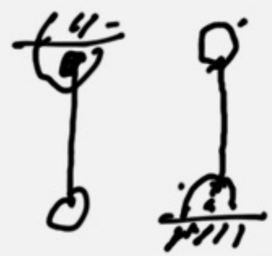
iam at 5060

**Pendulum:** $\ddot{\vartheta} + 2c\dot{\vartheta} + \sin\vartheta = 0$  $y_1 = \vartheta$, $y_2 = \dot{\vartheta}$

$$\frac{d}{dt}\begin{pmatrix}\vartheta \\ \dot{\vartheta}\end{pmatrix} = \vec{F} = \begin{pmatrix}\dot{\vartheta} \\ -\sin\vartheta - 2c\dot{\vartheta}\end{pmatrix}$$

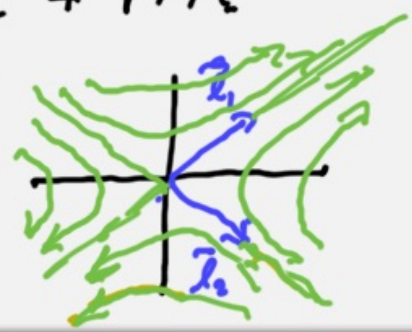


$\vec{F} = 0$ at $\dot{\vartheta} = 0$  $\vartheta = \begin{cases} 0 \\ \pi \end{cases} + k2\pi$

$$A_\pi = \left(\frac{\partial \vec{F}}{\partial \vartheta}, \frac{\partial \vec{F}}{\partial \dot{\vartheta}}\right)\Bigg|_\pi = \begin{pmatrix} 0 & 1 \\ -\cos\vartheta & -2c \end{pmatrix}_\pi = \begin{pmatrix} 0 & 1 \\ 1 & -2c \end{pmatrix}$$

$\lambda_1 = -c + \sqrt{1+c^2}$  $\lambda_2 = -c - \sqrt{1+c^2}$

$\lambda_1 > 0$  $\lambda_2 \le -1$



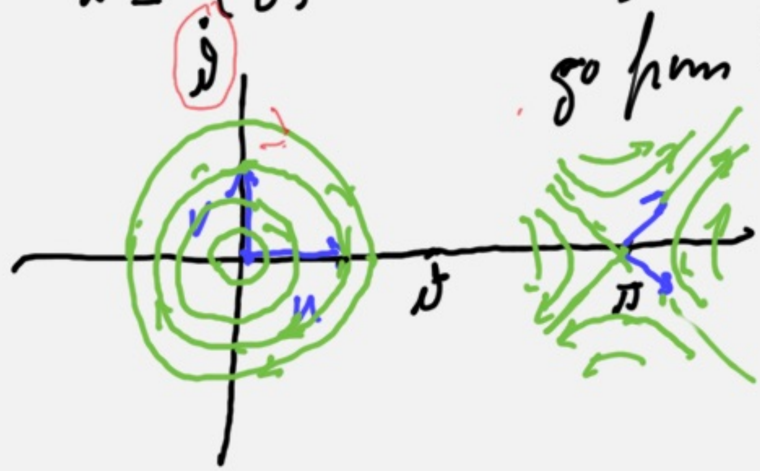
$$A_0 = \begin{pmatrix} 0 & 1 \\ -\cos\vartheta & -2c \end{pmatrix}_{\vartheta=0} = \begin{pmatrix} 0 & 1 \\ -1 & -2c \end{pmatrix}$$

$c=0$ $\quad A \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\quad (-\lambda)^2 = -1$ $\quad \lambda = \pm i$

$\lambda_R = 0$

center. $\quad \hat{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

go from $\hat{u}$ to $-\vec{v}$



Cases where the approximate analysis
($\frac{d\vec{x}}{dt} = A\vec{x}$) can lie:

1) Center $\lambda_r = 0$ $\mu > 0$
2) Improper node (defective A)
   If $\lambda > 0$ unstable, else not much be said
3) If one of the $\lambda$ values is zero

$A_0$ $\quad c>0$ $\quad \cancel{\lambda_1}$ $\quad \begin{pmatrix} 0 & 1 \\ -1 & -2c \end{pmatrix}$ $\quad \begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda-2c \end{vmatrix}$

$$\lambda^2 + 2c\lambda + 1 = 0$$

$$\lambda_{1,2} = -c \pm \sqrt{c^2-1}$$

$$= -c \pm i\sqrt{1-c^2} \quad \text{if } c<1$$

$$e^{\lambda_{1,2}t}\left[\ \ldots\ ---\ \right] \qquad \lambda_1 = c$$