hi 5060 } inhomogeneous

Always : solve the homogeneous equation first.

Methods 1) ~~Method of~~ undetermined coefficients = Guess

2) Variation of parameters

Undetermined coefficients

for Inhomogeneous term	Guess
$a + bx + \cdots + cx^n$	$A + Bx + \cdots + Cx^n + \cdots$
$\cos(ax)$, $\sin(ax)$	$A\cos(ax) + B\sin(ax)$
e^{ax}	Ae^{ax} or if H.S. xAe^{ax} or x^2Ae^{ax}, ...

General Solution = particular solution + homogeneous solution

Example

$$y'' - 4y' + 4y = 3e^{x} + 4e^{2x} + \cos(x)$$

homogeneous $y_h'' - 4y_h' + 4y_h = 0$

$$\lambda^2 - 4\lambda + 4 = 0 = (\lambda - 2)^2 \quad \lambda_1 = \lambda_2 = 2$$

$$y_h = C_1 e^{2x} + C_2 x e^{2x}$$

Guess: for $3e^{x}$, guess Ae^{x}

P.I.I. $Ae^{x} \cancel{- 4Ae^{x}} \cancel{+ 4Ae^{x}} = 3e^{x} \rightarrow A = 3$

For e^{2x}, guess $\cancel{De^{2x}}$ $\cancel{Bxe^{2x}}$ Bx^2e^{2x}

$$y'' - 4y' + 4y = 4e^{2x}$$

$$Bx^2e^{2x} = y_{p_2} \longrightarrow B(2x^2 + 2x)e^{2x} = y_{p_2}'$$

$$y_{p_2}'' = B(4x^2 + 8x + 2)e^{2x}$$

P.I.I

$$B(\cancel{4x^2} + \cancel{8x} + 2)e^{2x}$$

$$+B(\cancel{-8x^2} \cancel{-8x})e^{2x}$$

$$+B\cancel{4x^2}e^{2x}$$

$$= 4e^{2x} \Longrightarrow B = 2$$

$$\uparrow$$

$$y_{p_2} = 2x^2e^{2x}$$

$$y'' - 4y' + 4y = \cos(x)$$

$$\begin{array}{l|c|}
C\cos x + D\sin x = y_{p_3} & 4 \\
-C\sin x + D\cos x = y_{p_3}' & -4 \\
-C\cos x - D\sin x = y_{p_3}'' & 1
\end{array}$$

$$(3C - 4D)\cos x + (3D + 4C)\sin(x) = \cos x$$

$$\begin{array}{ll}
4C + 3D = 0 & -3 \\
3C - 4D = 1 & 4
\end{array} \qquad \left(\begin{array}{cc|c} 4 & 3 & 0 \\ 0 & -25 & 4 \end{array}\right)$$

$$D = -\frac{4}{25} \qquad C = \frac{3}{4}\cdot\frac{4}{25} = \frac{3}{25}$$

$$\frac{3}{25}\cos x - \frac{4}{25}\sin x = y_{p_3}$$

$$y_p = \frac{3}{25}\cos x - \frac{4}{25}\sin x + 2x^2 e^{2x} + 3e^x$$

$$y = \frac{3}{25}\cos x - \frac{4}{25}\sin x + 2x^2 e^{2x} + 3e^x + C_1 e^{2x} + C_2 x e^{2x}$$

~~Va~~..

Variation of parameters

Example: $y'' + y = \tan(x)$

homo sol: $y_h'' + y_h = 0$

$\lambda^2 + 1 = 0 \quad \lambda = \pm\sqrt{-1} = \pm i$

$y_h = C_1 e^{ix} + C_2 e^{-ix} \xrightarrow{\text{cleanup}}$

$y_h = A\cos x + B\sin x$

$y = A(x)\cos x + B(x)\sin x$

P.I.I $\quad y' = -A\sin x + B\cos x$

$+A'\cos x + B'\sin x$ / demand zero

$y' = -A\sin x + B\cos x + 0$

$y'' = \underline{-A\cos x - B\sin x} \cancel{+ A'\sin x}$

$-A'\sin x + B'\cos x$

last term : plug it all in

$y'' + y = \tan(x)$

$A'\cos x + B'\sin x = 0 \quad 0$

$-A'\sin x + B'\cos x + \overbrace{\cdots}^{0} = \tan(x)$

$$\begin{array}{ccc} A' & B' & \\ \left(\begin{array}{cc|c} \cos x & \sin x & 0 \\ -\sin x & \cos x & \tan(x) \end{array}\right) \end{array} \quad \begin{array}{l} \cdot\sin x \\ \\ \cdot\cos x \end{array}$$

$$\left(\begin{array}{cc|c} \cos x & \sin x & 0 \\ 0 & 1 & \sin x \end{array}\right) \quad B' = \sin x, \quad A' = -\frac{\sin^2 x}{\cos x}$$

$$B' = \sin x \qquad A' = \frac{-\sin^2 x}{\cos x} = \frac{-1 + \cos^2 x}{\cos x}$$

$$= \frac{-1}{\cos x} + \cos x$$

$$B = -\cos x + B_0$$

$$A = \sin x - \ln|\sec(x) + \tan(x)| + A_0$$

$$y = A\cos x + B\sin x$$

$$= \cancel{\sin x \cos x} \underbrace{- \cos x \ln|\sec(x) + \tan(x)|}_{y_p} + A_0 \cos x + B_0 \sin x - \cancel{\cos x \sin x}$$