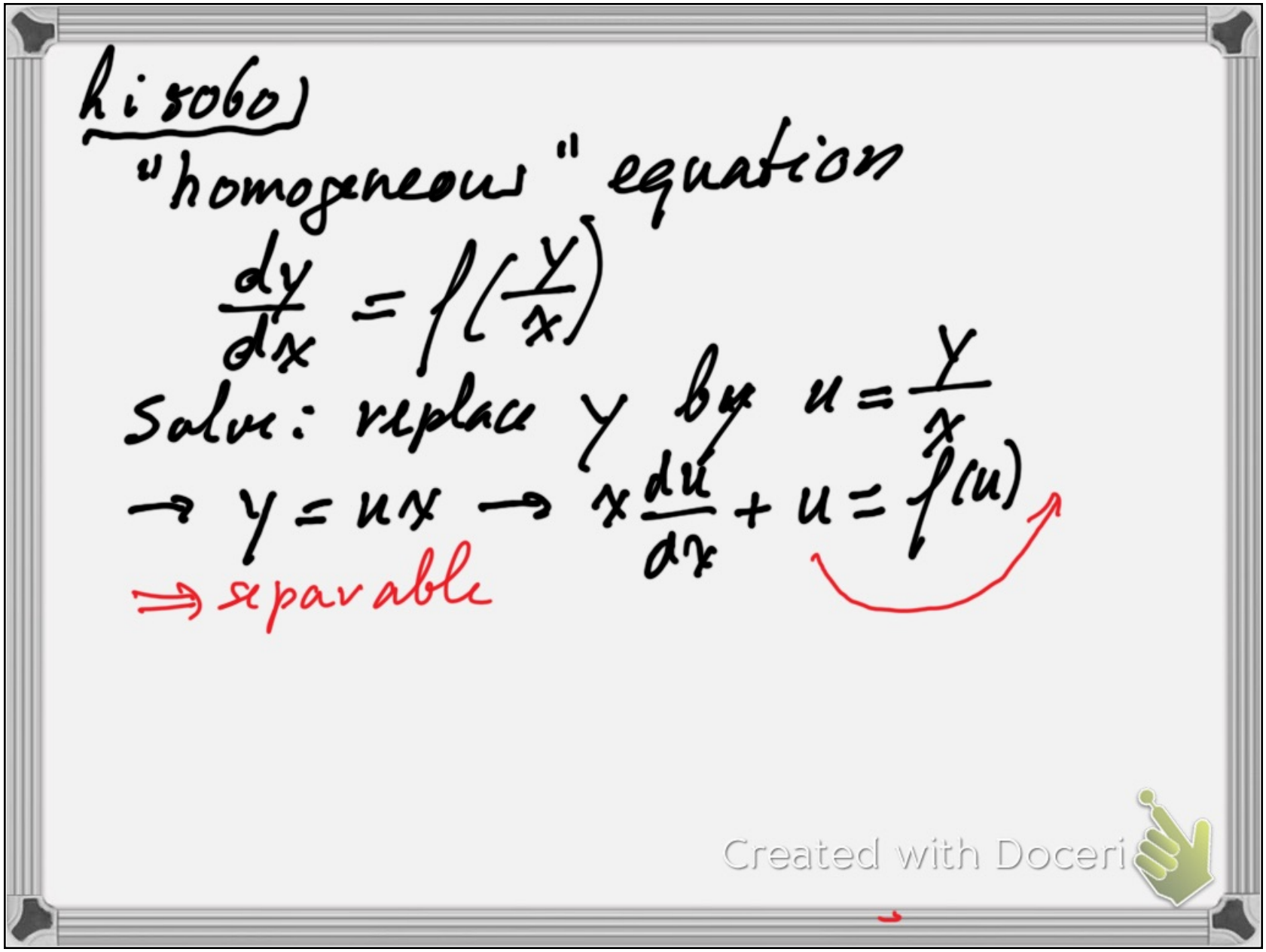
hi 5060)
"homogeneous" equation
$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$
Solve: replace y by $u = \frac{y}{x}$
$\rightarrow y = ux \rightarrow x\frac{du}{dx} + u = f(u)$
$\Rightarrow$ separable
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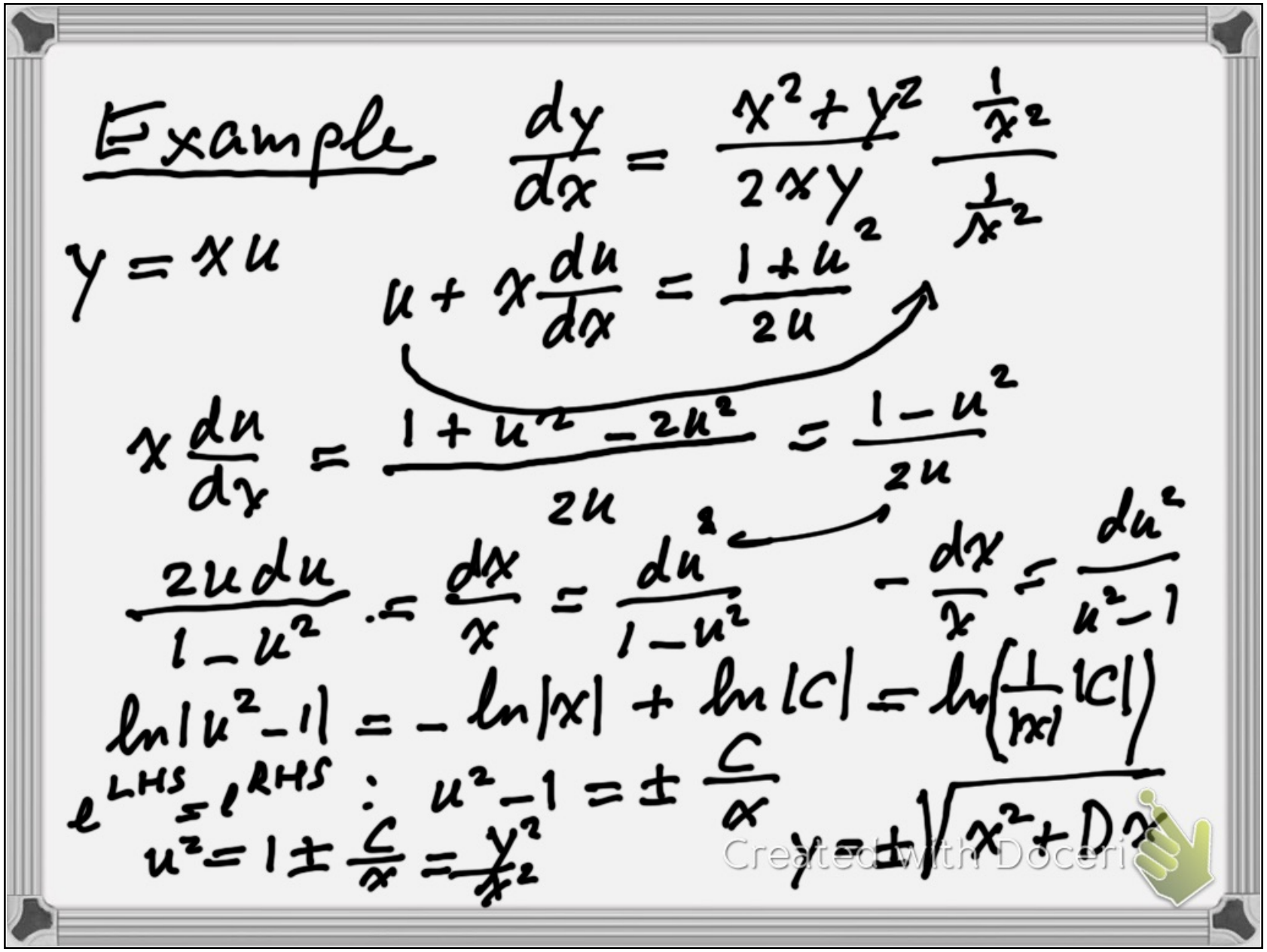
Example $\frac{dy}{dx} = \frac{x^2+y^2}{2xy} \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$
$y = xu$
$u + x\frac{du}{dx} = \frac{1+u^2}{2u}$
