EML 5060 **Analysis in Mechanical Engineering** **9/26/14**
Closed book *Van Dommelen* *12:30-1:20 pm*

Solutions should be fully *derived* showing all intermediate results, using class procedures. *Show all reasoning.* Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question. Answer exactly what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures. Give exact, fully simplified, answers where possible.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** Graphical depiction of a function is often an essential part to understand its properties.

   Question: Analyze and very neatly graph

   $$y = \frac{x^4}{x^3 - 8} \qquad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^3(x^3 - 32)}{(x^3 - 8)^2} \qquad \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{48x^2(x^3 + 16)}{(x^3 - 8)^3}$$

   Discuss x and y intercepts and extents, asymptotic behavior for large $|x|$, horizontal, oblique and vertical asymptotes, symmetries, local and global maxima and minima, concavity, inflection points, kinks, cusps, horizontal and vertical slopes and other singularities. Draw the function very neatly, clearly showing all features.

2. **Background:** Sometimes a formula can degenerate into the difference between two terms, each of which is ill-behaved by itself.

   Question: Find

   $$\lim_{x \to 0} \cot^4(x) - \csc^4(x)$$

   if it exists. Show all reasoning.

3. **Background:** Centroids of bodies are needed for applications such as dynamics, stability, center of pressure, etcetera.

   Question: Consider the body

   $$\text{above } z = 0 \qquad \text{below } z = y \qquad \text{inside } x^2 + y^2 = 2y$$

   taking the z axis to be upwards, the x axis towards you, and the y-axis to the right. Draw the body neatly in these axes. Now, *using cylindrical coordinates with the z-axis as their axis*, find the centroid integral

   $$\int z \, \mathrm{d}V$$

   Based in part on your picture, and the collapsed picture, give conclusive reasons why you want to do the integrations in a specific order. Find the integral. Hint: you will probably be able to find the final integral in the definite integrals section of your math handbook.