EML 4930/5930 | **Analysis in M.E. II** | **02/21/07**
Closed book | *Van Dommelen* | *9:40-10:30 am*

Show all reasoning and intermediate results leading to your answer, or credit will be lost. One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator and one handwritten letter-size single formula sheet.

1. Consider the following curve given in terms of a parameter t:

$$x = e^t \cos t \qquad y = e^t \sin t \qquad z = e^t$$

   Find the unit tangential vector to the curve in terms of t and find its curvature. Also, if $\vec{B}$ is the unit vector normal to the osculating plane of the curve, find the *direction* of the rate of change of $\vec{B}$ with respect to the arclength along the curve.

2. Consider the following force field:

$$\vec{F} = \hat{\imath}(x^2 + 2xy) + \hat{\jmath}(x^2 + y^2) + \hat{k} f(x)$$

   Find the most general possible form of function $f(x)$ for which this force field is conservative. Derive the *most general* expression for the potential energy, fully specifying all dependencies.

3. Boundary layer coordinates u_1, u_2, and u_3 are defined by the following expression for the position vector $\vec{r} = (x, y, z)$:

$$\vec{r} = \vec{r}_0 + \hat{\imath}_2 u_2 + \hat{\imath}_3 u_3$$

   Here $\hat{\imath}_3$ is a constant vector, but $\vec{r}_0$ and $\hat{\imath}_2$ depend on u_1. In particular,

$$\frac{\mathrm{d}\vec{r}_0}{\mathrm{d}u_1} = \hat{\imath}_1 \qquad \frac{\mathrm{d}\hat{\imath}_2}{\mathrm{d}u_1} = \kappa \hat{\imath}_1$$

   where κ is a function of u_1. The vectors $\hat{\imath}_1$, $\hat{\imath}_2$ and $\hat{\imath}_3$ are orthogonal unit vectors.

   (a) Show that boundary layer coordinates are orthogonal curvilinear coordinates.

   (b) Derive the scale factors or metric indices.

   (c) Derive the derivatives of the unit vectors with respect to the coordinate directions. Note the given data, such as that $\hat{\imath}_3$ is a constant vector.