$x\frac{du}{dx} = \frac{1+u^2-2u^2}{2u} = \frac{1-u^2}{2u}$
$\frac{2u\,du}{1-u^2} = \frac{dx}{x} = \frac{du^2}{1-u^2}$
$-\frac{dx}{x} = \frac{du^2}{u^2-1}$
$\ln|u^2-1| = -\ln|x| + \ln|C| = \ln\left(\frac{1}{|x|}|C|\right)$
$e^{LHS} = e^{RHS}$: $u^2-1 = \pm\frac{C}{x}$
$u^2 = 1 \pm \frac{C}{x} = \frac{y^2}{x^2}$
$y = \pm\sqrt{x^2+Dx}$
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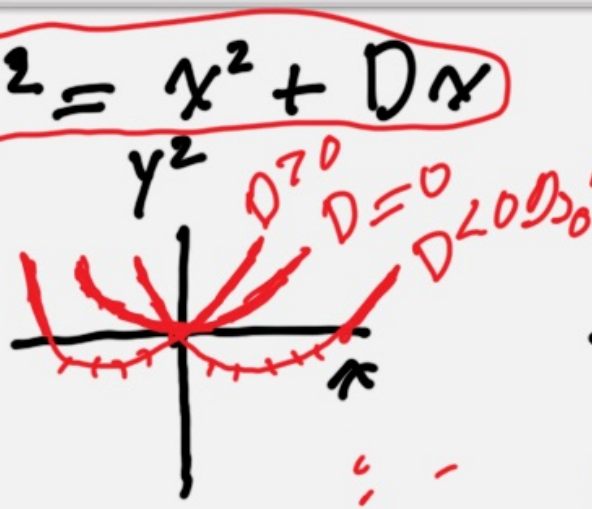
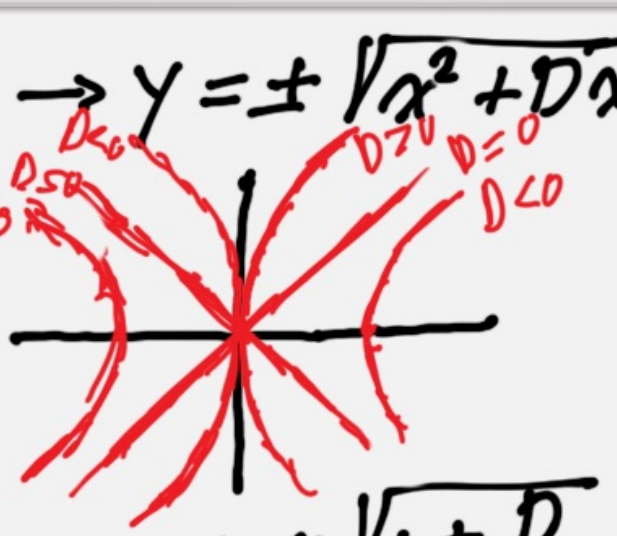
$y^2 = x^2 + Dx$

