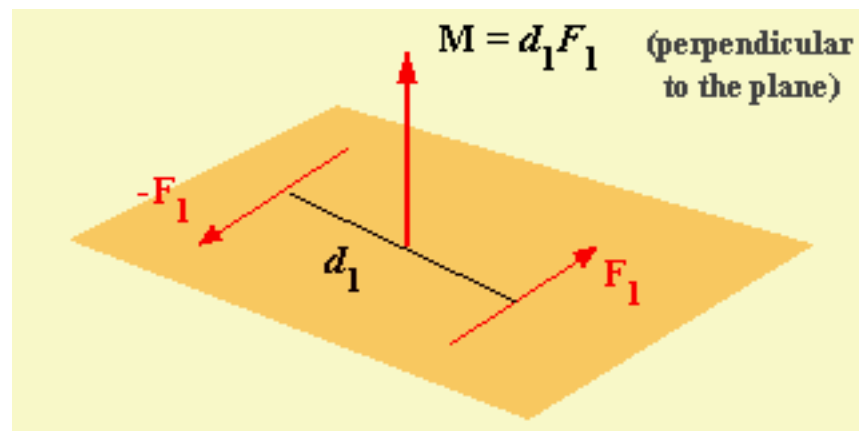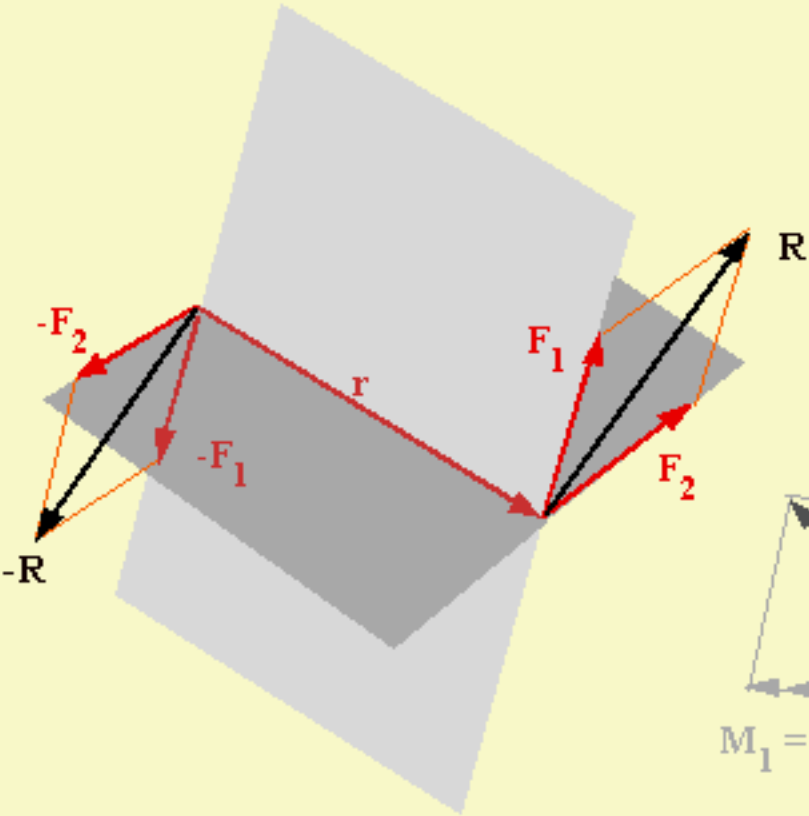
Couples-Equivalent Couples

