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$y'' + 4y = f(t)$ $\quad y(0) = 1$

$y'(0) = 0$

$f(t) \overset{P4}{=} \begin{cases} 0 & \text{if } t < 4 \\ 3 & \text{if } t > 4 \end{cases}$

$f(t) = 3\,H(t-4)\,1$

$\Updownarrow$ S1

$\frac{1}{s}$

P6

$\Leftrightarrow 3\,e^{-4s}\frac{1}{s}$

$s^2\hat{y} - s + 4\hat{y} = 3e^{-4s}\frac{1}{s}$

$\hat{y} = \frac{s}{s^2+4} + \frac{3}{s(s^2+4)}e^{-4s}$

$$Y = \frac{s}{s^2+4} + \frac{3}{s(s^2+4)}e^{-4s}$$

$\Downarrow$ S9 $\quad$ S1 $\quad$ S9 $\frac{3}{4} - \frac{3}{4}\cos 2t$

$\cos(2t)$ $\quad$ $\frac{3}{4}\frac{1}{s} - \frac{3}{4}\frac{s}{s^2+4} = F(s)$ $\quad$ $f(t)$

$$A\frac{1}{s} + \frac{Bs+C}{s^2+4} = \frac{3}{s(s^2+4)} = As^2 + 4A + Bs^2 + Cs$$

$O(s^2)$ $A = -B$ $\quad$ $O(s)$: $C = 0$ $\quad$ $O(1)$ $A = \frac{3}{4}$

P6 $\Rightarrow H(t-4)\frac{3}{4}[1 - \cos 2(t-4)] + \cos 2t = Y$

Must split up

$$y(t) = \begin{cases} = \cos 2t \\ = \cos(2t) + \frac{3}{4}[1 - \cos 2(t-4)] \end{cases}$$

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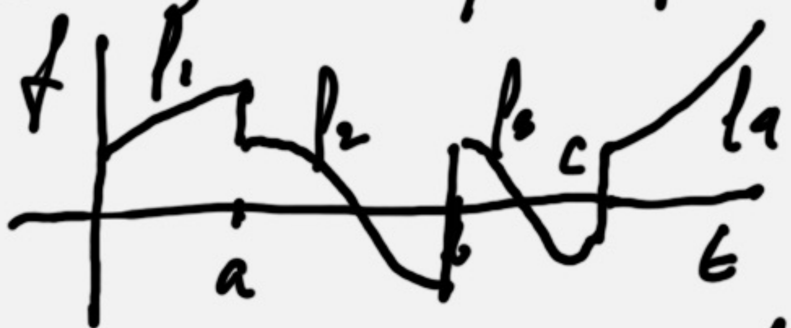
Dealing with jumps



$$f = f_1 + H(t-a)(f_2 - f_1) + H(t-b)(f_3 - f_2)$$
$$+ H(t-c)(f_4 - f_3) \ldots$$

e.g. Pulse!

$$f = H(t-a)C$$
$$- CH(t-b)$$

$g(t) = \cos(t+7) - 1$

$g(t-7) = \cos(t) - 1$

asked f

Example:

$$f(t) = \begin{cases} 1 & \text{for } 0 \le t \le 7 \\ \cos(t) & \text{for } 7 < t \end{cases}$$

$$f = 1 + H(t-7)\{\underbrace{\cos t - 1}_{\text{need}}\} \quad f^*(t-7)$$

$$= 1 + H(t-7)\{\cos(t-7+7) - 1\}$$

$\Downarrow$ S1 $\qquad \Downarrow$ P6

$\cos(t+7) - 1$

$$\Leftrightarrow \frac{1}{s} + e^{-7s}$$

$$\cos(t+7) = \cos(t)\cos 7 - \sin(t)\sin 7$$

$\frac{s}{s^2+1}$ $\qquad \frac{1}{s^2+1}$

$$\frac{1}{s} - e^{7s}\frac{1}{s} + e^{-7s}\left\{\cos(7)\frac{s}{s^2+1} - \sin(7)\frac{1}{s^2+1}\right\}$$

Convolution theorem P8 $\hat{f} \leftrightarrow f$ $\hat{g} \leftrightarrow g$

$$\underline{\hat{f}(s)\,\hat{g}(s)} \Longleftrightarrow \underbrace{\int_{\tau=0}^{t} f(\tau)\, g(t-\tau)\, d\tau}_{\text{do not write as } f*g\text{: illegal}}$$

Do not use unless unavoidable

Example

$$y'' - 5y' + 6y = f(t) \qquad \underline{y(0) = y'(0) = 0}$$

$$\Downarrow P4 \quad \Downarrow P9 \quad \Downarrow$$

$$s^2\hat{y} - 5s\hat{y} + 6\hat{y} = \hat{f}(s)$$

$$\hat{y} = \frac{\hat{f}(s)}{s^2 - 5s + 6}$$

Convolution is unavoidable here

$$\hat{g}(s) = \frac{1}{s^2 - 5s + 6}$$

if real roots: use Partial fraction

if complex roots complete the square

roots $s_{1,2} = \dfrac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 3 \\ 2 \end{cases}$

$$\frac{1}{s^2-5s+6} = \frac{1}{(s-2)(s-3)}$$

$$= \frac{A}{s-2} + \frac{B}{s-3} = \frac{As-3A+Bs-2B}{(s-2)(s-3)}$$

$$\left.\begin{array}{l} O(s)\quad A=-B \\ O(1)\quad B=1 \end{array}\right\} A=-1$$

$$\hat{g}(s) = \frac{1}{s^2-5s+6} = \frac{1}{s-3} - \frac{1}{s-2} \overset{S3}{\Longleftrightarrow} e^{3t} - e^{2t} = g(t)$$

$$\hat{f}(s)\hat{g}(s). \overset{P8}{\Longleftrightarrow} \int_0^t f(\tau)\underbrace{\left\{e^{3(t-\tau)} - e^{2(t-\tau)}\right\}}_{G(t-\tau)}d\tau$$

Green's function