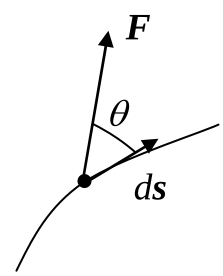
(- In class temperature, pressure, units, example.)

Lecture 3 Work, Energy, and Heat (Prof. G. Buzyna with modifications by J.C. Ordonez)

Work

Work is a mechanical concept given by the expression:



$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{s}$$

Note:

- F is a force and s is a displacement
- work is a scalar product
- $\vec{F} \cdot d\vec{s} = (F \cos\theta) ds$, where θ is the angle between the force and displacement vectors
- only force components along the displacement vector do work
- force components perpendicular to the displacement vector do no work

Mechanical Energy

We shall consider two components of mechanical energy, kinetic and potential energy. A relationship between kinetic energy and work is obtained from Newton's second law of motion.

In scalar form

$$\sum F = ma = m\frac{dV}{dt} = m\frac{dV}{ds}\frac{ds}{dt} = m\frac{dV}{ds}V$$

rewriting

$$\sum F ds = mVdV = \frac{1}{2} md(V^2)$$

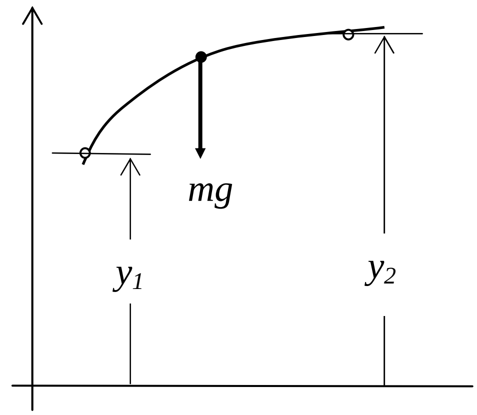
For a displacement from s_1 to s_2

$$\int_1^2 \vec{F} \cdot d\vec{s} = \int_1^2 \tfrac{1}{2} m d\left(V^2\right) = \tfrac{1}{2} m V_2^2 - \tfrac{1}{2} m V_1^2 = (KE)_2 - (KE)_1$$

Work = change in kinetic energy!

An expression for the gravitational potential energy is obtained from the work of a weight.

$$\int_1^2 \vec{F}\cdot d\vec{s} = \int_1^2 (-mg)dy = -mg(y_2 - y_1) = (PE)_1 - (PE)_2$$



Note that the negative sign inside the second integral indicates the opposite direction between the displacement and the force vectors. Hence work done to lift the weight is negative. Work done on the system is considered negative and by the system positive.

mgy = *PE* = gravitational potential energy

Work and energy

Return now to the work and kinetic energy expression derived above

$$\int_1^2 \vec{F}\cdot d\vec{s} = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

Let the force *F* in the above integral represent a sum of a gravitational force (F_g) and all forces other than gravitational, i.e. let $F = F_{other} + F_{g,}$, then

$$\int_1^2 \vec{F}_{other}\cdot d\vec{s} + \int_1^2 \vec{F}_g\cdot d\vec{s} = \int_1^2 \vec{F}\cdot d\vec{s} - (mgy_2 - mgy_1) = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

Rearranging:

$$\int_1^2 \vec{F}_{other}\cdot d\vec{s} = \left(\tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2\right) + (mgy_2 - mgy_1)$$

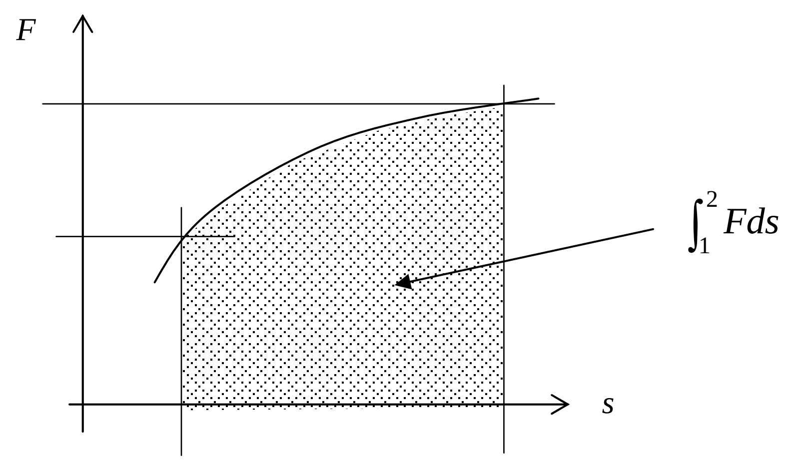
Work due to external forces other than gravitational forces = *Change in kinetic energy* + *Change in potential energy*

Some further notes of Work

The value of the integral

$$W_{12} = \int_1^2 F ds$$

depends on the path taken between the initial and final points. It's value is represented by the area under the curve representing the path, i.e. $F = f(s)$.



