

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet, plus the operator sheet.

Fully discuss the following scheme

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \alpha \frac{u_{j+2}^n - 2u_{j+1}^n + u_j^n}{(\Delta x)^2}$$

for the heat equation $u_t = \alpha u_{xx}$. Include stencil, CFL-conditions, consistency, accuracy, Von Neumann analysis. Make sure the reasons for your claims are clear, especially for the Von Neumann analysis.

If we provide an arbitrary initial condition, will the computed solution converge to the exact solution of the PDE problem when the mesh is refined enough? Are there any conditions on how the mesh must be refined, or can we refine it in any way?

Notes: Don't worry too much about how the second last point is computed; just assume that $u_{j-1}^n = u_j^n$. Use enough mesh points to sketch the mesh, at least 10 in the x -direction.