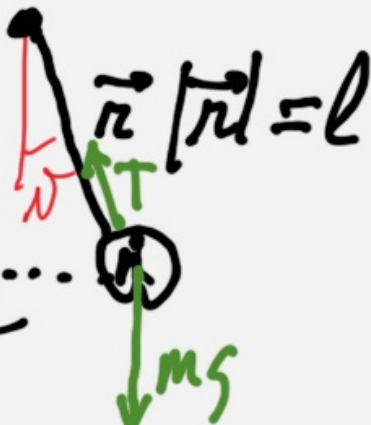


Risolvo $\dot{\vec{y}} = \vec{F}(\vec{y}, t)$ ^{autonomo}
 \downarrow
 2D

Pendulum




$\frac{ds}{dt} = M_2$

$ml^2 \ddot{\theta} = -mgl \sin \theta - c \dot{\theta}$

$\ddot{\theta} = -g \sin \theta - c^* \dot{\theta}$ $\sqrt{\frac{g}{l}} = \omega_n$

delve $t = \frac{\tau}{\omega_n}$

$\ddot{\theta} = -\sin \theta - \gamma \dot{\theta}$

Created with Doceri 


$\ddot{y} = -\sin y - \gamma \dot{y}$ $\gamma_1 = \dot{y}$
 $\gamma_2 = y$

$$\frac{dy_1}{dt} = \gamma_2$$

$$\frac{dy_2}{dt} = -\sin \gamma_1 - \gamma \gamma_2$$

Phase plane

$$\frac{d\vec{y}}{dt} = \vec{F}(\vec{y}) = \begin{pmatrix} \gamma_2 \\ -\sin \gamma_1 - \gamma \gamma_2 \end{pmatrix}$$



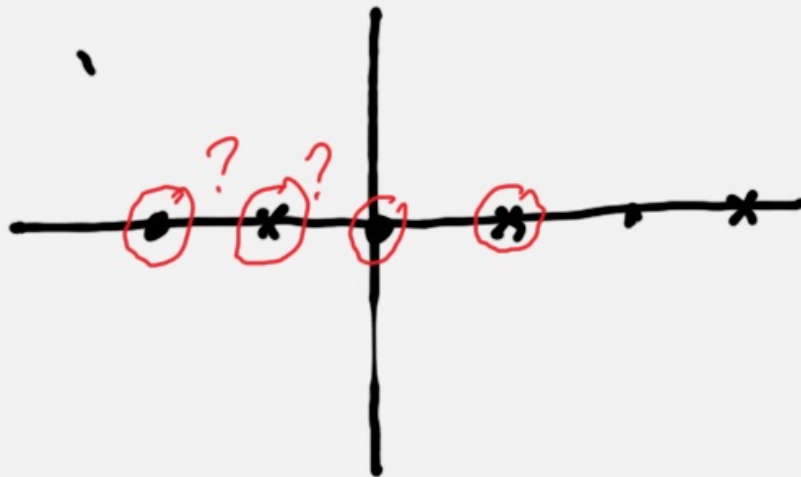
Being near a non sin value, nonzero \vec{F}

Created with Doceri

$$\vec{F} = \begin{pmatrix} -y_2 \\ r \sin y_1 - r y_2 \end{pmatrix}$$

Want the stationary points $\vec{F} = 0$

Here: $y_2 = 0$ $\sin(y_1) = 0$ $y_1 = 0, \pm\pi, \pm 2\pi$



Created with Doceri




Taylor series: near a stationary point write $\vec{y} \equiv \vec{y}_s + \vec{x}$ ($|\vec{x}| \ll 1$)

$$\frac{d\vec{y}}{dt} = \vec{F}(\vec{y}_s) + \left(\frac{\partial \vec{F}}{\partial \vec{y}} \right)_{\vec{y}_s} \vec{x} + \dots$$

$$\left(\frac{\partial \vec{F}}{\partial \vec{y}} \right)_{\vec{y}_s} = \begin{pmatrix} \frac{\partial F_1}{\partial y_1} & \frac{\partial F_1}{\partial y_2} & \dots \\ \frac{\partial F_2}{\partial y_1} & \frac{\partial F_2}{\partial y_2} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} = A_s$$

$$a_{ij} = \frac{\partial F_i}{\partial y_j}$$

Created with Doceri 

So Near the stationary point:

$$\frac{d\vec{x}}{dt} = A_s \vec{x}$$

Pendulum

$$\vec{F} = \begin{pmatrix} +y_2 \\ -l \sin(y_1) - \gamma y_2 \end{pmatrix} \quad A = \begin{pmatrix} 0 & 1 \\ -\cos(y_1) & -\gamma \end{pmatrix}$$

$$\begin{cases} y_1 = y_2 = 0 & A_s = \begin{pmatrix} 0 & 1 \\ -1 & -\gamma \end{pmatrix} \\ y_1 = \pi, y_2 = 0 & \dot{A}_s = \begin{pmatrix} 0 & 1 \\ 1 & -\gamma \end{pmatrix} \end{cases}$$

Created with Doceri



I/ A is non defective

$$\vec{x} = C_1 \vec{e}_1 e^{\lambda_1 t} + C_2 \vec{e}_2 e^{\lambda_2 t}$$

$$= C_1 e^{\lambda_1 t} \vec{e}_1 + C_2 e^{\lambda_2 t} \vec{e}_2$$

$y = y_5 + \vec{x}$

Case 1: Real eigen values unequal

Case 1a: λ_1, λ_2 are both positive $\lambda_1 > \lambda_2$

Left graph: $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ vs t . Arrows point to ∞ as $t \rightarrow \infty$.

Right graph: Phase plane plot with axes x_1 and x_2 . Trajectories are shown in blue with arrows pointing away from the origin. Eigenvectors e_1 and e_2 are shown as green arrows. Labels include C_1, C_2 and $C_2 = 0$.

Annotations: "stable node $\lambda_1, \lambda_2 < 0$ " (with arrows pointing left), "unstable node" (with arrows pointing right).

Created with Doceri

