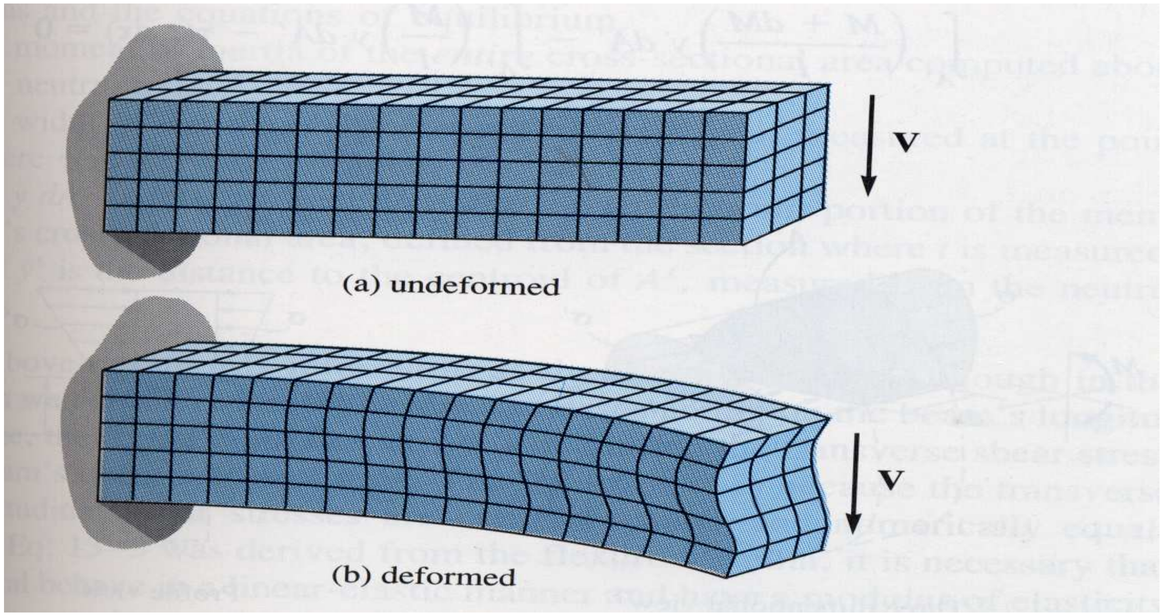
Mechanics & Materials 1

Chapter 13
Transverse Shear

FAMU-FSU College of Engineering
Department of Mechanical Engineering

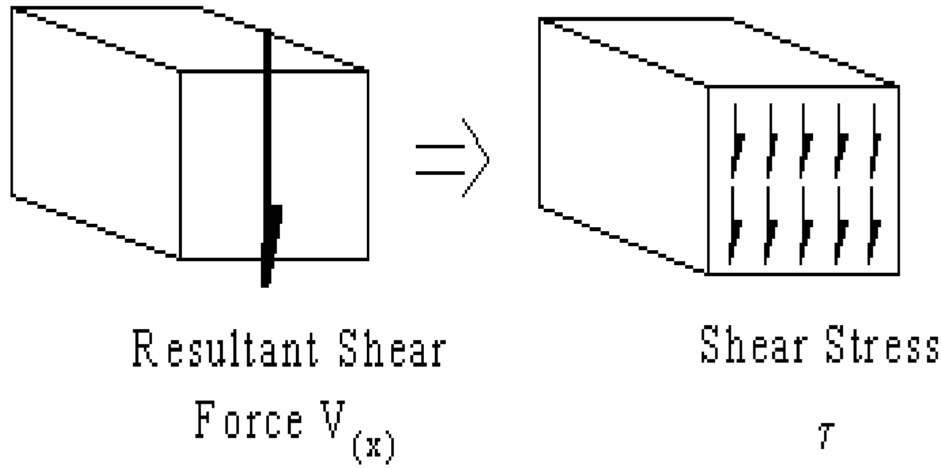
Transverse Shear



Transverse loads generate bending moments and shear forces.
Bending moments → bending stresses through the depth of the beam.
Shear forces → transverse shear-stresses distributed through the beam.

Transverse Shear

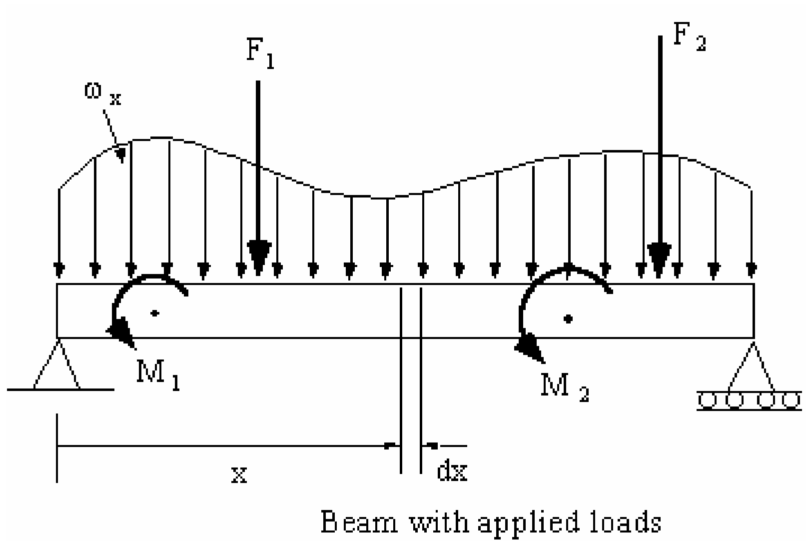
- Consider a typical beam section with a transverse load. The top and bottom surfaces of the beam carry no load, hence the shear stresses must be zero here.



Beam with transverse shear force showing the transverse shear stress generated by it.

