ai 5060 | General parti-al fractions

scattered around in Zill

$\frac{T}{B}$ polynomial in s degree less than of B
polynomial in s

If not, perform long division T/B as the fractional and redefine part.

Factor B in linear factors $(s-s_i)$ and quadratic ones $(s^2+a_is+b_i)$ {with complex roots)

For each factor $(s-s_i)^m$ in B,
assume (s_i must be real)

$$\frac{A_{i1}}{s-s_i} + \frac{A_{i2}}{(s-s_i)^2} + \cdots + \frac{A_{im}}{(s-s_i)^m}$$

For each factor $(s^2+a_i s+b_i)^m$
assume partial fractions

$$\frac{A_{i1}s+B_{i1}}{s^2+a_i s+b_i} + \frac{A_{i2}s+B_{i2}}{(s^2+a_i s+b_i)^2} + \cdots + \frac{A_{im}s+B_{im}}{(s^2+a_i s+b_i)^m}$$

Also, must "complete the squares"

$$s^2 + a_i s + b_i = \underbrace{\left(s + \frac{a_i}{2}\right)^2} + b_i - \frac{a_i^2}{4}$$

Example

$$y'' - 4y' + 4y = \cos t \quad y(0) = 1 \quad y'(0) = -1$$

Solution

$$y'' \xleftrightarrow{P4} s^2 \hat{y} - s y(0) - y'(0)$$

$$= s^2 \hat{y} - s + 1$$

$$-4y' \xleftrightarrow{P4} -4s\hat{y} + 4y(0) = -4s\hat{y} + 4$$

$$\cos(t) \xleftrightarrow{S} \frac{s}{s^2+1}$$

P.T.I.

$$\underbrace{(s^2-4s+4)}_{(s-2)^2}\hat{Y} = \frac{s}{s^2+1} + s - 5$$

$$= \frac{s + s^3 + s - 5s^2 - 5}{s^2+1}$$

$$\hat{Y} = \frac{s^3 - 5s^2 + 2s - 5}{(s^2+1)(s-2)^2}$$

$$= \frac{As+B}{s^2+1} + \frac{C}{s-2} + \frac{D}{(s-2)^2}$$

s^3

$s^3 + P_5$

$s^2 + P_7$

$$= \frac{As^3 - 4As^2 + 4As + Bs^2 - 4Bs + 4B + Cs^3 + Cs - 2Cs^2 - 2C + Ds^2 + D}{(s^2+1)(s-2)^2}$$

compare powers of s

$$\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
s^3 & \left(\begin{array}{cccc|c}
1 & 0 & 1 & 0 & 1 \\
-4 & 1 & -2 & 1 & -5 \\
4 & -4 & 1 & 0 & 2 \\
0 & 4 & -2 & 1 & -5
\end{array}\right) & \xrightarrow{G.E} & &
\end{array}$$

$D = -\frac{13}{5}$ $\quad C = \frac{22}{25}$ $\quad B = -\frac{4}{25}$ $\quad A = \frac{3}{25}$

$$y = \underbrace{-\frac{13}{5} t e^{2t} + \frac{22}{25} e^{2t}}_{\text{homo.}} \underbrace{- \frac{4}{25} \sin t + \frac{3}{25} \cos t}_{\text{particular}}$$

Shifting Theorems + Heaviside Step Functions

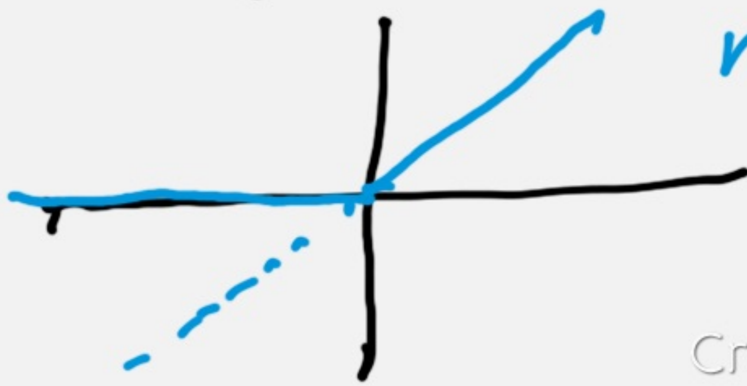
$S_1 : \quad 1 \Longleftrightarrow \frac{1}{s}$

true back transform of $\frac{1}{s}$

"Heaviside step function" $H(t)$

back transform of $\frac{1}{s^2}$ table: t

really: $H(t)\,t$



So really $H(t)f(t) \Longleftrightarrow \hat{f}(s)$

P7: First shifting theorem

assumed $f(t) \Longleftrightarrow \hat{f}(s)$

then $e^{\sigma t}f(t) \Longleftrightarrow \hat{f}(s-\sigma)$

P6: Second shifting theorem

$H(t-\tau)f(t-\tau) \Longleftrightarrow e^{-\tau s}\hat{f}(s)$