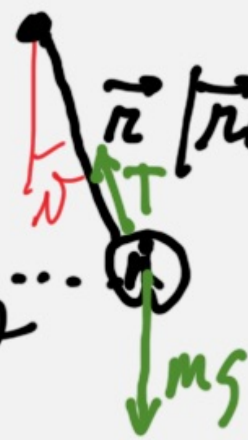
Ri 5060

$$\dot{\vec{y}} = \vec{F}(\vec{y}, \cancel{t})$$ autonomous

↓

2D

Pendulum



$|\vec{r}| = l$

$$\frac{d\ell}{dt} = M_z$$

$$m l^2 \ddot{\vartheta} = -mgl\sin\vartheta - C\dot{\vartheta}$$

$$\ddot{\vartheta} = -\frac{g}{l}\sin\vartheta - c^*\dot{\vartheta} \qquad \sqrt{\frac{g}{l}} = \omega_n$$

define $t = \dfrac{\tau}{\omega_n}$

$$\ddot{\vartheta} = -\sin\vartheta - \gamma\dot{\vartheta}$$

$$\ddot{\vartheta} = -\sin\vartheta - \gamma\dot{\vartheta}$$

$y_1 = \vartheta.$

$y_2 = \dot{\vartheta}$

$$\boxed{\begin{aligned} \frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= -\sin y_1 - \gamma y_2 \end{aligned}}$$

phase plane

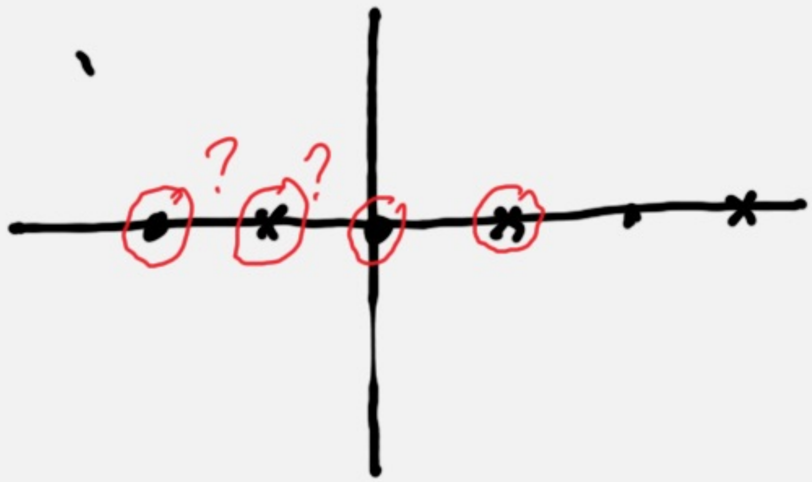
$$\frac{d\vec{y}}{dt} = \vec{F}(\vec{y}) = \begin{pmatrix} y_2 \\ -\sin y_1 - \gamma y_2 \end{pmatrix}$$

Boring near a non singular, nonzero $\vec{F}$

$$\vec{F} = \begin{pmatrix} -y_2 \\ -\sin y_1 - \gamma y_2 \end{pmatrix}$$

Want the stationary points $\vec{F} = 0$

Here: $y_2 = 0$ $\sin(y_1) = 0$ $y_1 = 0, \pm\pi, \pm 2\pi$



Taylor series: near a stationary point write $\vec{y} = \vec{y}_s + \vec{x}$ $\quad |\vec{x}| \ll 1$

$$\frac{d\vec{y}}{dt} = \cancelto{0}{\vec{F}(\vec{y}_s)} + \left(\frac{\partial \vec{F}}{\partial \vec{y}}\right)_s \vec{x} + \cdots$$

$$\frac{d\vec{x}}{dt}$$

$$\left(\frac{\partial \vec{F}}{\partial \vec{y}}\right)_s = \boxed{\begin{pmatrix} \frac{\partial F_1}{\partial y_1} & \frac{\partial F_1}{\partial y_2} & \cdots \\ \frac{\partial F_2}{\partial y_1} & \frac{\partial F_2}{\partial y_2} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}} = A_s$$

$$\boxed{a_{ij} = \frac{\partial F_i}{\partial y_j}}$$

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So Near the stationary point:

$$\frac{d\vec{x}}{dt} = A_s \vec{x}$$

Pendulum

$$\vec{F} = \begin{pmatrix} +y_2 \\ -\sin(y_1) - \gamma y_2 \end{pmatrix} \quad A = \begin{pmatrix} 0 & 1 \\ -\cos(y_1) & -\gamma \end{pmatrix}$$

$$\begin{cases} y_1 = y_2 = 0 & A_s = \begin{pmatrix} 0 & 1 \\ -1 & -\gamma \end{pmatrix} \\ y_1 = \pi \quad y_2 = 0 & A_s = \begin{pmatrix} 0 & 1 \\ 1 & -\gamma \end{pmatrix} \end{cases}$$