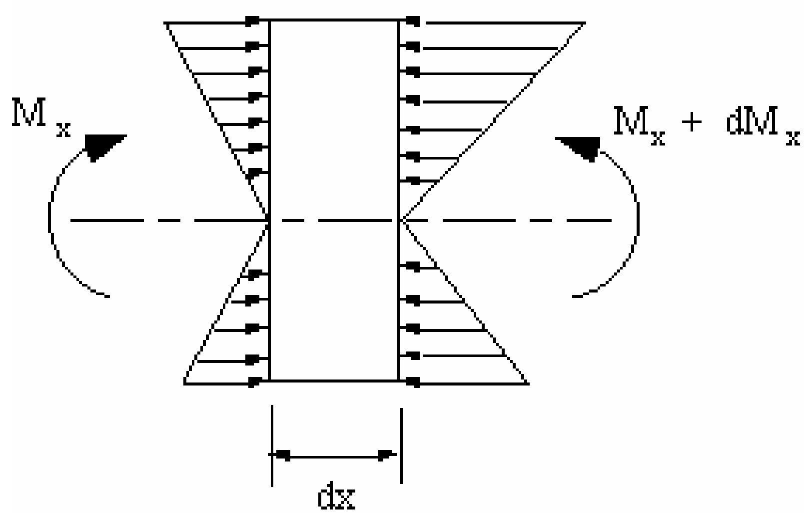
Shear Formula

- To determine the shear stress distribution consider a loaded beam:
- Consider a FBD of the element dx with the bending moment stress distribution only:



Shear Formula

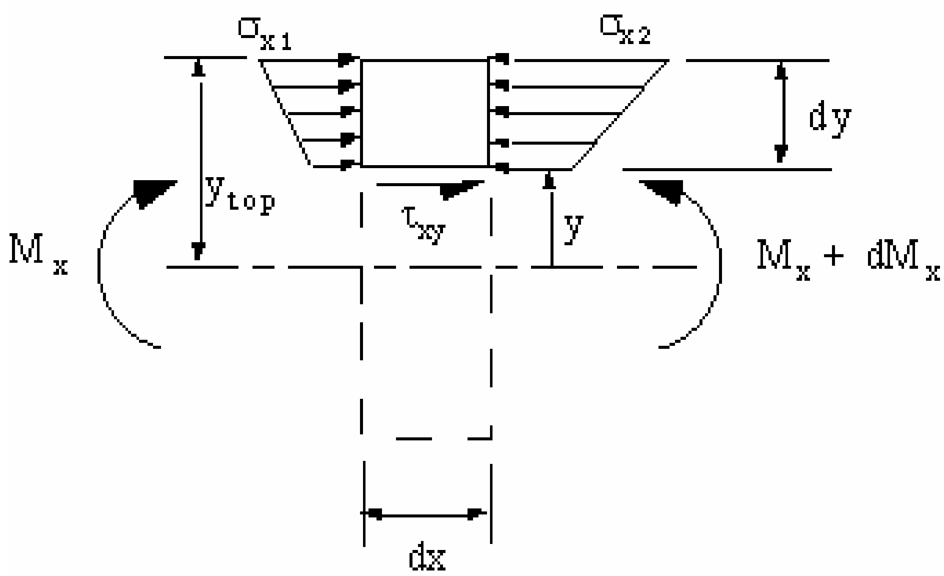
- Summing the forces horizontally on this element, the stresses due to the bending moments only form a couple, therefore the force resultant is equal to zero.



Length of beam dx with normal stress distribution due to bending moment

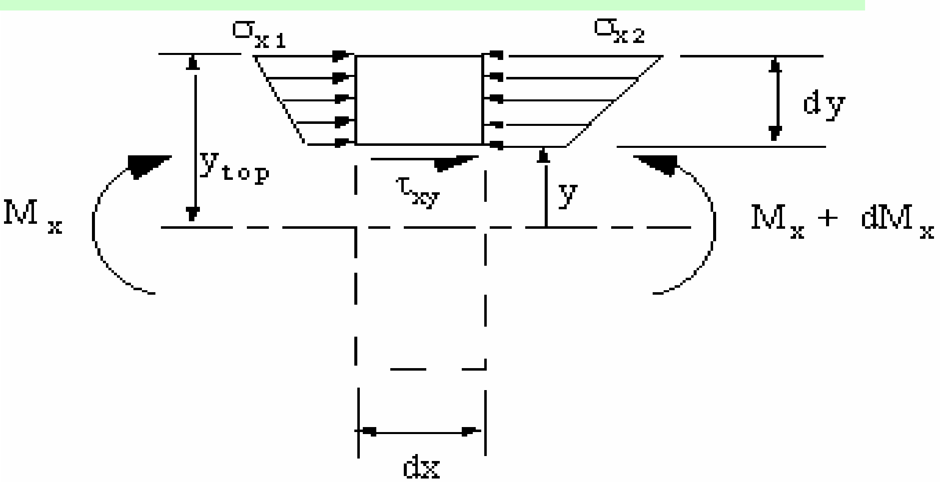
Shear Formula

- Consider now a segment of this element from a distance y above the Neutral Axis to the top of the element. For it to be in equilibrium,a shear stress τ_{xy} must be present.



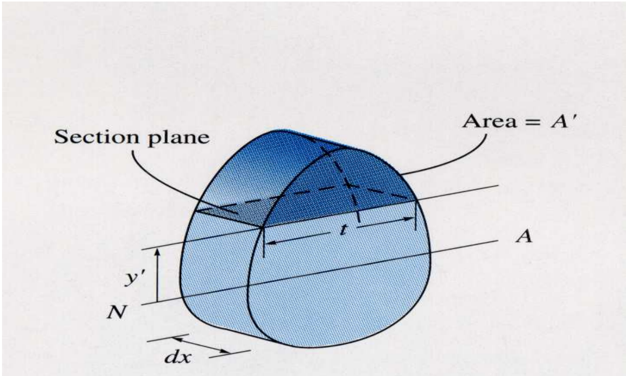
Segment of length dx cut a distance y from NA, with equilibrating shear stress τ_{xy}

Shear Formula



Segment of length dx cut a distance y from NA, with equilibrating shear stress τ_{xy}

- Let the width (into the page) of the section at a distance y from the NA be a function of y and call it 't'.



(b)

- Applying the horizontal equilibrium equation, gives:

$$\sum F_x = 0 \Rightarrow \int_y^{y_{top}} \sigma_{x1} t dy - \int_y^{y_{top}} \sigma_{x2} t dy + \tau_{xy} t dy = 0$$

Shear Formula

- Substituting for the magnitude of the stresses using flexure formula gives:

$$\tau_{xy} = \frac{dM_x}{dx}\frac{1}{It}\int_{y}^{y_{top}} ytdy$$

- Simplifying and dividing by dx and t gives:

$$\int_{y}^{y_{top}} \frac{M_x y}{I} tdy - \int_{y}^{y_{top}} \frac{(M_x + dM_x)y}{I} tdy + \tau_{xy} tdx = 0$$

- Considering the relation between shear and bending moment

$$V_x = \frac{dM_x}{dx}$$

$$\tau_{xy} = \frac{V_x}{It}\int_{y}^{y_{top}} ytdy$$

First Moment of Area, Q

- The integral $$\int_{y}^{y_{top}} y\,t\,dy = \int_A y\,dA$$
- Represents the first moment of area A about the Neutral axis.This quantity is termed as Q
- The centroid is given as $$\bar{y} = \frac{\int y\,dA}{A}$$
- So the first Moment of Area is given as

$$Q = \int_A y\,dA = \bar{y}A$$

- The units for Q are $(\text{length})^3$; m^3, in^3, etc..

