<u>hi5060)</u> symmetric (Hermitian matrices) → ~~rotate the coord~~ take at new basis, the orthonormal eigenvectors → matrix simplifies to diagonal

Quadratic forms (2D)

By example

$$-5x_1^2+4x_1x_2+3x_2^2=2$$

$\rightarrow$ Linear algebra

$$(x_1\ x_2)\begin{pmatrix}-5 & 2\\ 2 & 3\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=2 \qquad (\vec{x}^T A\vec{x}=c)$$

$$(x_1)(x_2)\begin{pmatrix}a_{11}x_1+a_{12}x_2\\ a_{21}x_1+a_{22}x_2\end{pmatrix}$$

$$\vec{x}=E\vec{x}''$$

Now change basis $\vec{x}\Rightarrow\vec{x}'=\begin{pmatrix}x_1'\\ x_2'\end{pmatrix}$

$$\vec{x}^TA\vec{x}=\vec{x}'^TE^TAE\vec{x}' \qquad A'=E^{-1}AE$$

E is orthonormal $\rightarrow E^{-1}=E^T$

$$A = \begin{pmatrix} -5 & 2 \\ 2 & 3 \end{pmatrix} \quad |A - \lambda I| = \begin{pmatrix} -5-\lambda & 2 \\ 2 & 3-\lambda \end{pmatrix}$$

$$= \lambda^2 + 2\lambda - 19$$

$$\lambda_{1,2} = \frac{-2 \pm \sqrt{4 + 4 * 19}}{2}$$

$$= -1 \pm \sqrt{20}$$

$$\lambda_1 = -1 + \sqrt{20} \quad (A - \lambda I) = \begin{pmatrix} -4-\sqrt{20} & 2 \\ \cancel{2}\ 0 & \cancel{4} - \sqrt{20} \end{pmatrix} \quad 4+\sqrt{20}$$

$$e_1 = \frac{2}{4+\sqrt{20}} e_2 = \frac{1}{2+\sqrt{5}} \qquad \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2+\sqrt{5}} \\ 1 \end{pmatrix} e_2$$

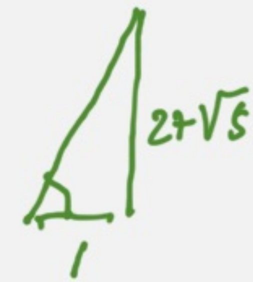
$$\vec{e}_1^{\,*} = \begin{pmatrix} 1 \\ 2+\sqrt{5} \end{pmatrix}$$

$$\lambda_2 = -1 - \sqrt{20} \quad \vec{e}_2^{\,*} = \begin{pmatrix} -2-\sqrt{5} \\ 1 \end{pmatrix} \quad \vec{e}_1^{\,*} = \begin{pmatrix} 1 \\ 2+\sqrt{5} \end{pmatrix}$$

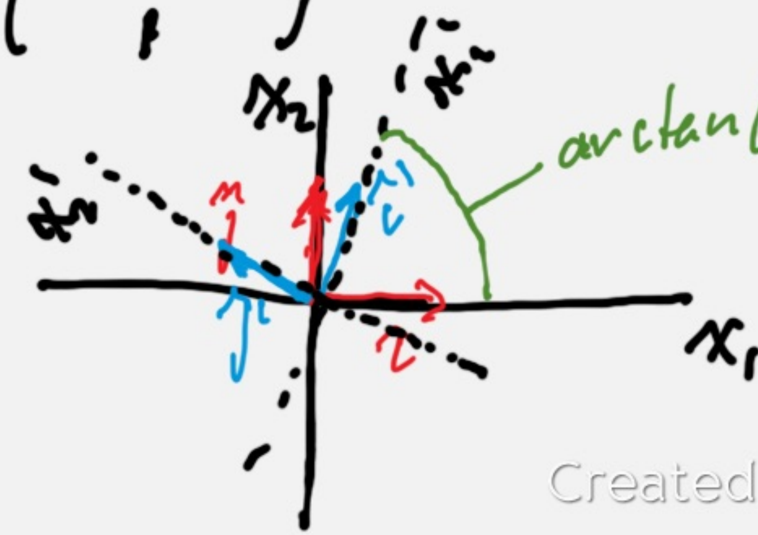
$$\vec{e}_1 = \hat{\imath}' = \vec{e}_1^{\,*} \Big/ \sqrt{1^2 + (2+\sqrt{5})^2}$$

$$\hat{\imath}' = \begin{pmatrix} 1 \\ 2+\sqrt{5} \end{pmatrix} \Big/ \sqrt{10 + 4\sqrt{5}}$$

$$\hat{\jmath}' = \begin{pmatrix} -2-\sqrt{5} \\ 1 \end{pmatrix} \Big/ \sqrt{10 + 4\sqrt{5}}$$



$$\arctan\left(\frac{2+\sqrt{5}}{1}\right) \approx 76.7^\circ$$



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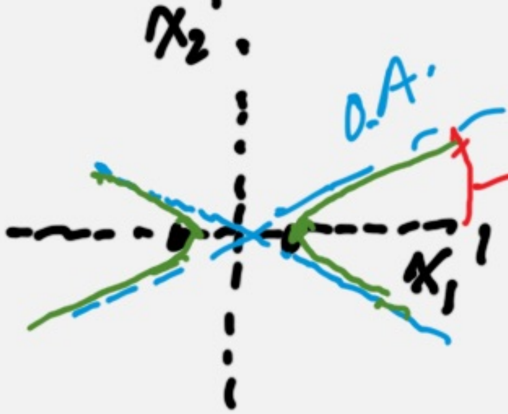
In the new coordinate system



arctan(m)

