iamat5060 $\frac{d\vec{y}}{dt} = \vec{F}(\vec{y})$ stationary points: $\vec{F}(\vec{y}_s) = 0$ Nearby $\vec{y} = \vec{y}_s + \vec{x}$

Taylor series $A_s = \left(\frac{\partial \vec{F}}{\partial \vec{y}}\right)_s$

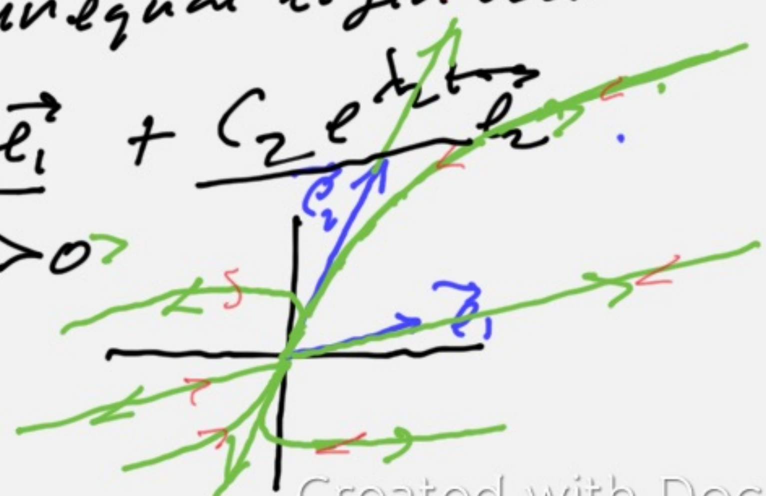
$\frac{d\vec{x}}{dt} = A\vec{x}$

Case 1: real unequal eigenvalues

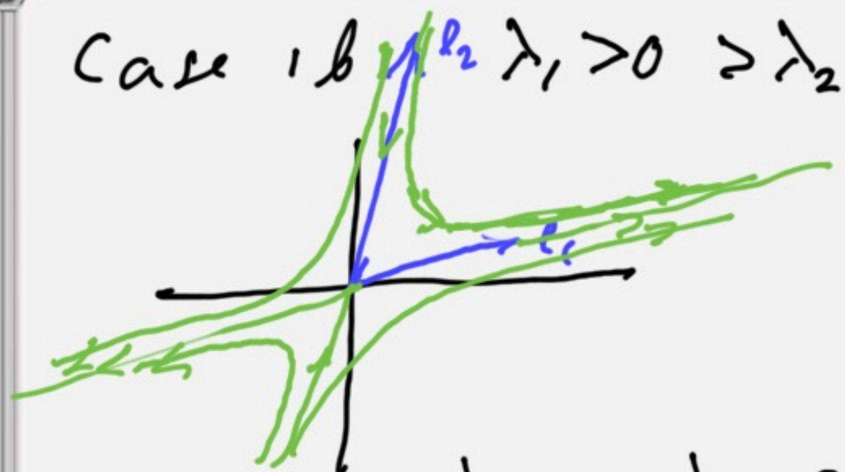
$\vec{x} = c_1 e^{\lambda_1 t} \vec{e}_1 + c_2 e^{\lambda_2 t} \vec{e}_2$