Transverse Shear Formula

$$\tau = \frac{VQ}{It}$$

τ: the shear stress in the member at a point located y from N.A.

V: the internal Resultant shear force (from equilibrium and shear diagram)

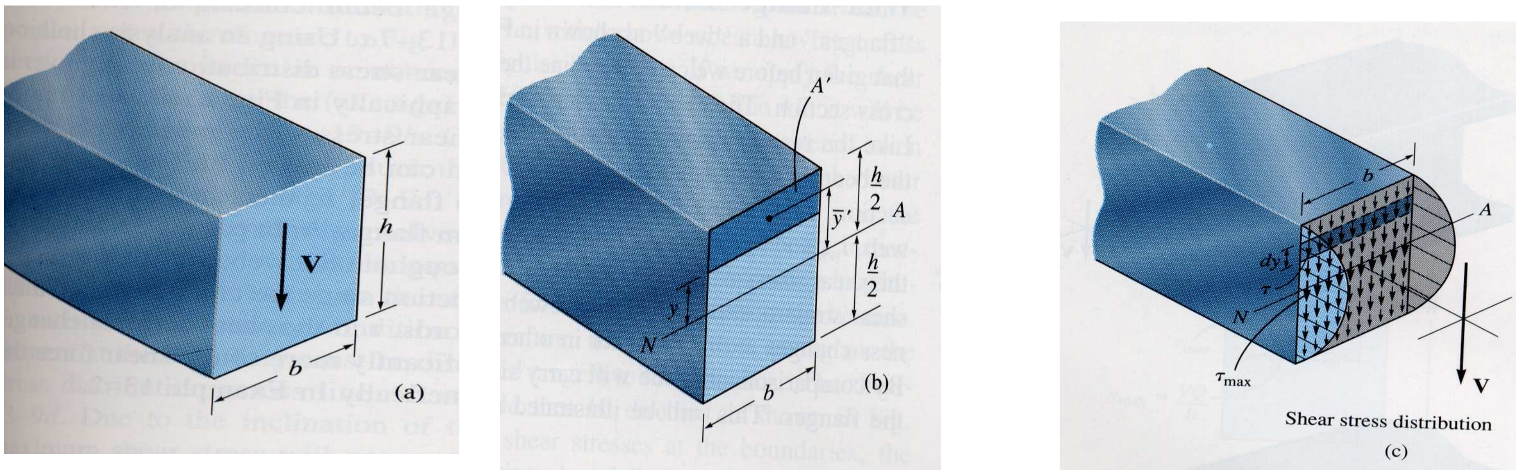
I: moment of Inertia of the entire cross section about the N.A

t: the width of the member's cross sectional area, measured at the point where the shear is to be determined

$$Q = \int_A y\,dA = \bar{y}A$$

- Where A is the area of the portion of the member's cross section from the section where t is measured to either the top or the bottom and $\bar{y}$ is the distance to the centroid of A, measured from N.A

Shear Stress in Beams: Rectangular Cross Section



- For beams with rectangular cross section

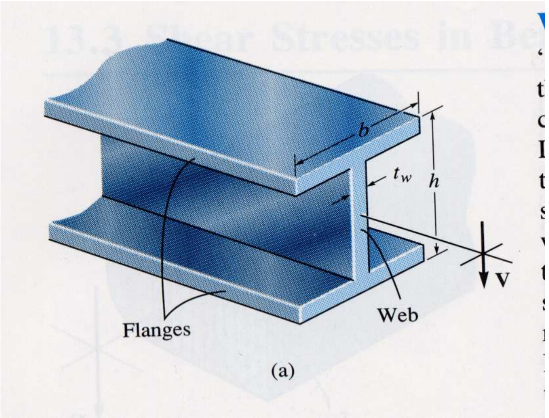
$$\tau = \frac{VQ}{It} = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

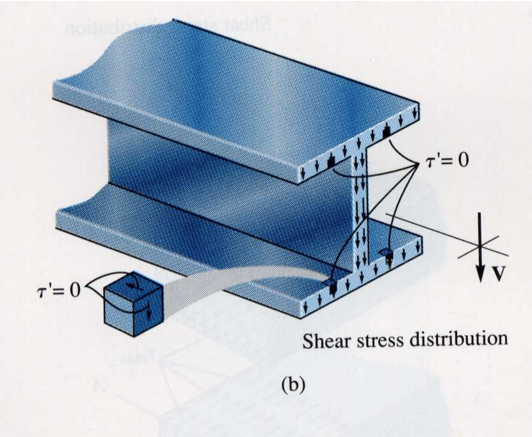
y measured from N.A

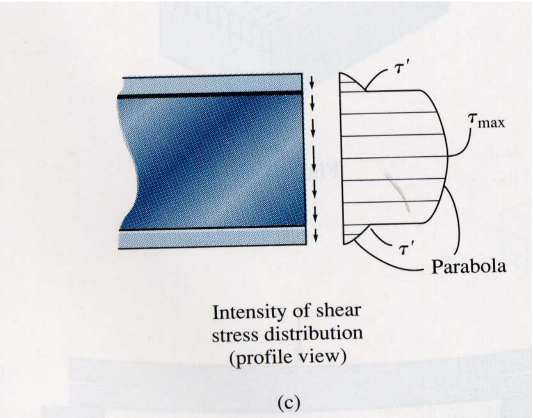
$$\tau_{\max} @ y = 0 \Rightarrow \tau_{\max} = 1.5\frac{V}{h}$$

