

hi 5060 Laplace transforms
Zill Chap 4

is an association of function
 $f(t)$ for $0 \leq t < \infty$

$\hat{f}(s)$

\longleftrightarrow

$f(s)$

s complex

$\hat{f}(s) = \int_0^{\infty} f(t) e^{-st} dt$

$f(t) = \int_{\gamma} \hat{f}(s) e^{st} ds$

Back $\hat{f}(s) = \mathcal{L}[f(t)]$

$f(t) = \mathcal{L}^{-1}[\hat{f}(s)]$

Diagram of the complex s -plane. The horizontal axis is the real axis and the vertical axis is the imaginary axis. A vertical red line is drawn at $s = \sigma_c$. To the right of this line, there is a branch cut on the real axis from s_2 to s_R . The region to the left of the vertical line is shaded, representing the region of convergence for the Laplace transform.


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\mathcal{L} chearity P2 $\mathcal{L} f(t) \rightarrow \mathcal{L} \hat{f}(s)$
 $f_1(t) + f_2(t) \rightarrow \hat{f}_1(s) + \hat{f}_2(s)$

$\mathcal{L} 1 \rightarrow \mathcal{L} \frac{1}{s}$

Example:
 $2 \sinh(t) - 4 \longleftrightarrow$
 $\begin{matrix} \updownarrow s_{12} \\ 2 \frac{1}{s^2-1} \end{matrix} - \begin{matrix} \updownarrow s_1 \\ \frac{4}{s} \end{matrix}$

must show table entries & do 1 at a time

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Example: $y' + 4y = 1$, $y(0) = -3$

$y' \xrightarrow{P4} s\hat{y} - 1y(0)$ \rightarrow $\begin{matrix} -4x \\ C e^{-4x} \\ + \\ -3 \frac{1}{4} \end{matrix}$

$\hat{y} \leftrightarrow \frac{1}{s}$ \downarrow $s\hat{y} + 3 + 4\hat{y} = \frac{1}{s}$

$(s+4)\hat{y} = \frac{1}{s} - 3$ $\hat{y} = \frac{1}{s(s+4)} - \frac{3}{s+4}$

$\frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$ **Partial fraction** $s_1 \uparrow \sigma = -4$

$As + 4A + Bs = 1$

$0(s): B = -A$ $0(s^0): A = \frac{1}{4}$

~~$\hat{y} = \frac{1}{4s} - \frac{3}{s+4}$~~

~~$y = \frac{1}{4}e^{0t} - 3e^{-4t}$~~


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$$\hat{y} = \frac{1}{s(s+4)} - \frac{3}{s+4} \overset{s^3}{\curvearrowright} - 3e^{-4t}$$

$$\begin{array}{c} \downarrow \\ \frac{1}{4} - \frac{1}{4} \\ \frac{1}{s} - \frac{1}{s+4} \\ \downarrow \quad \downarrow \\ \frac{1}{4} - \frac{1}{4} e^{-4t} \end{array}$$

$$y = \frac{1}{4} - \frac{1}{4} e^{-4t} - 3e^{-4t}$$

$$y = -3\frac{1}{4} e^{-4t} + \frac{1}{4}$$

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Example: $y' + 4y = \cos t$ $y(0) = 1$

$y' + 4y \xrightarrow{\text{prev}} s\hat{y} - y(0) + 4\hat{y}$

$\xrightarrow{\text{prev}} s\hat{y} + 4\hat{y} - 1$

$\cos t \xrightarrow{\text{prev}} \frac{s}{s^2+1}$

$(s+4)\hat{y} = 1 + \frac{s}{s^2+1}$

$\hat{y} = \frac{1}{s+4} + \frac{s}{(s^2+1)(s+4)}$

$\downarrow e^{-4t}$ $\frac{A}{s+4} + \frac{Bs+C}{s^2+1}$

~~$A \cos t + B \sin t$~~
 ~~$A \sin t + B \cos t$~~
 ~~$\cos t$~~

$4A + B = 1$
 $4B - A = 0$

may not introduce complex coefficients
 → may not say $s^2+1 = (s+i)(s-i)$

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$$\frac{1s + 0s^2 + 0s^1}{(s+4)(s^2+1)} = \frac{A}{s+4} + \frac{Bs+C}{s^2+1}$$

$$= \frac{As^2 + A + Bs^2 + 4Bs + Cs + 4C}{(s+4)(s^2+1)}$$

$$\begin{aligned} O(s^2): & A + B = 0 \\ O(s) & 4B + C = 1 \\ O(s^0) & A + 4C = 0 \end{aligned}$$

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