Case 1a $\lambda_1 > \lambda_2 > 0$

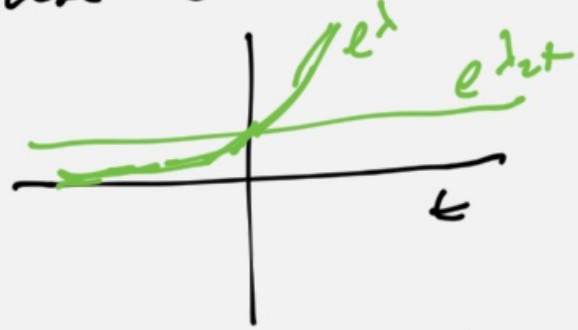
Case 1A $\lambda_1 < \lambda_2 < 0$

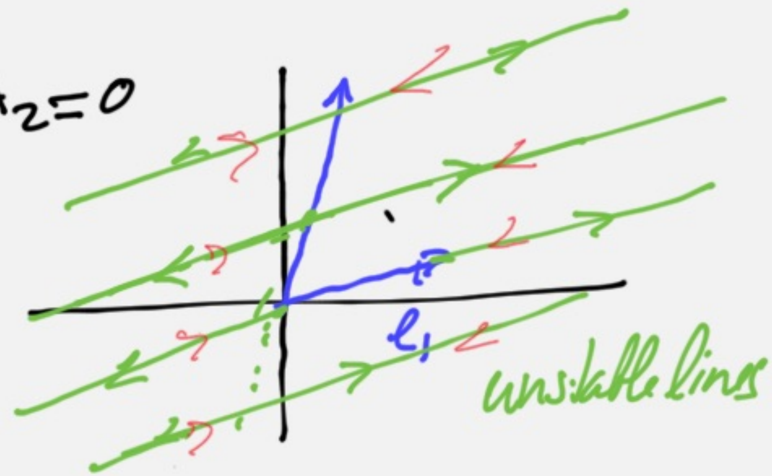


Case 1b $\lambda_1 > 0 > \lambda_2$



case 1c : $\lambda_1 > 0 \quad \lambda_2 = 0$



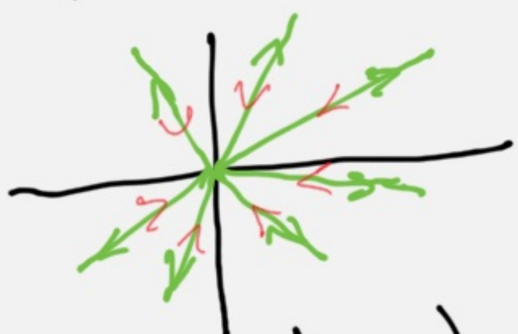


case 1C $\lambda_1 < 0 \quad \lambda_2 = 0$

Case 2: real equal eigenvalues, 2 eigenvectors

case 2a: $\lambda_1 = \lambda_2 = \lambda > 0$ $\quad \vec{A} = \lambda I$

$$\vec{x} = e^{\lambda t}\left(C_1 \vec{e}_1 + C_2 \vec{e}_2\right)$$

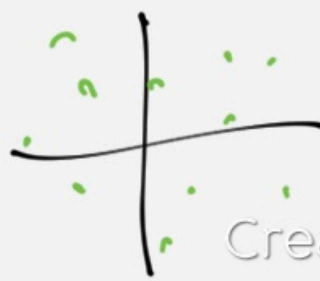


unstable star

case 2A $\lambda_1 = \lambda_2 = \lambda < 0$ > stable star

case 2b: $\lambda_1 = \lambda_2 = 0$ $\quad \vec{x} = C_1 \vec{e}_1 + C_2 \vec{e}_2$