$$\tau = 0 @ y_{top} \,\&\, y_{bottom} \xrightarrow{because} Q = 0$$

Shear Stress In Beams: Wide Flange Beam



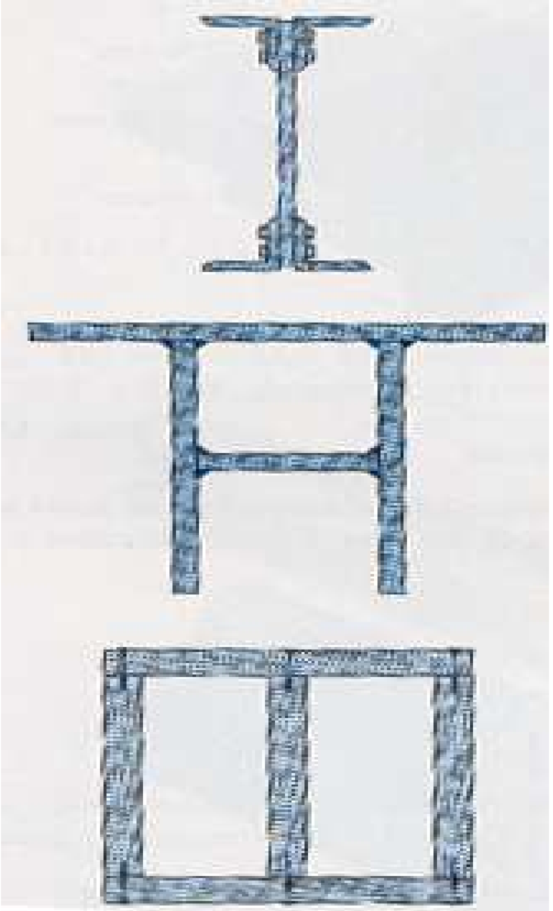




- The wide –flange beam consists of two flanges and a web.
- We can use the same analysis as before, but the shear distribution encounters a jump because of the cross sectional area thickness (t) change at the point where the web and the flange are connected
- So when the cross section is short and flat, or the cross section suddenly changes, then the shear formula shouldn't be applied. Instead, an integration scheme needs to be applied

Shear Flow in Built-up Members: q

- Members usually built up from several parts.
- These parts are fastened together by bolts, welding,or glue.
- These fasteners resist shear force along the members length.
- This loading per unit length is called shear flow, **q**



Shear Flow in Built-up Members: q

- Considering the shear flow along the juncture where the composite part is connected to the flange.
- As in the shear formula

$$dF = \frac{dM}{I}\int_{A'} y\,dA'$$

- The integral is the first moment of area,

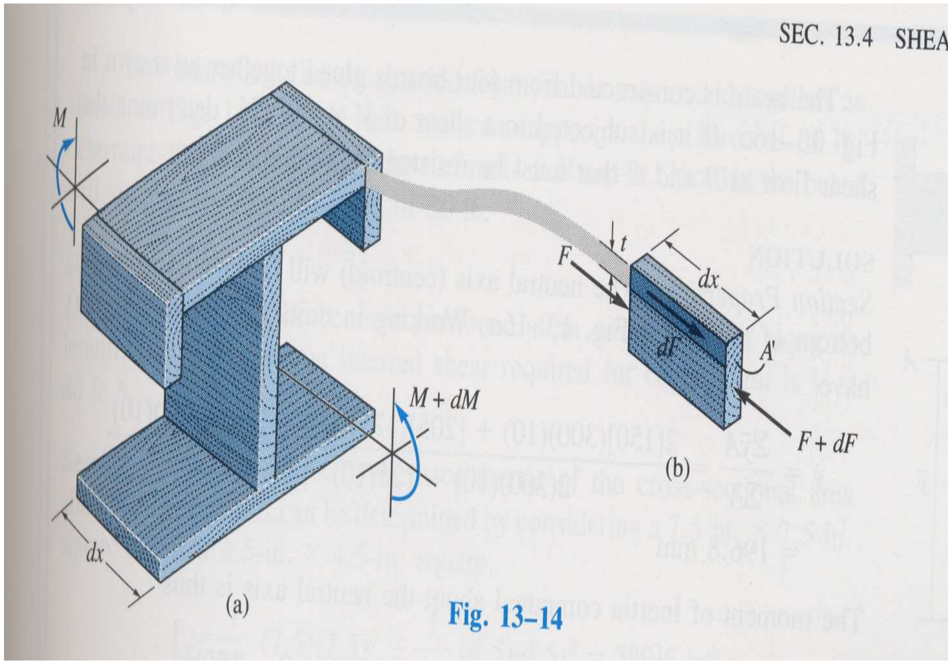


Fig. 13–14

Shear Flow in Built-up Members: q

$$q = \frac{dF}{dx} = \frac{dM}{dxI}\int_{A'} y\,dA'$$

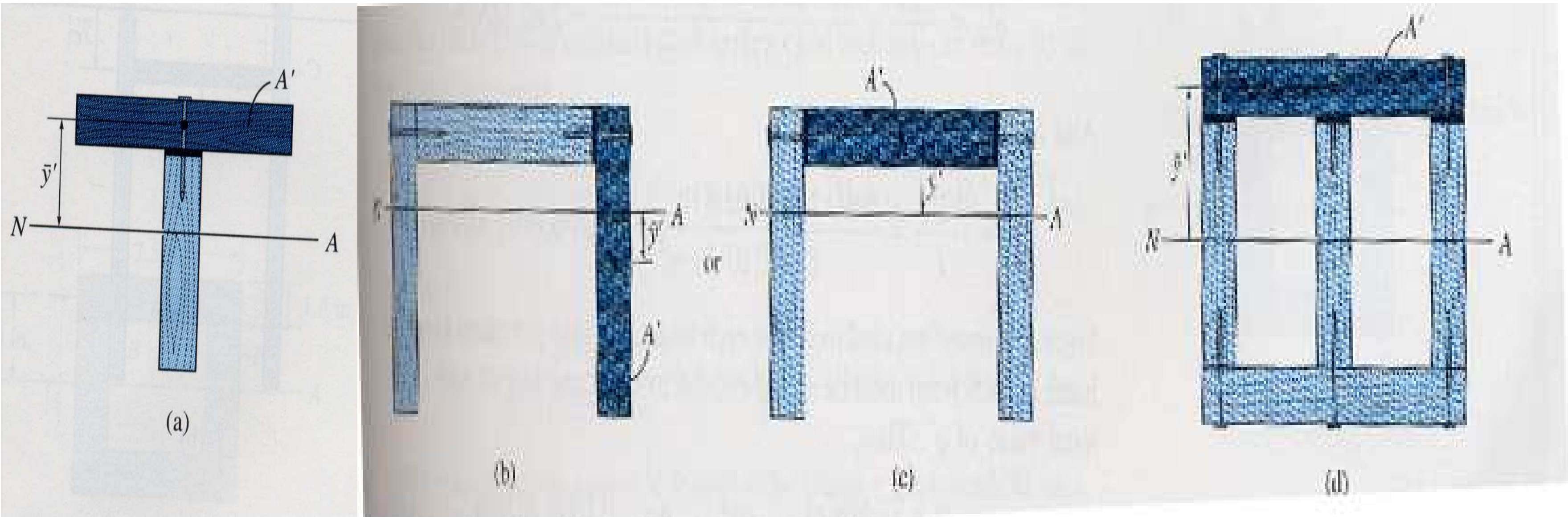
$$but\ \frac{dM}{dx} = V\ \&\ \int_{A'} y\,dA' = Q$$