$\rightarrow y = \pm\sqrt{x^2 + Dx}$





large $|x|$

$$y = \pm x\sqrt{1 + \frac{D}{x}}$$

$$\simeq \pm x\left[1 + \pm\frac{1}{2}\frac{D}{x} + \cdots\right]$$

$$= \pm\left(x + \frac{1}{2}D\right)$$

Bernoulli's equation

$$\frac{dy}{dx} + p(x)y = q(x)y^n \qquad n \neq 0, 1$$

Solution: divide by y^n

$$\frac{1}{y^n}\frac{dy}{dx} + p(x)\frac{1}{y^{n-1}} = q(x)$$

$$\frac{-1}{n-1}\frac{d}{dx}\frac{1}{y^{n-1}} \qquad u = \frac{1}{y^{n-1}}$$

$$\frac{-1}{n-1}\frac{du}{dx} + p(x)u = q(x)$$

Example: $xy' + y = xy^3$

$\cdot \frac{1}{y^3}$ $\qquad \frac{x}{y^3}y' + \frac{1}{y^2} = x$

$\frac{dy^{-2}}{dx} = -2\frac{1}{y^3}\frac{dy}{dx}$ $\qquad -\frac{1}{2}x\frac{dy^{-2}}{dx} + y^{-2} = x$

$\ln a + \ln b = \ln ab$

$u = y^{-2}$

$-\frac{1}{2}x\frac{du}{dx} + u = x$

Solve homogeneous equation

$-\frac{1}{2}x\frac{du_h}{dx} + u_h = 0$

$\frac{du_h}{u_h} = +\frac{2}{x}dx$ $\qquad \ln|u_h| = \underbrace{2\ln|x|}_{\ln x^2} + \ln|C|$

$|u_h| = |C|x^2 \qquad \pm C = D$

$u_h = Dx^2$

Use variation of parameter

$u = E(x)x^2$ P.I.

