

From Last Time

$$\delta Q = dE + \delta W$$

$$\delta Q = dU + d(PE) + d(KE) + \delta W$$

$${}_1Q_2 = U_2 - U_1 + mg(z_2 - z_1) + \frac{1}{2}m(v_2^2 - v_1^2) + {}_1W_2$$

and for systems where $\Delta PE = \Delta KE = 0$

$${}_1Q_2 = U_2 - U_1 + {}_1W_2 \quad \text{or} \quad {}_1Q_2 = \Delta U + {}_1W_2$$

Consider a constant pressure process

$$\begin{aligned} {}_1W_2 &= \int_1^2 P dV = P \int_1^2 dV \\ &= P(V_2 - V_1) = PV_2 - PV_1 \end{aligned}$$

$${}_1W_2 = P_2V_2 - P_1V_1$$

$${}_1Q_2 = \Delta U + {}_1W_2$$

$$= U_2 - U_1 + P_2V_2 - P_1V_1$$

$$= (U_2 + P_2V_2) - (U_1 + P_1V_1)$$

$$\boxed{\text{ENTHALPY} \quad H = U + PV} \quad \text{kJ, Btu}$$

$$\text{specific enthalpy } h \equiv \frac{H}{m} = \frac{U}{m} + \frac{PV}{m}$$

$$h = u + Pv$$

h is a thermodynamic state property (like P, T, v, x, u)
and for mixtures of sat liq & sat vap

$$h = h_f + x(h_g - h_f)$$

$$h = h_f + x h_{fg}$$

note: for constant-pressure quasi-equilibrium processes
where $\Delta PE = \Delta KE = 0$ ${}_1Q_2 = H_2 - H_1$

Changing gears...

Sometimes we are interested in the

Heat per unit mass required to raise the temperature of a substance 1 degree

assuming $\Delta PE = \Delta KE = 0$

$$\delta Q = dU + \delta W = dU + PdV$$

for a CONSTANT VOLUME PROCESS, $dV = 0$

$$\delta Q = dU + P dV \rightarrow 0$$

$$\left(\frac{\delta Q}{\delta T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$\frac{1}{m} \left(\frac{\delta Q}{\delta T}\right)_V = \frac{1}{m} \left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{\partial u}{\partial T}\right)_V$$

we call this $C_V \equiv \left(\frac{\partial u}{\partial T}\right)_V$ constant volume specific heat

for a CONSTANT PRESSURE PROCESS

$$\delta Q = dU + PdV = dH$$

$$\left(\frac{\delta Q}{\delta T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P$$

$$\frac{1}{m} \left(\frac{\delta Q}{\delta T}\right)_P = \frac{1}{m} \left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial h}{\partial T}\right)_P$$

we call this $C_P \equiv \left(\frac{\partial h}{\partial T}\right)_P$ constant pressure specific heat

Both C_V & C_P have units of $\frac{kJ}{kg \cdot K}$ or $\frac{Btu}{lb \cdot ^\circ R}$

for solids & liquids:

$$\frac{dv}{dP} \text{ is small (they are fairly incompressible)}$$

$$\text{so } C_p \approx C_v$$

Note: C_p & C_v are functions of Temperature

however, for small ΔT C_p & C_v may be considered to be constant with temperature

for solids & liquid for which you have no tables

$$du \approx dh \approx C_p dT$$

$$\int_1^2 du \approx \int_1^2 dh \approx \int_1^2 C_p dT$$

& if C_p is const with T

$$\Delta u \approx \Delta h \approx C_p (T_2 - T_1)$$

Values for C_p can be found in Table

A.3 solids

A.4 liquids

for ideal gases

$u = f(T)$ u is a function of Temperature only
 \therefore if T is constant (isothermal process),
 u is constant

$$\left(\frac{\partial u}{\partial T}\right)_v \Rightarrow \left(\frac{du}{dT}\right)_v = C_{v0}$$

↑ 0 indicates ideal gas

$$du = C_{v0} dT$$

$$dU = mC_{v0} dT$$

remember, $Pv = RT$

so, $h = u + Pv$

$$h = u + RT$$

and h is a function of T only, also

$$\left(\frac{\partial h}{\partial T}\right)_p \Rightarrow \left(\frac{dh}{dT}\right)_p = C_{p0}$$

$$dh = C_{p0} dT$$

$$dH = mC_{p0} dT$$

$$h = u + Pv = u + RT$$

$$dh = du + R dT$$

$$C_{p0} dT = C_{v0} dT + R dT$$

$$C_{p0} = C_{v0} + R$$

$$\overline{C}_{p0} = \overline{C}_{v0} + \overline{R}$$

where $\overline{C}_{p0} = C_{p0} \cdot M$

Note: C_{p0} varies with temperature see fig 5.11 pg 137

for ideal gases

C_{v0} & C_{p0} may vary with temperature

$$du = C_{v0} dT$$

$$dh = C_{p0} dT$$

To find ΔU (listed in order of increasing accuracy)

(I) Assume C_{v0} is constant w/ T

$$\Delta U = \int_1^2 C_{v0} dT = C_{v0} \int_1^2 dT$$

$$\Delta U = C_{v0} (T_2 - T_1) \quad C_{v0} \text{ in A.5}$$

(II) Get C_{v0} as a function of T & integrate

$$\Delta U = \int_1^2 C_{v0} dT$$

$$\left[\int_{T_1}^{T_2} C_{p0} dT \right] - R(T_2 - T_1)$$

C_{p0} as function of T in A.6

(III) Get actual values of u

$$\Delta U = u_{T_2} - u_{T_1}$$

u_T from A.7, A.8

likewise Δh can be found

(I) assuming constant C_{p0}

$$\Delta h = C_{p0} (T_2 - T_1) \quad \text{A.5}$$

(II) C_{p0} as function of T

$$\Delta h = \int_{T_1}^{T_2} C_{p0} dT$$

A.6

(III) actual h values

$$\Delta h = h_{T_2} - h_{T_1}$$

A.7, A.8, A.9

About Table A.6

gives C_{p0} as function of θ

$$\text{where } \theta = \frac{T}{1000}$$

$$\frac{d\theta}{dT} = \frac{1}{1000}$$

$$1000 d\theta = dT$$

So

$$\Delta h = \int_{T_1}^{T_2} C_{p0} dT$$
$$= \int_{\theta_1 = \frac{T_1}{1000}}^{\theta_2 = \frac{T_2}{1000}} C_{p0} 1000 d\theta$$

$$\Delta h = 1000 \int_{\theta_1}^{\theta_2} C_{p0} d\theta$$

$$\Delta u = \Delta h - R\Delta T$$

$$\Delta u = \left[1000 \int_{\theta_1}^{\theta_2} C_{p0} d\theta \right] - R(T_2 - T_1)$$