Hence,

$$q = \frac{VQ}{I}$$

The definitions of V, Q, and I are exactly the same as in the shear formula

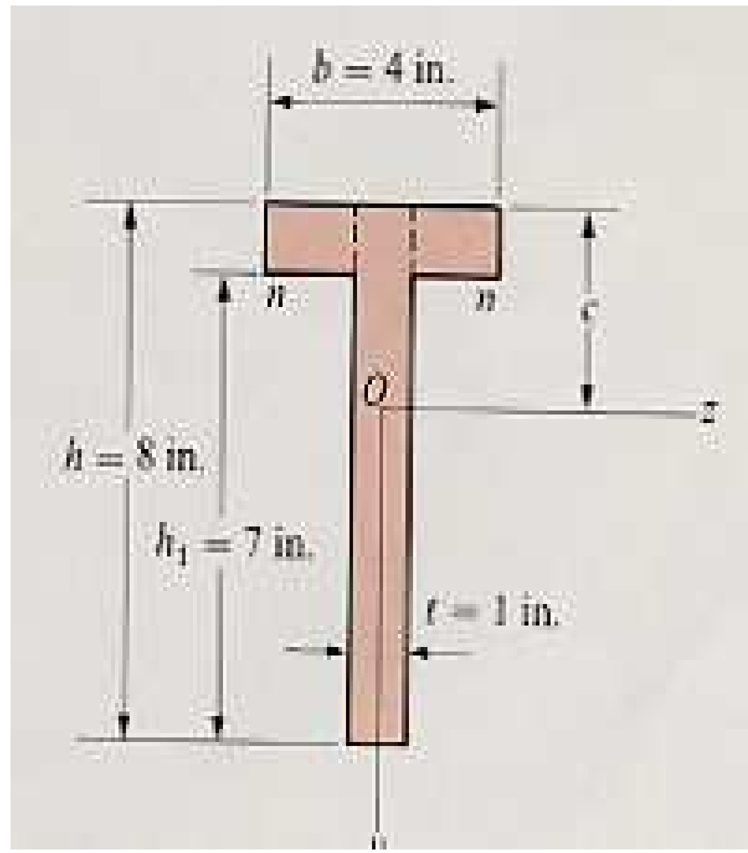
Shear Flow in Multiple Members



- The shear flow might be carried by a single, double or triple fasteners, so after we calculate q we divide the value by the number of fasteners.

Example

- Determine the maximum shear stress in the web of a T-shaped cross-section shown if b=4in., t=1in., h=8in., and V = 10000lb.



Solution

- Find the location of the N.A

$$c = \frac{(3in.)(1in.)(0.5in.) + (8in.)(1in.)(4in.)}{(3in.)(1in.) + (8in.)(1in.)}$$
$$= \frac{33.5in^3}{11.0in^2} = 3.045in.$$

- Moment of inertia I:

$$I = \frac{1}{3}(4in.)(1in.)^3 + \frac{1}{3}(1in.)(7in.)^3 - (11.0in.^2)(2.045in.)^2 = 69.66in^4$$

Find the first moment of area

Then use the shear formula to

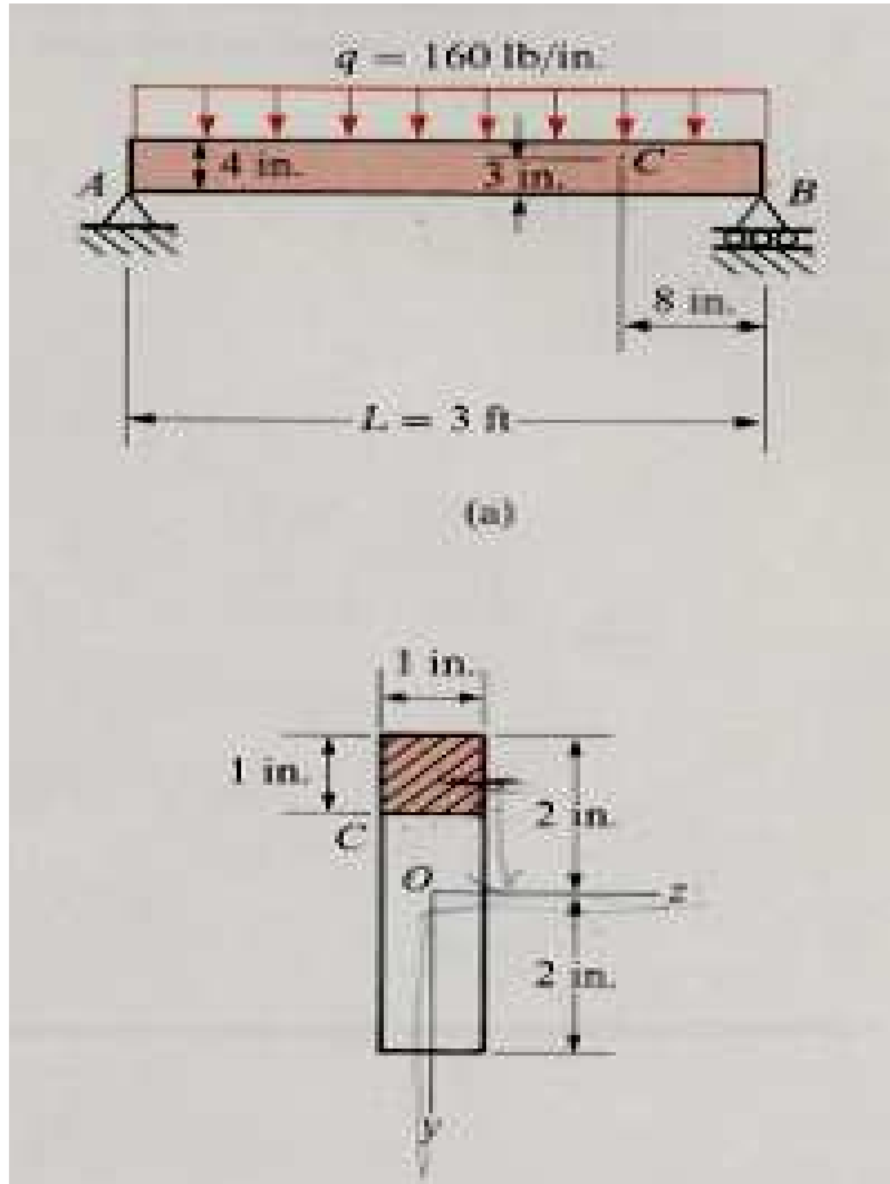
find the shear stress

$$Q = (1in.)(4.955in)^2\left(\frac{1}{2}\right) = 12.28in^3$$

$$\tau = \frac{VQ}{It} = \frac{(10000lb)(12.28in.^3)}{(69.66in.^4)(1in.)} = 1760\,psi$$

Example

- Calculate the normal and shear stresses acting at point C in the steel beam AB shown.



