

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad A^T = A$$

orthonormal
eigenvectors
required

$$|A - \lambda I| = \begin{vmatrix} -\lambda & 0 & 0 & 0 \\ 0 & 1-\lambda & -2 & 0 \\ 0 & -2 & 1-\lambda & 0 \\ 0 & 0 & 0 & -\lambda \end{vmatrix} = -\lambda \left[(1-\lambda)^2 - 4 \right]$$

$\lambda = 1 \pm 2$
 $(1-\lambda)^2 = 4$

$$= -\lambda \left[(1-\lambda)^2 - 4 \right]$$

$$= -\lambda^2 \left[(1-\lambda)^2 + 4 \right]$$

$\lambda_1 = \lambda_2 = 0$
 $\lambda_3 = -1 \quad \lambda_4 = 3$

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Handwritten mathematical work on a whiteboard background showing the row reduction of a matrix $(A \cdot 0I)$.

The initial matrix is:

$$(A \cdot 0I) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Row operations are indicated by arrows and numbers 1 and 2. The matrix is transformed into:

$$\begin{pmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Further row operations lead to:

$$\begin{pmatrix} 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The final matrix is shown as a sum of row operations:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

Labels l_1, l_2, l_3, l_4 are used to identify the rows. A note at the bottom right says "Created with Doceri" with a hand icon.

In general, you will have to make \vec{e}_1^* and \vec{e}_2^* orthonormal:

(Procedure: Gram-Schmidt:

$$\hat{e}_1 = \frac{\vec{e}_1^*}{|\vec{e}_1^*|} \quad \vec{e}_2^{**} = \vec{e}_2^* - \hat{e}_1 (\vec{e}_2^* \cdot \hat{e}_1)$$

$$\hat{e}_2 = \frac{\vec{e}_2^{**}}{|\vec{e}_2^{**}|}$$

~~QR algorithm~~
Power method

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arbitrary starting vector:

$$\vec{v} = v_1' \vec{e}_1 + v_2' \vec{e}_2 + v_3' \vec{e}_3 + \dots$$

$$A \vec{v} = \lambda_1 v_1' \vec{e}_1 + \lambda_2 v_2' \vec{e}_2 + \lambda_3 v_3' \vec{e}_3 + \dots$$

$$A^2 \vec{v} = \lambda_1^2 v_1' \vec{e}_1 + \lambda_2^2 v_2' \vec{e}_2 + \lambda_3^2 v_3' \vec{e}_3 + \dots$$

$$A = Q R \rightarrow \begin{array}{l} \text{upper triangular} \\ \text{orthonormal matrix} \end{array}$$

$$A^* = \bar{A} Q^{-1} \quad A^* = Q R Q^{-1}$$



Quadratic forms in 3D

$$\lambda_1 x'^2 + \lambda_2 y'^2 + \lambda_3 z'^2 = \dots$$

all λ values same sign
 \rightarrow all positive
 ellipsoids

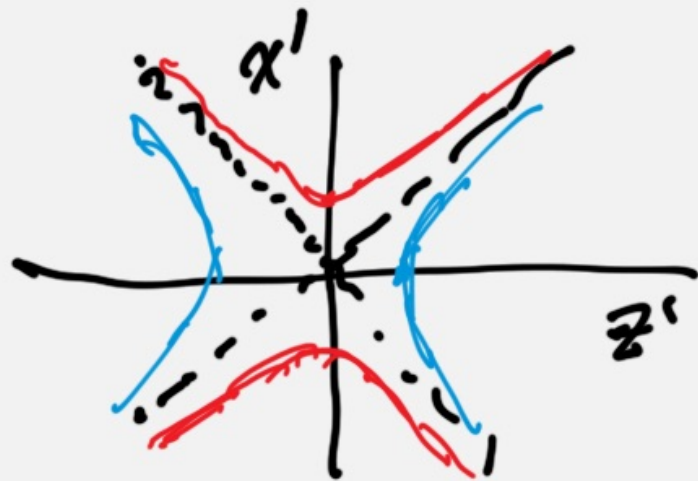
(equivalent to

$$\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2 = \dots$$

$$\tilde{x} = \sqrt{\lambda_1} x' \quad \tilde{y} = \sqrt{\lambda_2} y' \quad \tilde{z} = \sqrt{\lambda_3} z'$$

if two eigenvalues equal "spheroid"

$$x'^2 + y'^2 - z'^2 = -3$$



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