Equivalent couples are simple to visualize in a 3-D model.
 The resulting couple of 30 lb. -in. for the first case is equivalent to the other two.
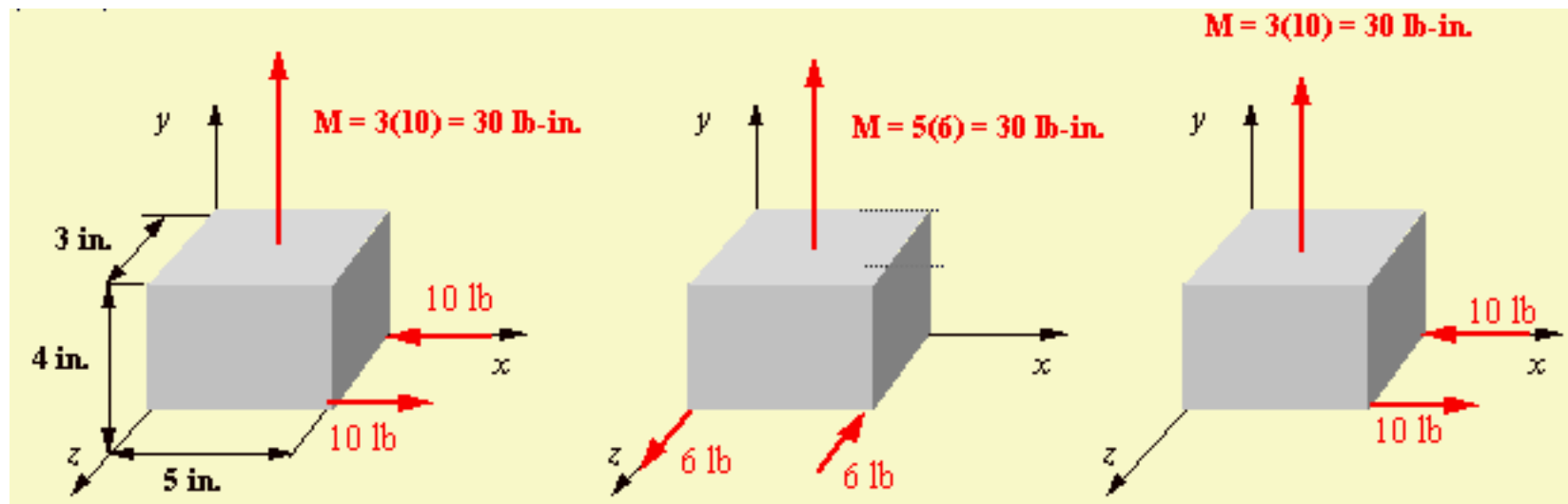
Couples as Vectors

Couples are vectors and can be added. Consider the two couples F_1 and F_2 in the two planes.

The moment vector of the couple is

$$M = r \times R = r \times (F_1 + F_2)$$

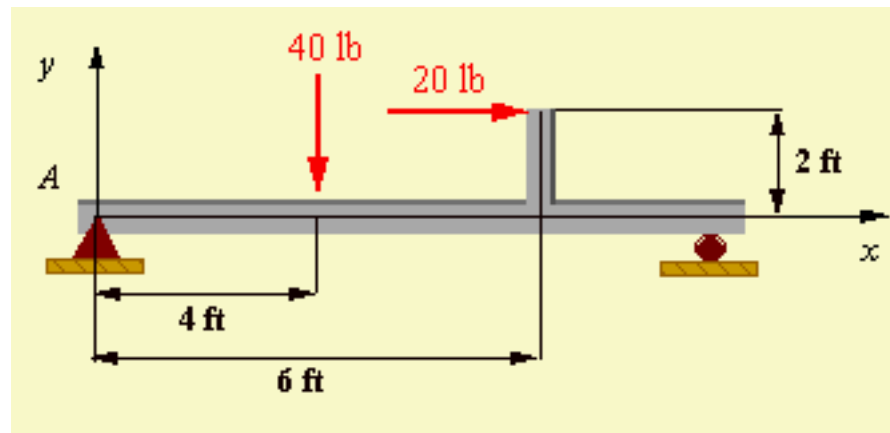$$= r \times F_1 + r \times F_2$$

$$= M_1 + M_2$$

The conclusion to be drawn from this is that moments resulting from two (or more) couples can be expressed as the vector sum of the individual moments.