Solution

•From static equilibrium:

$$M = 17920\text{in.-lb}$$
$$V = \text{-}1600\text{lb}$$

•Inertia of the cross section:

$$I = \frac{bh^3}{12} = \frac{1}{12}(1.0in.)(4.0in.)^3 = 5.333in.^4$$

•Bending stress at point C;

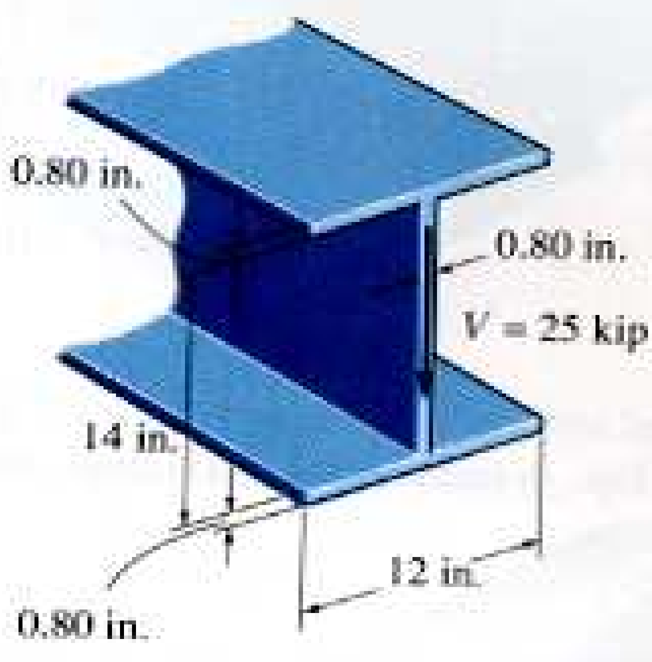
$$\sigma_x = \frac{My}{I} = \frac{(17920)(1.0in.)}{5.333in.} = -3360\,psi$$

Find the first moment of area
Then use the shear formula to
find the shear stress

$$Q = (1in.)(1in.)(1.5in.) = 1.5in^3$$

$$\tau = \frac{VQ}{Ib} = \frac{(1600lb)(1.5in^3)}{(5.333in^4)(1.0in)} = 450\,psi$$

Problem



Prob. 13–3

- If the wide-flange beam is subjected to a shear of V=25 kip, determine:
- A) the max shear stress in the beam
- B) the average shear stress in the web

Solution

$$I = \frac{1}{12}(6)(15.6)^3 - \frac{1}{12}(11.2)(14^3)$$

$$= 1235.35 in^4$$

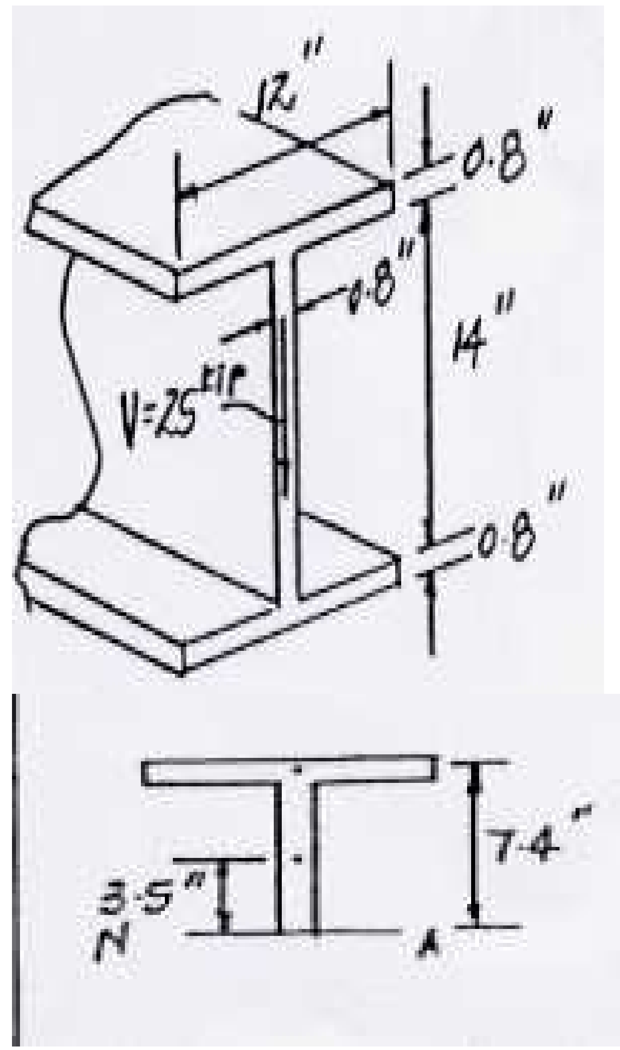
$$Q_{max} = \tilde{y}'A' = 7.4*(0.8)*12 + 3.5(7)(0.8)$$

