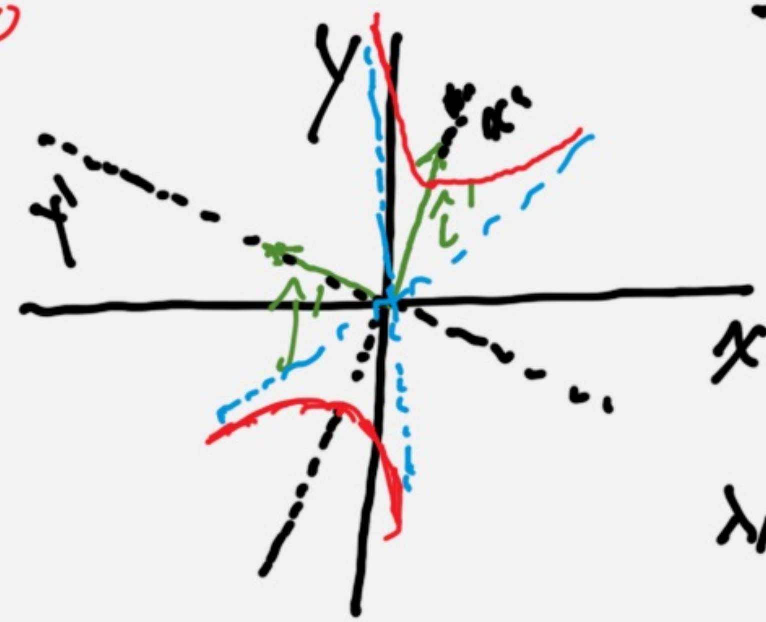
hi5060



$$-5x^2+4xy+3y^2=2$$

$$A=\begin{pmatrix}-5 & 2\\ 2 & 3\end{pmatrix}$$

$$\lambda_1 x'^2+\lambda_2 y'^2=2$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$A^T = A$

orthonormal eigenvectors required

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 0 & 0 & 0 \\ 0 & 1-\lambda & -2 & 0 \\ 0 & -2 & 1-\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{vmatrix} = -\lambda \left| \; \right|$$

$\lambda = 1 \pm 2$

$(1-\lambda)^2 = 4$

$$= -\lambda \left[(1-\lambda)^2(-\lambda) + 4\lambda \right]$$

$$= -\lambda^2 \left[(1-\lambda)^2 + 4 \right]$$

$\lambda_1 = \lambda_2 = 0$

$\lambda_3 = -1 \quad \lambda_4 = 3$

$$(A - 0I) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{matrix} 2 \\ \\ 1 \\ \\ \end{matrix} \begin{pmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{array}{cccc} l_1 & l_2 & l_3 & l_4 \\ \end{array} \begin{pmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$-3 l_3 = 0 \rightarrow l_3 = 0 \rightarrow l_2 - 2 \times 0 = 0$$

$$\begin{pmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{l_1^*} l_1 + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{l_2^*} l_4$$

In general, you will have to make $\vec{e}_1^{\,*}$ and $\vec{e}_2^{\,*}$ orthonormal;

Procedure: Gram-Schmidt:

$$\hat{i}' = \frac{\vec{e}_1^{\,*}}{|\vec{e}_1^{\,*}|} \qquad \vec{e}_2^{\,**} = \vec{e}_2^{\,*} - \hat{i}'\,\underbrace{(\vec{e}_2^{\,*}\cdot\hat{i}')}$$

$$\hat{j}' = \frac{\vec{e}_2^{\,**}}{|\vec{e}_2^{\,**}|}$$

~~Q·R algorithm~~

Power method

arbitrary starting vector:

$$\vec{v} = v_1'\vec{e_1} + v_2'\vec{e_2} + v_3'\vec{e_3} + \cdots$$

$$A\vec{v} = \underbrace{\lambda_1 v_1'}\vec{e_1} + \lambda_2 v_2'\vec{e_2} + \lambda_3 v_3'\vec{e_3} + \cdots$$

$$A^2\vec{v} = \lambda_1^2 v_1'\vec{e_1} + \lambda_2^2 v_2'\vec{e_2} + \lambda_3^2 v_3'\vec{e_3} + \cdots$$

$A = QR$

$R \rightarrow$ upper triangular matrix

$Q \rightarrow$ orthonormal matrix

$A^* = AQ^{-1}$ $\qquad$ $A^* = QRQ^{-1}$

Quadratic forms in 3D

$$\lambda_1 x'^2 + \lambda_2 y'^2 + \lambda_3 z'^2 = \dots$$

all λ values same sign

$\rightarrow$ all positive

ellipsoids

(equivalent to

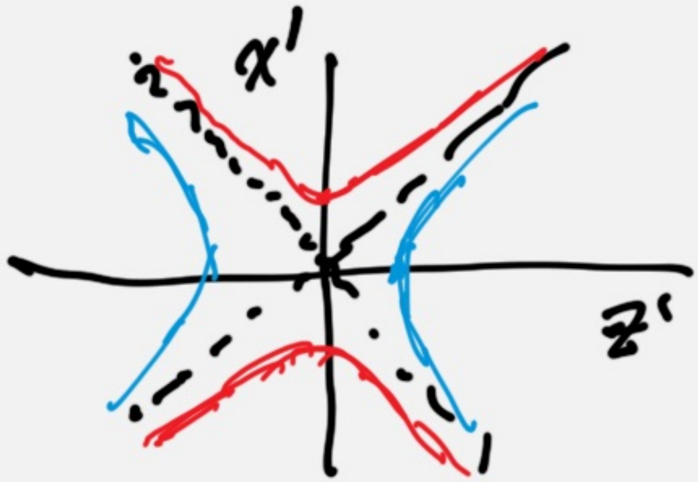
$$\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2 = \dots$$

$\tilde{x} = \sqrt{\lambda_1}\, x'$ $\tilde{y} = \sqrt{\lambda_2}\, y'$ $\tilde{z} = \sqrt{\lambda_3}\, z'$

if two eigenvalues equal spheroid

$$-3$$

$$\overbrace{x'^2 + y'^2}^{r^2} - z'^2 = \cancel{8}\ 3$$

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