$$M = r \times R$$

$$M_2 = r \times F_2$$

$$M_1 = r \times F_1$$
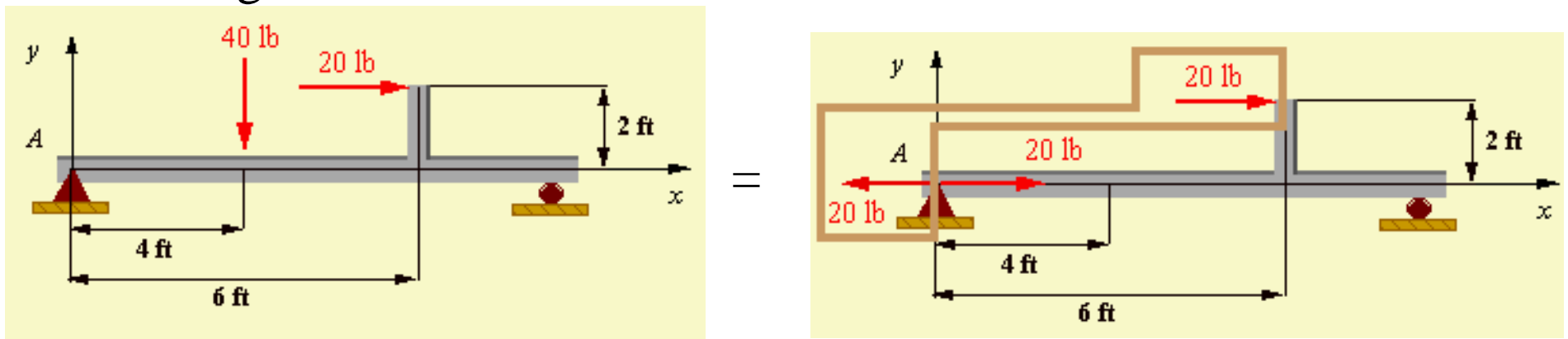
Note: these moments are vectors.

Couples-Example

Consider the two 40 lb and 20 lb forces. Replace the resultant moment by a force-couple system around around A.
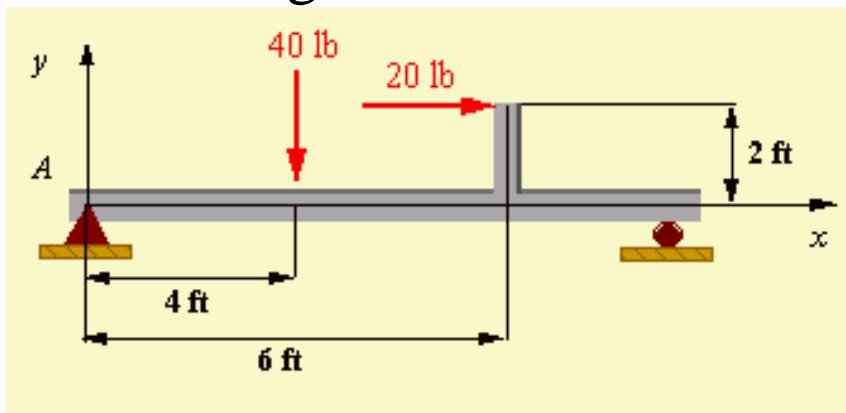
Couples-Example

Let's concentrate on the 20 lb. Force. We can add a positive and negative 20 lb. Force at A.
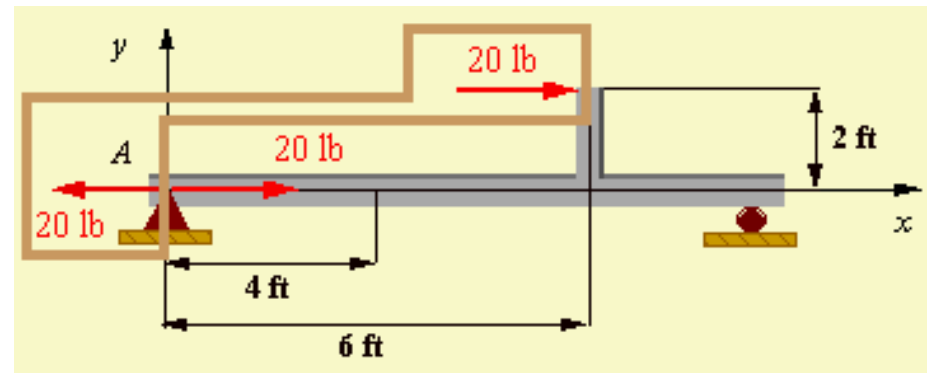