$$= 90.64 in^3$$

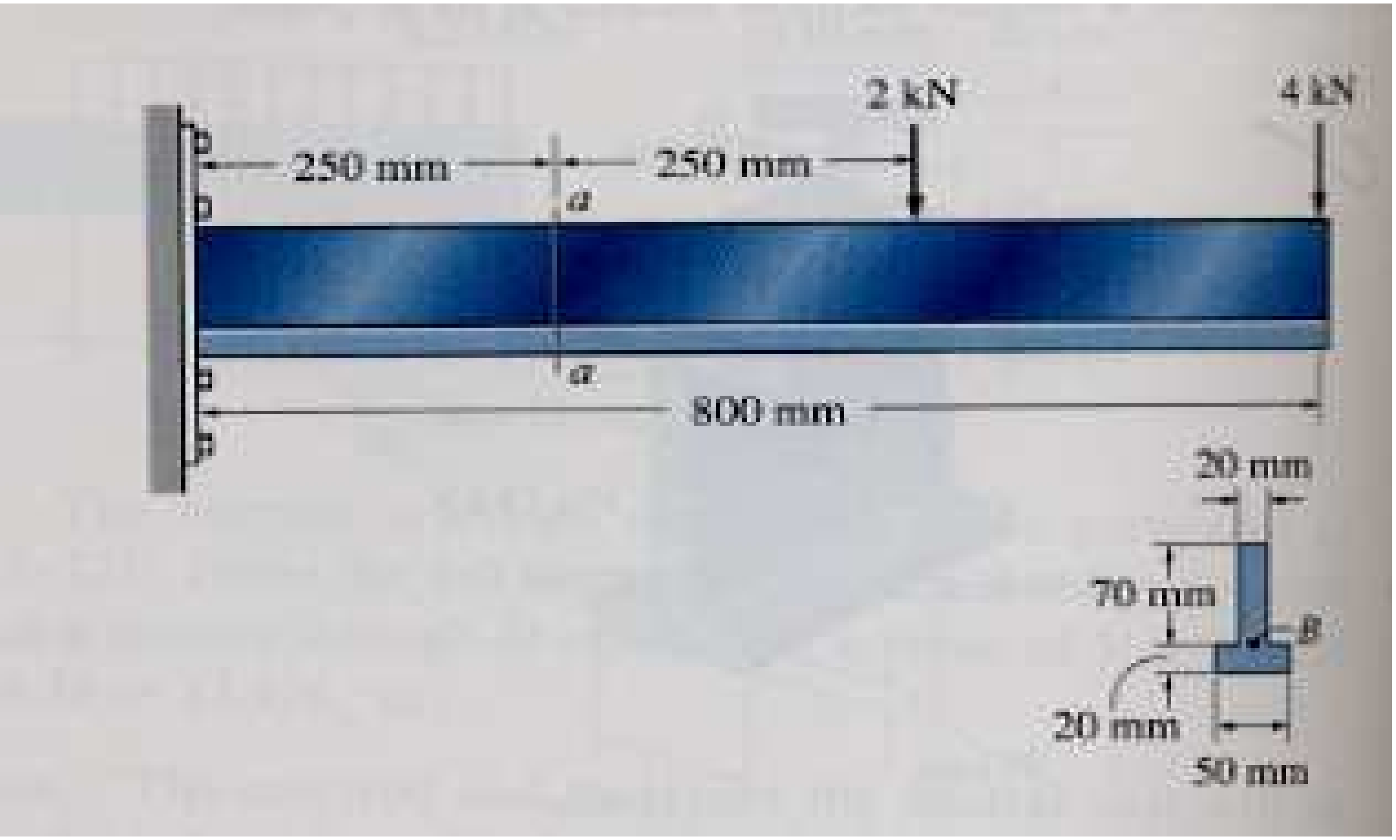
$$\tau_{max} = \frac{VQ_{max}}{It}$$

$$= \frac{25(90.64)}{1235.35(0.8)} = 2.29 Ksi$$

$$b)\tau_{avg} = \frac{V}{td} = \frac{25}{0.8(15.6)} = 2.00 Ksi$$



Problem



- Determine the maximum shear stress acting at section a-a in the beam.

Solution

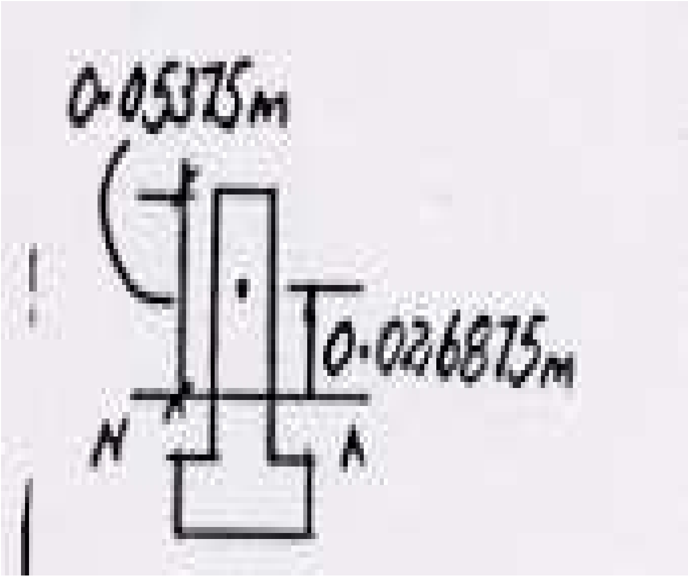
$$\bar{y} = \frac{(.05)(.02)(.01) + (.07)(.02)(.055)}{(.05)(.02) + (.07)(.02)}$$

$$= .03625m$$

$$I_{NA} = \frac{1}{12}(.05)(.02)^3 + 0.05(.02)(.02625)^2$$

$$+ \frac{1}{12}(.02)(.07)^3 + .02(.07)(.01875)^2$$

$$= 1.786 * 10^{-6} m^4$$



Solution

$$I_{NA} = 1.78625(10^{-6})m^4$$

From shear diagram $\Rightarrow V = 6kN$

$$Q_{\max} = \tilde{y}'A' = (0.026875)(0.05375)(0.02)$$

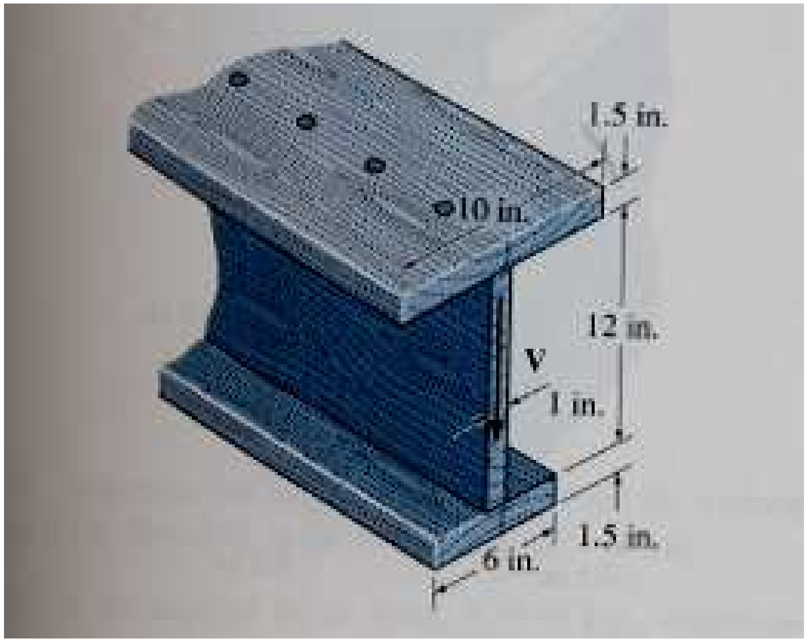
$$= 28.8906(10^{-6})m^3$$

$$\tau_{\max} = \frac{VQ_{\max}}{It}$$

$$= \frac{6(10^3)\left(28.8906\right)(10^{-6})}{1.78625(10^{-6})(0.02)}$$

$$= 4.85MPa$$

Problem 13



- The beam is constructed of three boards.
- Determine the maximum shear V that it can sustain if the allowable shear stress for the wood is 400psi.
- What is the spacing s of the nails if each nail can resist the shear force of 400lb.