$\arctan\left(\sqrt{\frac{\sqrt{20}-1}{\sqrt{20}+1}}\right) \approx 38.5^\circ$

$$A' = \begin{pmatrix} -1+\sqrt{20} & 0 \\ 0 & -1-\sqrt{20} \end{pmatrix}$$

quadratic form:

$$\lambda_1 x_1'^2 + \lambda_2 x_2'^2 = 2$$

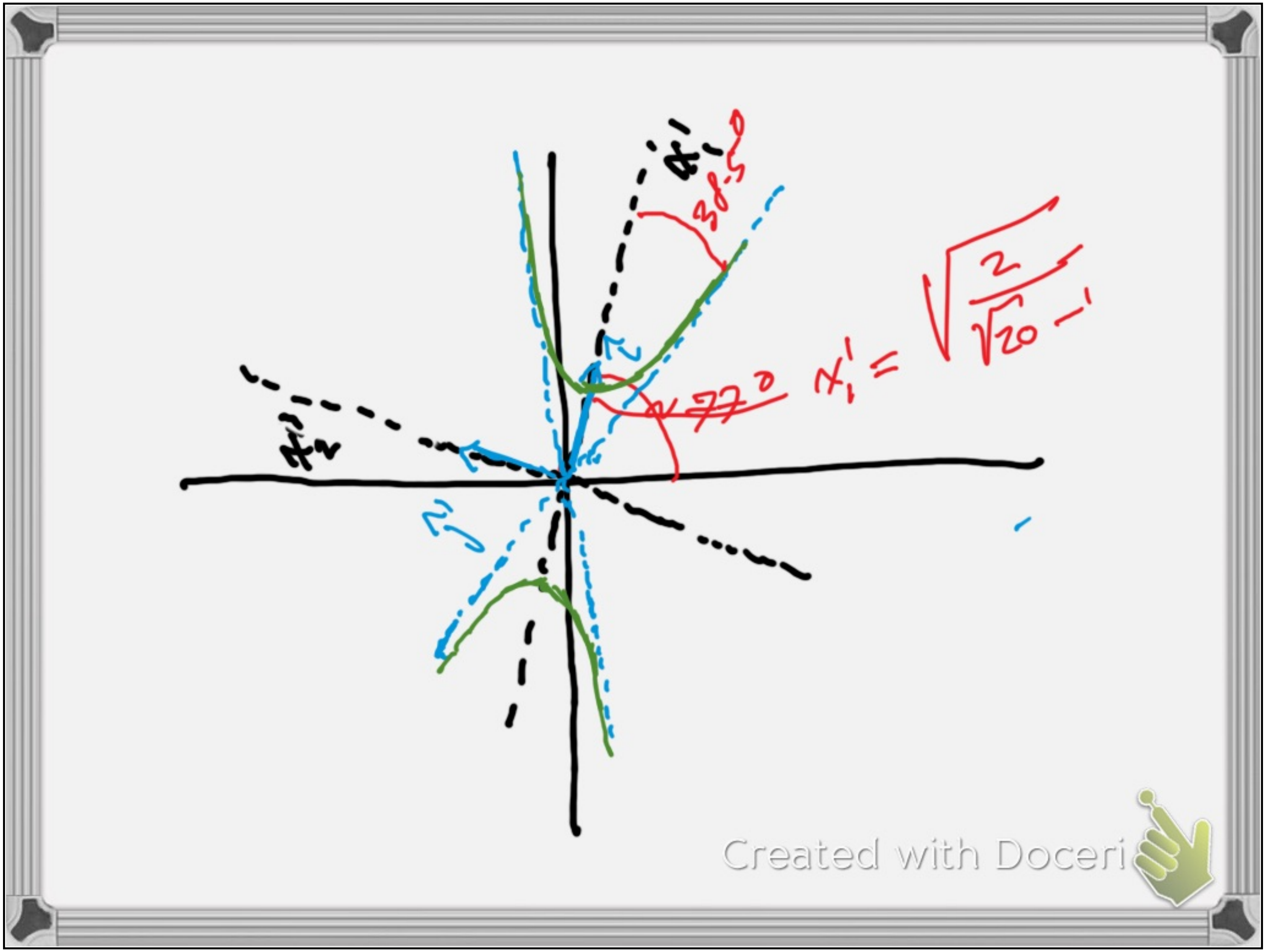
$$(\sqrt{20}-1)\, x_1'^2 - (\sqrt{20}+1)\, x_2'^2 = 2$$

hyperbolas ; x_1' intercepts

$$x_1' = \sqrt{\frac{2}{\sqrt{20}-1}}$$

x_2' intercepts

large x_1': $x_2' = \sqrt{\frac{(\sqrt{20}-1)x_1'^2 - \cancel{2}}{\sqrt{20}+1}} \approx \pm\sqrt{\frac{\sqrt{20}-1}{\sqrt{20}+1}}\, x_1' + \ldots$


x'₁
38.5°
77°
$x_1' = \sqrt{\frac{2}{\sqrt{20}-1}}$
x'₂
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