Only for special situations can we write the work as the difference between two quantities associated with the final and initial state. Such is the case when the force represents the weight, $F = mg$. In that case,

$$W_{12} = (PE)_2 - (PE)_1$$

We are careful to indicate variables that are path dependent by writing two subscripts, such as W_{12}. **Note there is no such thing as W_1 of W_2, only W_{12}.**

Work is defined only for a process and is thus not a state variable (not a property)

$KE = ½\, mV^2$ and $PE = mgz$ are state variables and therefore can be defined for a given state. Kinetic and potential energies are state properties.

Work is not a state variable, it is not a property, it is an energy transfer process. It represents energy transfer by mechanical means.

The process must be specified to evaluate work.

To distinguish between state and process variables, we denote their differential values by different symbols:

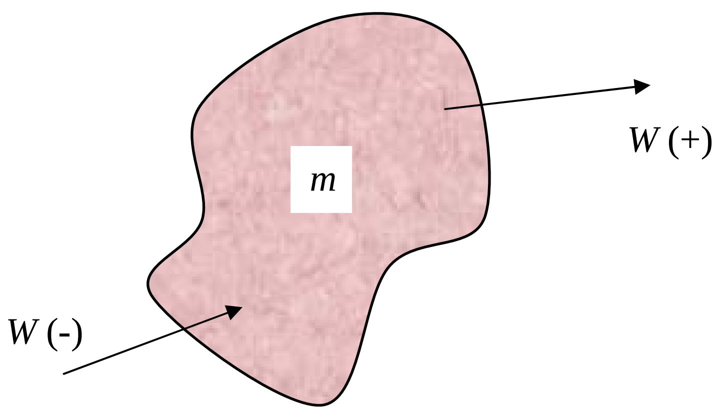
- a state variable (property) is designated by dX, where X is some property, and d represent an "exact" differential
- a process variable is designated by δW, where W represents a function dependent on the process, W is a function of s, and δ represent an "inexact" differential

$\int_1^2 \delta W = W_{12} \neq W_2 \text{-} W_1 \quad \rightarrow$ must define process $\int_1^2$

$\int_1^2 dX = X_2 - X_1 \quad \rightarrow$ X is a property, a state variable

Sign convention for work:

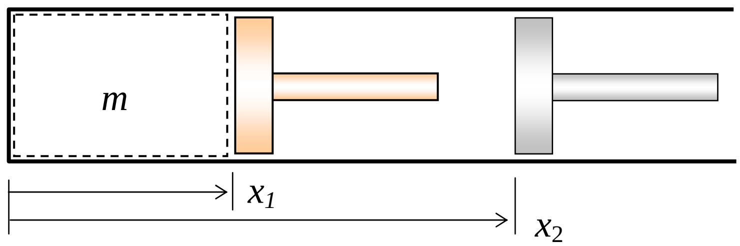
- $W_{12} > 0$, work done <u>by</u> the system is positive
- $W_{12} < 0$, work done <u>on</u> the system is negative



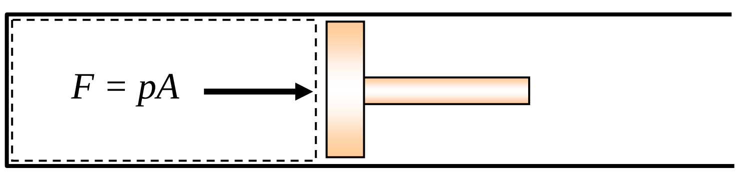
Evaluation of energy transfer by work

Work by expansion

Consider work by expansion in a piston cylinder device.



Consider a fluid mass m going through an expansion process from x_1 to x_2. The system is identified as the fluid enclosed by the dashed region inside the cylinder.



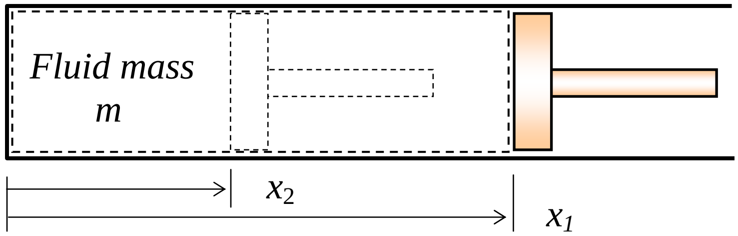
The fluid exerts a force on the piston and moves the piston in the positive x direction (in the same direction as the pressure force). The force is $F = pA$.

The work done by the fluid is

$$W_{12} = \int_1^2 F dx = \int_1^2 pA dx = \int_1^2 p dV$$

Note: for expansion $W_{12} > 0$, work done by the fluid (system).