Solution

From problem 13-22 I_{NA}=119.643 in^4

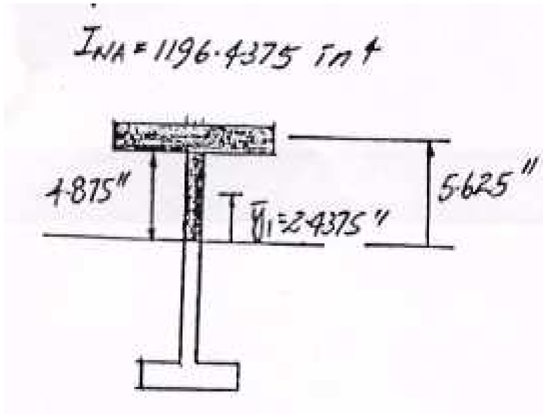
$$Q_{\max} = \sum \tilde{y}A = (5.625)(10)(1.5) + (2.4375)(4.875)(1)$$

$$= 96.258\, in^3$$

$$\tau_{\max} = \tau_{allow} = \frac{VQ_{\max}}{It}$$

$$0.4 = \frac{V(96.258)}{1196.4375(1)}$$

$$V = 4.97\, kip$$



From problem 7-30

$$Q_t = 84.375\, in^3$$

$$Q_b = 70.875\, in^3$$

$$q_t = \frac{4.9718\,(10^{-3})(84.375)}{1196.4375} = 350.62\, lb/in$$

$$q_b = \frac{4.9718\,(10^{-3})(70.875)}{1196.4375} = 294.52\, lb/in$$

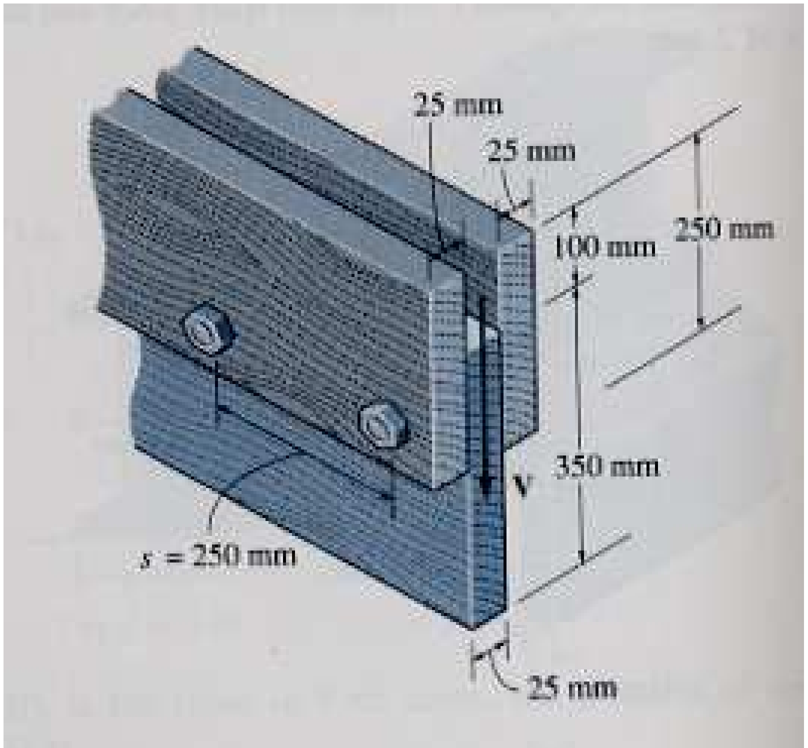
$$S = \frac{F}{q}$$

$$S_t = \frac{400}{350.62} = 1.14\, in$$

$$S_b = \frac{400}{294.52} = 1.36\, in$$

Problem 13.28

- A beam is constructed from three boards bolted together as shown.
- Determine the shear force developed in each bolt if the bolts are spaced s = 250mm apart and the applied shear V= 35kN.



Prob. 13–28

Solution

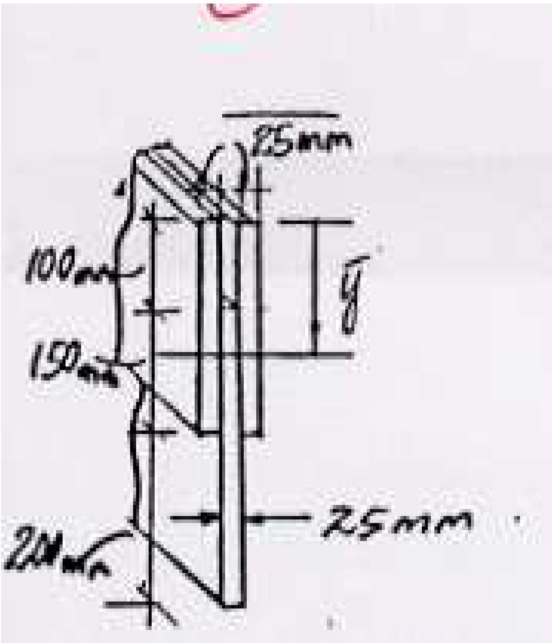
$$\bar{y} = \frac{2(0.125)(0.25)(0.025) + (0.275)(0.35)(0.025)}{2(0.25)(0.025) + 0.35(0.025)}$$

$$= 0.1876\,m$$

$$I_{NA} = 2\left(\frac{1}{12}\right)(0.025)(0.25^3) + 2(0.025)(0.25)(0.06176^2)$$

$$+ \frac{1}{12}(0.025)(0.35^3) + (0.025)(0.35)(0.08824^2)$$

$$= 0.270236(10^{-3})m^4$$



$$Q = \bar{y}'A' = 0.067176\ (0.025)(0.25) = 0.386(10^{-3})m^3$$

$$q = \frac{VQ}{I} = \frac{35(0.386)(10^{-3})}{0.270236\ (10^{-3})}$$

$$= 49.993\ kN/m$$

$$F = q(s) = 49.993(0.25)$$

$$= 12.5\,kN$$