I/ A is nondefective

$$\vec{x} = C_1 \vec{e_1} e^{\lambda_1 t} + C_2 \vec{e_2} e^{\lambda_2 t}$$

$$= C_1 e^{\lambda_1 t} \vec{e_1} + C_2 e^{\lambda_2 t} \vec{e_2}$$

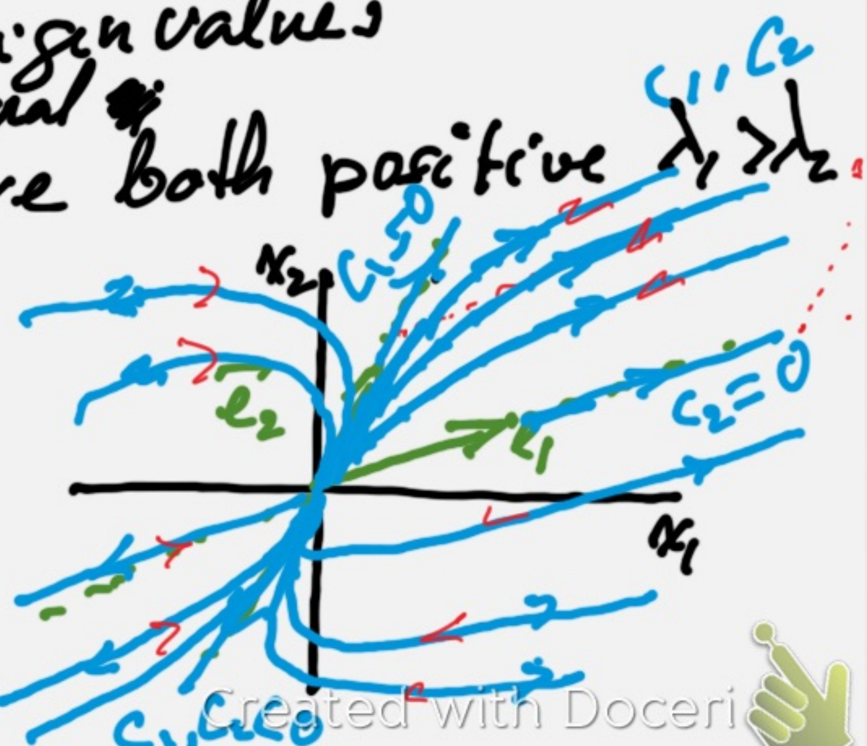
$y = y_s + \vec{x}$

$\boxed{\text{Case 1}}$: Real eigenvalues unequal

Case 1a: λ_1, λ_2 are both positive $\lambda_1 > \lambda_2$

stable node $\lambda_1, \lambda_2 < 0$

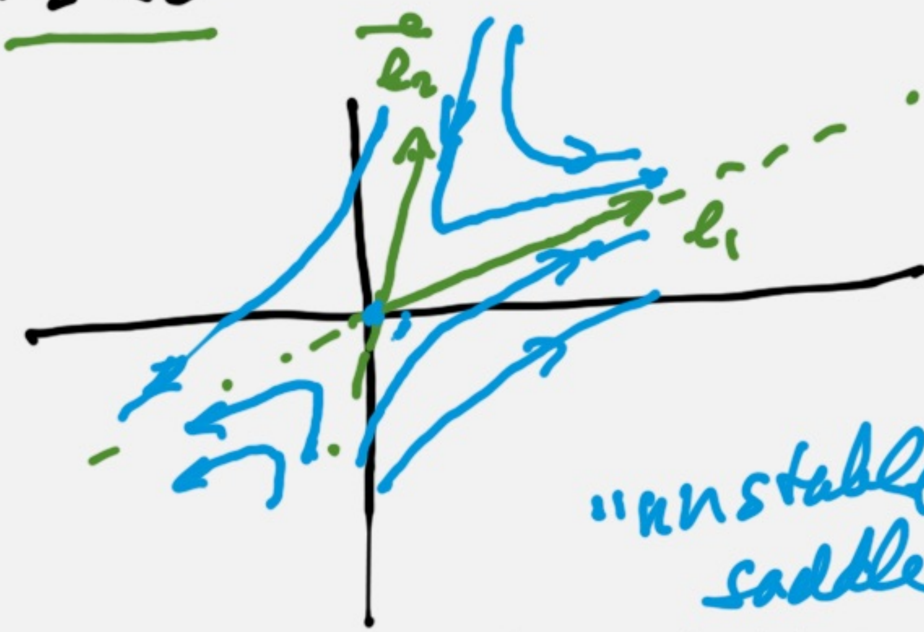
"unstable node"



$\lambda_1, \lambda_2 < 0$ λ $e^{\lambda_1 t}$

invert the arrows

$\lambda_1 > 0$, $\lambda_2 < 0$



"unstable saddle"