EML 5060	**Analysis in Mechanical Engineering**	**12/11/07**
Closed book	*Van Dommelen*	*10:00-12:00 EDT*

Solutions should be fully *derived* showing all intermediate results, using class procedures. Show all reasoning. Bare answers are absolutely not acceptable, because I will assume they come from your calculator (or the math handbook, sometimes,) instead of from you. You must state what result answers what part of the question if there is any ambiguity. Answer what is asked; you do not get any credit for making up your own questions and answering those. Use the stated procedures.

One book of mathematical tables, such as Schaum's Mathematical Handbook, may be used, as well as a calculator, and a handwritten letter-size formula sheet.

1. **Background:** The following ODE arises in the analysis of hedging of variable annuities:

$$xy' - py = 1 \qquad x \geq 0 \qquad p > 0$$

Question:

(a) Derive the general solution of the homogeneous equation.

(b) Derive the general solution of the inhomogeneous equation.

(c) Very neatly draw an exhaustive set of solution curves, and then some more, of the inhomogeneous equation for the case that $p = \frac{1}{2}$. (Note that $x \geq 0$.)

(d) Find the solution when $p = \frac{1}{2}$ and $y(1) = 0$.

(e) Comment on the initial value problem in which $p = \frac{1}{2}$ and $y(0) = 0$.

2. **Background:** Vibrations of simple systems are governed by the ODE:

$$y'' + cy' + y = F(t)$$

where $F(t)$ is some given time-varying force acting on the system.

Question: Derive the solution of this equation if $c = 1$, $y(0) = 1$, and $y'(0) = 2$, by means of Laplace transformation. Work out completely; there may not be a $*$ or H in the answer. You must use the completion of the square method in this problem.

3. **Background:** The following nonlinear system describes a biased Van der Pol oscillator:

$$x' = y \qquad y' = -4(x^2 - 2x)y - x + 1$$

Question:

(a) Explain why we can learn information about the solutions of this system by studying the following system:

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

In particular, identify x_1 and x_2.

(b) Write the general solution for x_1 and x_2.

(c) Very neatly and quantitatively correct, draw an exhaustive set of solution curves in the x_1, x_2-plane. Make sure all qualitative features are clearly visible.

(d) Classify the point precisely.

(e) Explain whether or not these solution curves are relevant to the solution curves of the original problem. If they are, in what sense they are relevant. If the relevance is not quite certain, indicate so.

(f) What can be said about the long-time behavior of typical solutions of the original system based on the obtained results?