EML 5060 | **Analysis in Mechanical Engineering** | **09/25/20**

Closed book | *Van Dommelen* | *12:30-1:20 pm*

Solutions should be fully *derived* showing all intermediate results, using class procedures. *Show all reasoning.* Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. *You must state what result answers what part of the question.* Answer what is asked; you do not get any credit for making up your own questions and answering those. Ask if clarification of what is asked is needed. Use the stated procedures. Give exact, fully simplified, answers where possible.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

Write on only the front side of the page.

1. **Background:** Graphical depiction of a function is often an essential part to understand its properties.

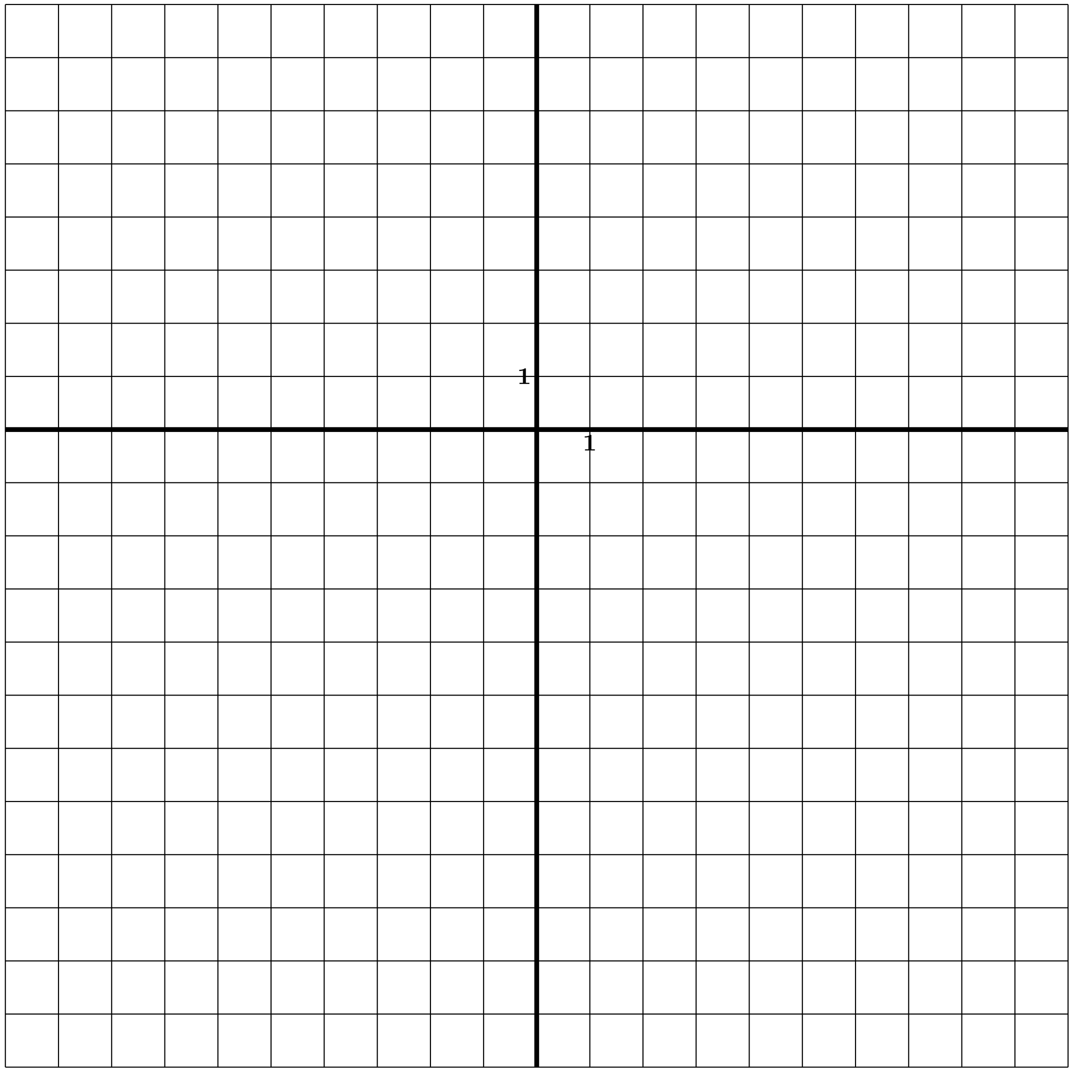
   Question: Analyze and very neatly graph

   $$y = \frac{x^2 - 2x + 1}{2x + 2} \qquad \Longrightarrow \qquad \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{8}{(x+1)^3}$$

   Discuss x and y intercepts and extents, asymptotic behavior for large positive x, horizontal, oblique and vertical asymptotes, local and global maxima and minima, concavity, inflection points, kinks, cusps, vertical slopes and other singularities.

   Draw the function very neatly and precisely, on suitably labelled axes, clearly showing all features.

   Hint: the quadratic in the top of the correct $\mathrm{d}y/\mathrm{d}x$ should have integer unequal roots.



2. **Background:** Sometimes you may need to evaluate a function at a position where the expression for it is singular. A limit then needs to be taken.

   Question: Derive
   $$\lim_{x\to 1}\left(\frac{1}{\ln x^2} - \frac{1}{x-1}\right)$$

3. **Background:** Centroids of plates are important for dynamics, hydrostatics, etcetera.

   Question: Consider the region

   $$\text{inside } x^2 + y^2 = 2x \qquad \text{and} \qquad \text{above } y = x$$

   For this region, we want the x position of the centroid, which requires doing the integral $\int x \,\mathrm{d}A$. This integral is to be done in the standard polar coordinates ρ, θ from the origin.

   First draw the region to integrate over in the x, y-plane twice. In the first graph, draw the lines of first integration if you do ρ first. In the second graph, draw the lines of first integration if you do θ first. Then for each case, write the complete two-dimensional integral to do with all limits of integration. Actually do the one that seems easiest to you.