Work by compression



$$W_{12} = \int_1^2 pdV < 0$$

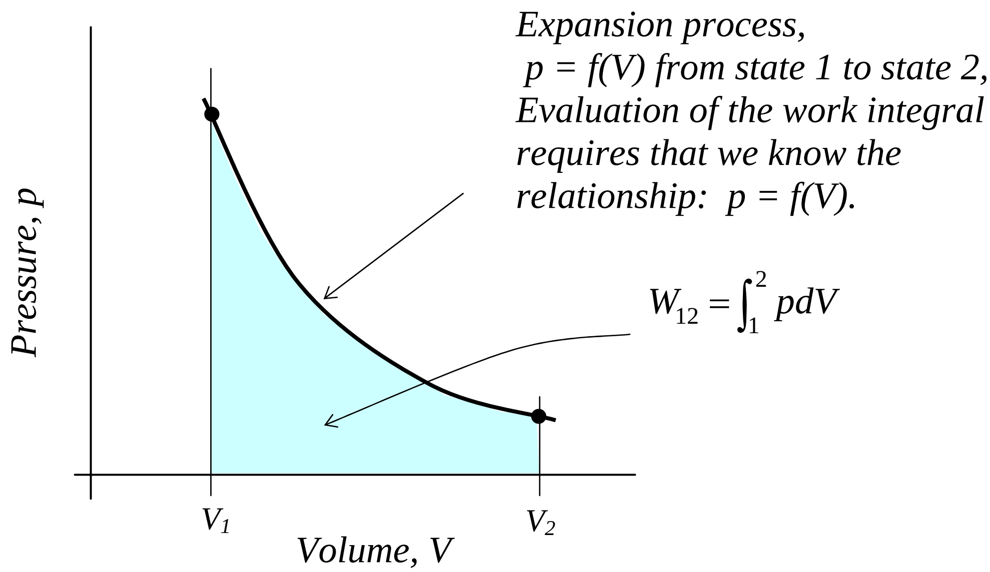
Note since $V_2 < V_1$, and $dV < 0$, the work represents work done <u>on</u> the system.

In the above piston expansion/compression examples, the mass of the system remained unchanged (constant) throughout the process. Such a system is called a

The integral expression:

$$W_{12} = \int_1^2 pdV$$

represents the area under the process curve $p = f(V)$.



Note that the evaluation of the work integral

$$W_{12} = \int_1^2 \delta W = \int_1^2 p dV$$

requires that we know $p = f(V)$.

Some common process:

1) CONSTANT VOLUME PROCESS, $V = constant$, $dV = 0$

2) CONSTANT PRESSURE PROCESS, $p = constant$

3) POLYTROPIC PROCESS, pV^n = constant, $n = constant \neq 1$

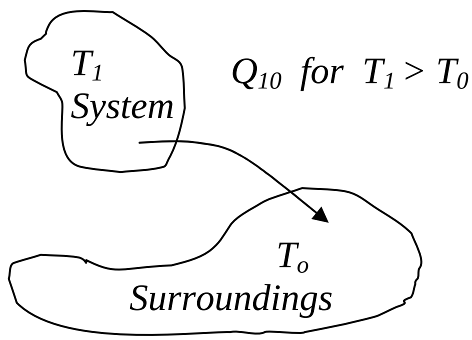
4) PROCESS WITH $pV = constant$

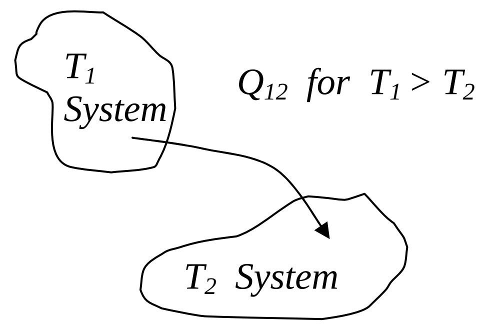
5) LINEAR RELATIONSHIP BETWEEN PRESSURE AND VOLUME

$$p = a + bV$$

HEAT

Heat is an energy interaction between the system and its surroundings (across the boundary) due to a temperature difference (between the system and its surroundings).

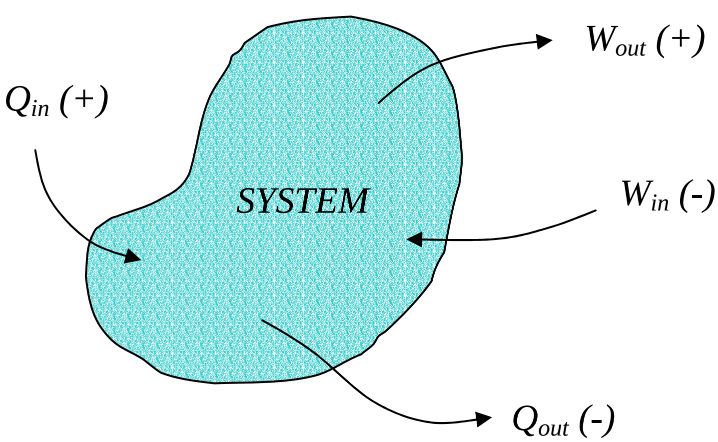




Note:

- A body does not contain heat
- Heat is defined only as it crosses the boundary
- Heat is a process variable – it depends on the particular process – it is a path function, it is not a property.
- $Q_{12} = \int_1^2 \delta Q$
- Units: <u>SI</u> *Jule = N m,* <u>U.S.</u> *Btu = 778 ft* lb_f

SIGN CONVENTION:



If no heat transfer, then

$Q_{12} = 0 \Downarrow$

ADIABATIC PROCESS