Example Project Description Question

Question:

Describe the project in which you plotted streamlines around an airfoil using a complex velocity potential. Include

1. Equations solved. Physical meaning of the equations.
2. Boundary conditions, if any. Physical meaning of the boundary conditions.
3. Results obtained in the project. What these results mean physically.

Ideal student answer

1. (a) *Equations solved*
   - Inviscid potential flow $\omega = \nabla \times \vec{v} = 0$, $\vec{v} = grad\phi$.
   - Incompressible, ρ constant, $\nabla \cdot \vec{v} = 0$, $u = \psi_y, v = -\psi_x$.
   - Written as a complex streamfunction $F = \phi + i\psi$.
   - Conformal mapping relating ideal flows in two coordinate systems: $\zeta = z - \epsilon + 1/(z - \epsilon)$.
   - $$F = U\left(z + \frac{r_0^2}{z}\right) - \mathrm{i}V\left(z - \frac{r_0^2}{z}\right) + \mathrm{i}\frac{\Gamma}{2\pi}\ln\left(\frac{z}{r_0}\right)$$
   - The Kutta condition is $\Gamma = 4\pi V r_0$.
   - Streamlines are found as lines of constant ψ.

   (b) *Physical meaning of the equations solved*
   - Viscous effects are ignored.
   - The incoming air is free of vorticity.
   - Flows around circles in the z-plane are mapped to airfoils in the ζ-plane.
   - Small constant ϵ is a measure of the thickness of the airfoil. A flat plate airfoil is found for $\epsilon = 0$.
   - The airfoils are called Joukowski airfoils.
   - The Kutta condition prevents the potential flow going around the traling edge of the airfoil.
   - The Kutta condition ensures that the trailing edge point is a stagnation point in the z-plane.
   - Boundary layers will simply not go around a trailing edge.

2. (a) *Boundary conditions, if any*
   - On the surface of the airfoil: $\vec{v} \cdot \vec{n} = 0$, the airfoil is a streamline $\psi =$ constant.
   - At large distances, the horizontal component of the air velocity is the constant U, and the vertical component is V.

   (b) *Physical meaning of the boundary conditions*
   - There is no penetration of air into the surface of the airfoil, nor motion away from it that would leave a vacuum.
   - However, the flow will slip along the surface, in violation of the no-slip condition of viscous fluids.
   - As a result, there are no boundary layers.

- The far-field boundary condition can be thought of as making this the flow field of an airfoil moving through an atmosphere otherwise at rest (but seen by an observer moving along with the airfoil, not by one at rest compared to the far-field atmosphere.
- V/U is the angle of attack, the angle between the chord and the motion.

3. (a) *Results obtained*

    - The flow around a reasonable airfoil shape was obtained.
    - The airfoil trailing edge was a cusp, not a wedge.
    - The airfoil was symmetric.
    - When the circulation was not correct, the streamlines bend around the trailing edge.
    - When it was, the streamline left the trailing edge tangentially.
    - The effects of the airfoil on the streamlines die out quite quickly with distance.
    - The streamlines really bunch up near the leading edge.

   (b) *Physical meaning of the results*

    - Structurally, a Joukowski airfoil is not ideal.
    - There must be a suction peak near the leading edge.