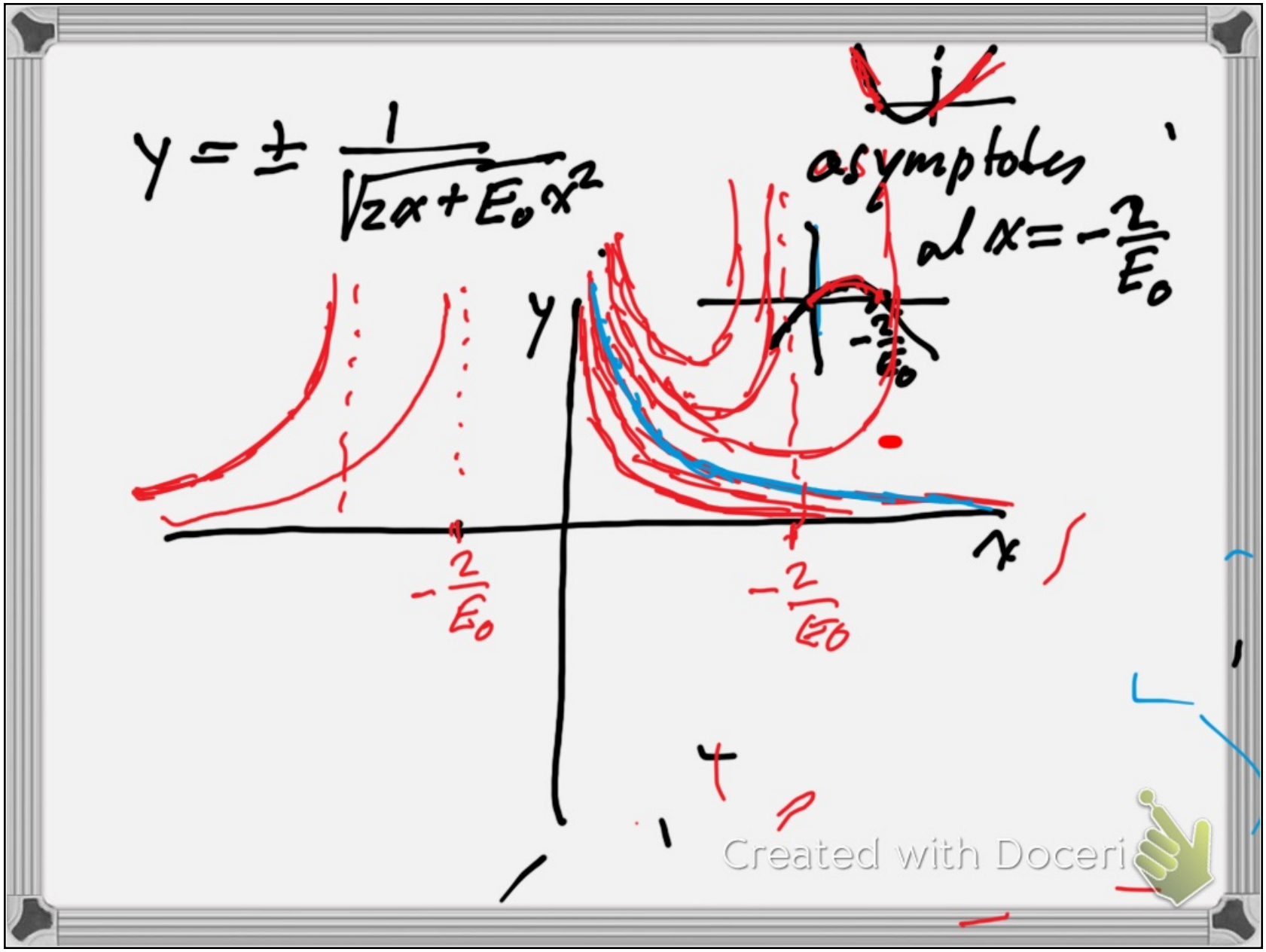
$-\frac{1}{2}x\frac{du}{dx} + u = x$

$-\frac{1}{2}x\left(E'x^2 + 2Ex\right) + Ex^2 = x$

$-\frac{1}{2}xE'x^2 = x \qquad E' = -\frac{2}{x^2}$

$E = \frac{2}{x} + E_0 \qquad u = Ex^2$

$u = 2x + E_0x^2 = \frac{1}{y^2} \qquad y = \pm\frac{1}{\sqrt{2x+E_0x^2}}$


$y = \pm \frac{1}{\sqrt{2x + E_0 x^2}}$
asymptotes
at $x = -\frac{2}{E_0}$
y
x
$-\frac{2}{E_0}$
$-\frac{2}{E_0}$
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