hi5060 | Laplace transforms

Zill Chap 4

is an association of function

$f(t)$ for $0 \leq t < \infty$

$\Longleftrightarrow \hat{f}(s)$

s complex

Book $\longrightarrow$ $\hat{f}(s) = L[f(s)]$

$\hat{f}(s) = \int_0^\infty f(t) e^{-st} dt \quad f(t) = \int_{s_r - i\infty}^{s_r + i\infty} \hat{f}(s) e^{st} ds$

Linearity P2

$c\, f(t) \to c\, \hat{f}(s)$

$f_1(t) + f_2(t) \to \hat{f}_1(s) + \hat{f}_2(s)$

$$C\,1 \to C\,\frac{1}{s}$$

Example:

$2\sinh(t) - 4 \Longleftrightarrow$

$\Updownarrow$ S12 $\Updownarrow$ S1

must show table entries & do 1 at a time

$$2\,\frac{1}{s^2-1} - \frac{4}{s}$$

Example: $y' + 4y = 1$ $\quad y(0) = -3$

$\sout{Ce^{-4x} + \frac{1}{4}}$ $\quad -3\frac{1}{4}$

$y' \overset{P4}{\Longleftrightarrow} s\hat{y} - 1\,y(0)$

$\hat{y} \Longleftrightarrow \hat{y}$

$1 \Longleftrightarrow \frac{1}{s} \quad (S1)$

$$s\hat{y} + 3 + 4\hat{y} = \frac{1}{s}$$

$$(s+4)\hat{y} = \frac{1}{s} - 3 \qquad \hat{y} = \frac{1}{s(s+4)} - \frac{3}{s+4}$$

Partial fractions $\quad S3 \Updownarrow a=-4 \quad -3e^{-4t}$

$$\frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4} = \frac{As + 4A + Bs}{s(s+4)}$$

$O(s)$: $B = -A$ $\quad O(s^0)$: $A = \frac{1}{4}$

not $\widehat{f_1(s) f_2(s)}$

$$\hat{Y} = \frac{1}{s(s+4)} - \frac{3}{s+4} \quad \overset{S3}{\Longleftrightarrow} \quad -3e^{-4t}$$

$$\frac{1}{s(s+4)} \Downarrow$$

$$\frac{\frac{1}{4}}{s} - \frac{\frac{1}{4}}{s+4}$$

$$S1 \Updownarrow \qquad \Updownarrow S3$$

$$\frac{1}{4} - \frac{1}{4}e^{-4t}$$

$$y = \frac{1}{4} - \frac{1}{4}e^{-4t} - 3e^{-4t}$$

$$y = -3\frac{1}{4}e^{-4t} + \frac{1}{4}$$

Example: $y' + 4y = \cos(t)$ $\quad y(0) = 1$

$$y' + 4y \overset{\text{prev}}{\Longleftrightarrow} s\hat{y} - y(0) + 4\hat{y} = s\hat{y} + 4\hat{y} - 1$$

$$\cos t \Longleftrightarrow \frac{s}{s^2+1}$$

$$(s+4)\hat{y} = 1 + \frac{s}{s^2+1}$$

$$\hat{y} = \frac{1}{s+4} + \frac{s}{(s^2+1)(s+4)}$$

$$\frac{1}{s+4} \to e^{-4t}$$

$$\frac{s}{(s^2+1)(s+4)} \Updownarrow \frac{A}{s+4} + \frac{Bs+C}{s^2+1}$$

$4(A\cos t + B\sin t)$

$-A\sin t + B\cos t$

$= \cos t$

$4A + B = 1$

$4B - A = 0$

~~cannot~~ may not introduce complex coefficients

→ may not say $s^2+1 = (s+i)(s-i)$

$$\frac{1s + 0s^2 + 0 \cdot 1}{(s+4)(s^2+1)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+1}$$

$$= \frac{As^2 + A + Bs^2 + 4Bs + Cs + 4C}{(s+4)(s^2+1)}$$

$O(s^2)$: $A + B = 0$

$O(s)$ $4B + C = 1$

$O(s^0)$ $A + 4C = 0$