

Solutions should be fully *derived* showing all intermediate results, using class procedures. Show all reasoning. Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question if there is any ambiguity. Answer exactly what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures. Give exact, fully simplified, answers where possible.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** Graphical depiction of a function is often an essential part to understand its properties.

Question: Analyze and very neatly graph

$$y = \sqrt{\sqrt{x^4 - 16}}$$

Discuss x and y intercepts and extents, behavior for large $|x|$, horizontal, oblique and vertical asymptotes, symmetries, local and global maxima and minima, kinks, cusps, horizontal and vertical slopes and other singularities.

2. **Background:** Finding optimal solutions is what practical engineering is all about.

Question: Find the cheapest box to hold 100 candles. The length of the box should be the length of the candles, 0.5 ft. You can approximate 100 candles to occupy a volume of 0.2 ft³. The sides of the box cost \$0.30/ft², while the top and bottom cost \$0.50/ft². Find the width and height and cost of the cheapest box. Note: of course the height separates top and bottom.

3. **Background:** Multiple integrals are needed for material costs, dynamic moments, beam stiffnesses, etcetera in mechanical engineering.

Question: Find

$$\int_{\mathbf{R}} x e^y \, dx dy$$

if \mathbf{R} is the *first quadrant* region satisfying $y \leq x$ and $y \geq x^2 - 2$. Discuss also in some detail what is the best order of integration, and why.