This combination of forces is a couple. It produces a moment with a magnitude of $M = 20(2) = 40$ lb-ft.   Using the right-hand rule sign convention, it is a negative moment ($\mathbf{M} = -40\mathbf{k}$ lb-ft).

Couples-Example

Let's concentrate on the 20 lb. Force. We can add a positive and negative 20 lb. Force at A.

=>

This combination of forces is a couple. It produces a moment with a magnitude of $M = 20(2) = 40$ lb-ft.   Using the right-hand rule sign convention, it is a negative moment ($M = -40\mathbf{k}$ lb-ft).

Couples-Example

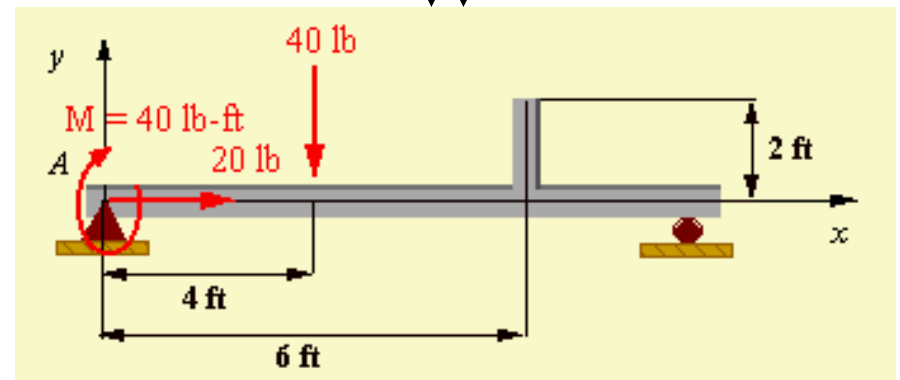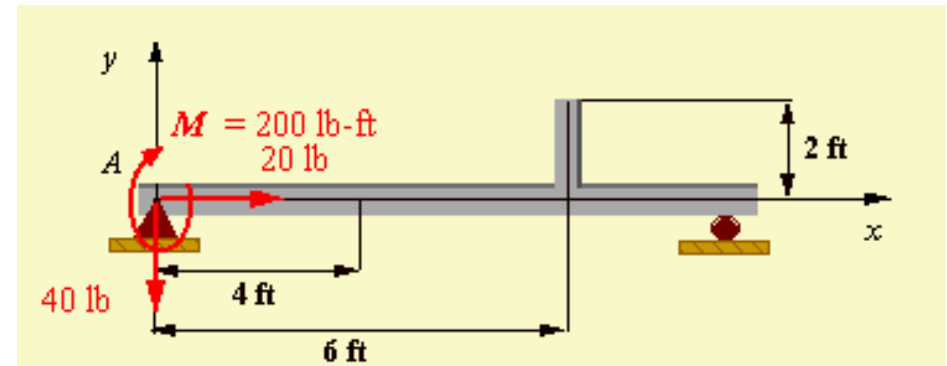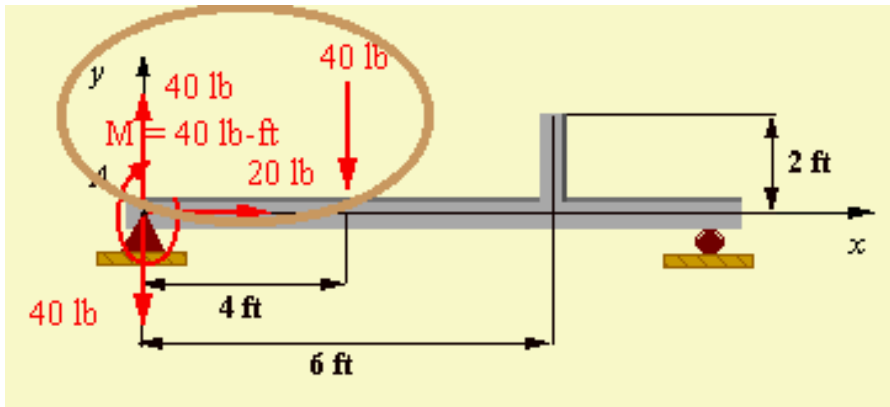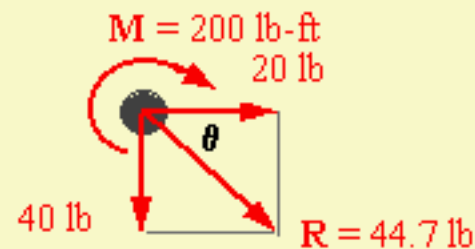Next, let's concentrate on the 40 lb. Force. We can do the same as
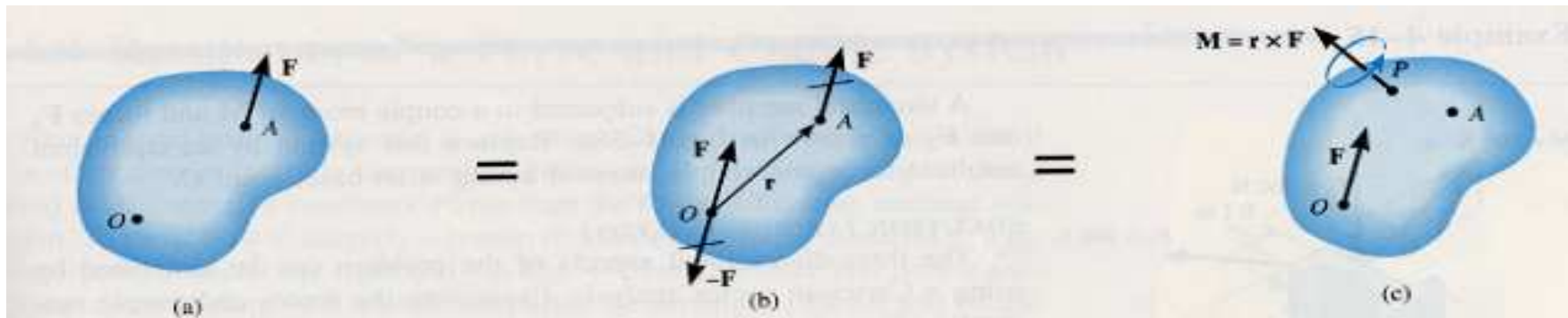What we did for the same case.

The moment formed by this couple is $M = 40 (4) = 160$ lb-ft. It is in the same direction as the moment caused by the 20-lb couple, and the two can be added to produce a single resultant moment ($M = -200\mathbf{k}$ lb-ft).

$\theta = \tan^{-1}(40/20) = 63.4^0$

Moving a Force on a Rigid Body

- Force **F** can be moved to point **O** without changing the external effect on the body by applying equal but opposite forces **F** and **-F** at point **O**.

- The two forces will introduce a moment **M** $= r \times F$

- **Q:** What will happen if point **O** is on the line of action of the force **F**?

Reducing Force and Moment System

- Resultant(Equivalent) Force:

$$F_R = \sum F$$

- Resultant Moment:

$$M_{Ro} = \sum M + \sum M_o$$

$$= \sum M + \sum \bar{r} \times \bar{F}$$