hi 5060 SPCI

$$(A-\lambda I)\ell = e$$

$$(A-\lambda I)(A-\lambda I)\ell = (A-\lambda I)e$$

$$e^{At} = 1 + At + \frac{1}{2}t^2A^2 \cdots$$

Inhomogeneous constant coefficient problems

$$\vec{x}' = A\vec{x} + \vec{g}(t) \qquad \vec{x}(0) = \vec{x}_0$$

given

Variation of parameters

$$\vec{x}_h = \Omega \vec{C} \Rightarrow \vec{x} = \Omega \vec{D}(t) \quad \text{P.I.I}$$

$$\cancel{\Omega'\vec{D}} + \Omega\vec{D}' = \cancel{A\Omega\vec{D}} + \vec{g}$$

solve for $\vec{D}'$, then integrate to get $\vec{D}$

Undetermined coefficients:

$$\vec{g} = \vec{C}_1 e^{at} \xrightarrow{\text{guess}} \vec{x}_p = \vec{d}\, e^{at}$$

Diagonalizing A (if not defective)

$$\vec{x}' = \underbrace{E \Lambda E^{-1}}_{A} \vec{x} + \vec{g} \qquad E^{-1}\vec{x} = \vec{y}$$

E^{-1}

$$\vec{y}' = \Lambda \vec{y} + \underbrace{E^{-1}\vec{g}}_{\vec{h}}$$

$$y_1' = \lambda_1 y_1 + h_1$$

$$y_2' = \lambda_2 y_2 + h_2$$

Example

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 5 & -4 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 2e^t \\ 2e^t \end{pmatrix} \qquad x(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$|A - \lambda I| = \lambda^2 - 2\lambda + 1 \rightarrow \lambda_1 = \lambda_2 = 1$$

$$\begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix} \rightarrow \begin{pmatrix} 4 & -4 \\ 0 & 0 \end{pmatrix} \quad \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e_2$$

$$\vec{e} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \longrightarrow \vec{x}_{h_1} = c_1 \vec{e}\, e^t$$

$$(A - \lambda I)\vec{T} = \vec{e} \qquad \left(\begin{array}{cc|c} 4 & -4 & 1 \\ 4 & -4 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 4 & -4 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

$$f_1 = f_2 + \frac{1}{4}$$

take $f_2 = 0$

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} f_2 + \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix} \quad \vec{T} = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix}$$

$$\vec{x}_h = C_1 \begin{pmatrix}1\\1\end{pmatrix} e^t + C_2^* \left\{ t \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}\frac{1}{4}\\0\end{pmatrix} \right\} e^t$$

define $C_2^* = 4C_2$

$$= C_1 \begin{pmatrix}1\\1\end{pmatrix} e^t + C_2 \begin{pmatrix}4t+1\\4t\end{pmatrix} e^t$$

$$\Omega = \begin{pmatrix} e^t & (4t+1)e^t \\ e^t & 4t\,e^t \end{pmatrix}$$

$$\Omega \vec{D}' = \begin{pmatrix}2e^t\\2e^t\end{pmatrix}$$

$$\begin{pmatrix} \cancel{e^t} & (4t+1)\cancel{e^t} \\ \cancel{e^t} & 4t\,\cancel{e^t} \end{pmatrix} \begin{pmatrix}2\cancel{e^t}\\2\cancel{e^t}\end{pmatrix}$$

$$\begin{array}{cc|c} D_1' & D_2' & \\ \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 4t+1 & 2 \\ 0 & -1 & 0 \end{array}\right) \rightarrow D_2' = 0 \qquad D_1' = 2$$

$$D_2 = D_{20} \qquad D_1 = 2t + D_{10}$$

$$\vec{x} = (2t + \overset{3}{\cancel{D_{10}}})\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^t + \overset{-4}{\cancel{D_{20}}}\begin{pmatrix} 4t+1 \\ 4t \end{pmatrix} e^t$$

B.I I.C. $\vec{x}(0) = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \rightarrow$

$$\begin{array}{cc} D_{10} & D_{20} \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 1 & -1 \\ 1 & 0 & 3 \end{array}\right) \rightarrow D_{10} = 3 \qquad D_{20} = -4$$

$\boxed{\text{Nonlinear systems of first order ODE}}$

$$\dot{\vec{y}} = \vec{F}(\vec{y}, t) \quad \text{or}$$

$$\left.\begin{aligned} \dot{y}_1 &= F_1(y_1, y_2, \dots, y_n, t) \\ \dot{y}_2 &= F_2(y_1, y_2, \dots, y_n, t) \\ &\vdots \\ \dot{y}_n &= F_n(y_1, y_2, \dots, y_n, t) \end{aligned}\right\}$$

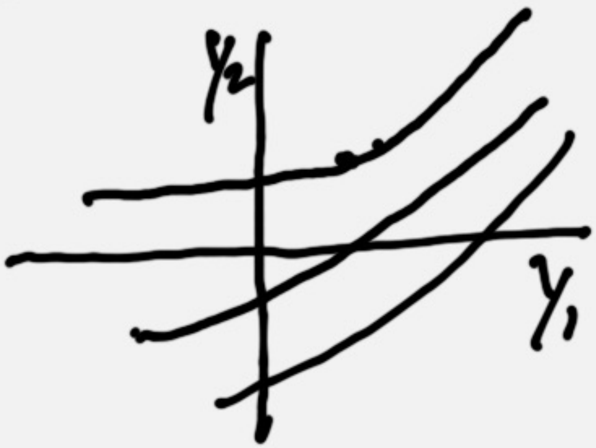
exact ~~analytical~~ Solution not possible

Autonomous systems $\vec{F}(\vec{y}, t) = \vec{F}(\vec{y})$

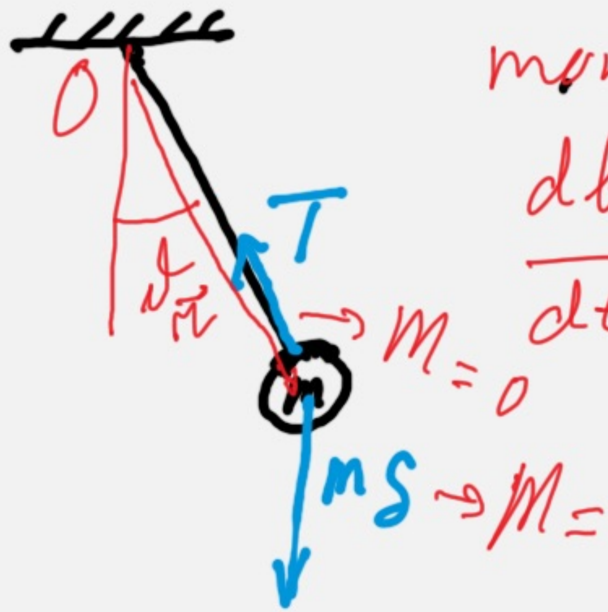
Non autonomous system $n \times n$

Define $y_{n+1} = t \longrightarrow \dot{y}_{n+1} = 1$

2D autonomous systems y_1, y_2



Example viscously damped non-linear pendulum.



momentum about O

$$\frac{dL_z}{dt} = M_z$$

$M_z \rightarrow \vec{r} \times \vec{F}$

$$L_z = \vec{r} \times m\vec{v}$$

$\vec{v} \rightarrow r\dot{\vartheta}\hat{i}_\vartheta$

$$L_z = m r^2 \dot{\vartheta}$$