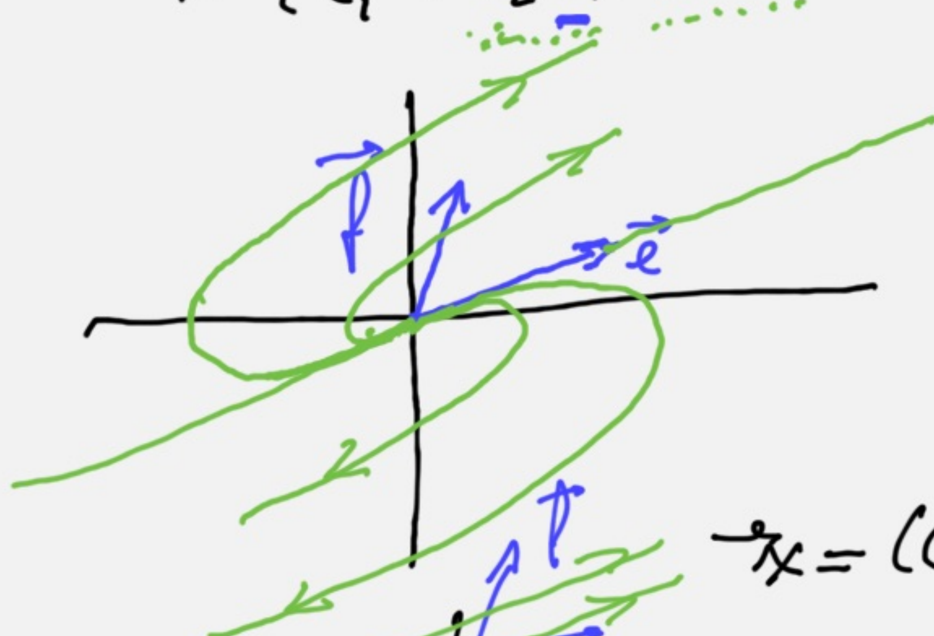
stable points



Case 3: $\lambda_1 = \lambda_2$, 1 eigenvector (defective A)

$$\vec{x} = c_1 e^{\lambda t}\vec{e} + c_2(t\vec{e} + \vec{f})e^{\lambda t}$$

$$= (c_1 + c_2 t)e^{\lambda t}\vec{e} + c_2 e^{\lambda t}\vec{f}$$

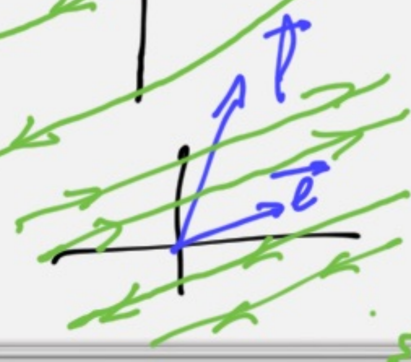


case 3a $\lambda > 0$

case 3A $\lambda < 0$

invert the arrows and $\vec{f}$

case 3b $\lambda = 0$



$$\vec{x} = (c_1 + c_2 t)\vec{e} + c_2\vec{f}$$

Case 4: Complex conjugate eigenvalues

$$\vec{x} = d_1 e^{\lambda_R t}\left[\vec{u}\cos\mu t - \vec{v}\sin\mu t\right] + d_2 e^{\lambda_R t}\left[\vec{u}\sin\mu t + \vec{v}\cos\mu t\right]$$

$$= e^{\lambda_R t}\left[(d_1\cos\mu t + d_2\sin\mu t)\vec{u} + (\overset{D\sin\alpha}{d_2}\cos\mu t - \overset{D\cos\alpha}{d_1}\sin\mu t)\vec{v}\right]$$

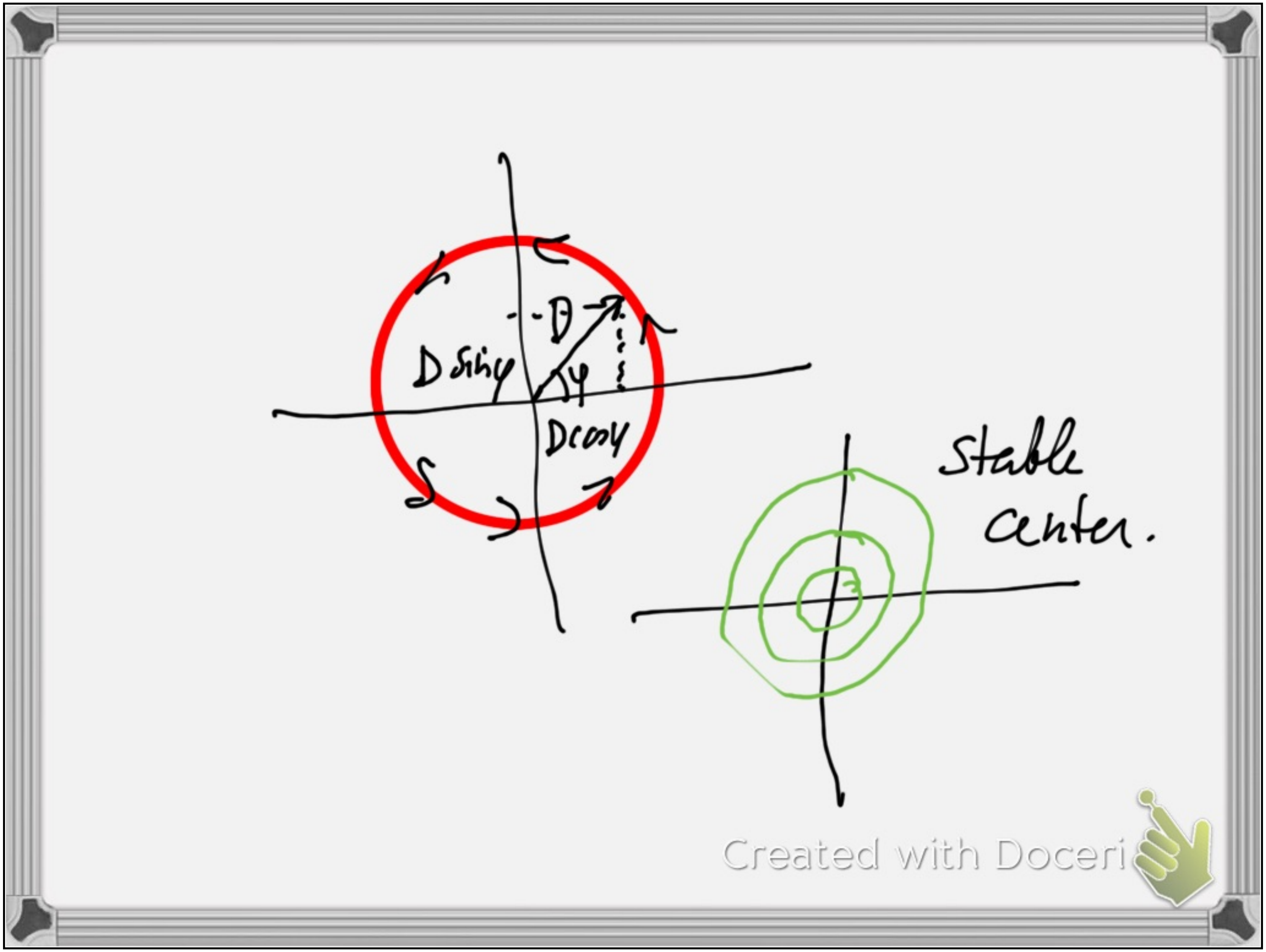
Write $d_1 = D\cos\alpha$ $d_2 = D\sin\alpha$

$$\vec{x} = e^{\lambda_R t}\left[D\cos(\underbrace{\mu t - \alpha}_{\varphi})\vec{u} - D\sin(\overbrace{\mu t - \alpha}^{\varphi})\vec{v}\right]$$

φ increases with t

Case 4A ~~Suppose~~ $\lambda_R = 0$ then $\vec{x} = D\cos\varphi\,\vec{u} - D\sin\varphi\,\vec{v}$

if $\vec{u} = \hat{i}$ $\vec{v} = \hat{j}$


D
$D\sin\varphi$
φ
$D\cos\varphi$
Stable center.
Created with Doceri

Case 4a

$\lambda_1 = 0$

$$\vec{x} = D\cos\varphi\,\vec{u} - \underline{D\sin\varphi}\,\vec{v}$$