hi 50601 $A\vec{e} = \lambda \vec{e}$ $\vec{e} \neq 0$

If $A^{(m \times n)}$ is not defective, then it has n independent eigenvectors

$\longrightarrow$ can use as basis, $\longrightarrow$ A becomes diagonal matrix $A' = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix}$

Examples $\quad v = \underset{E}{P}\vec{v}' \quad \vec{v}' = \underset{E^{-1}}{P^{-1}}\vec{v}$

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} \longrightarrow P = \begin{pmatrix} 3 & -3 \\ \sqrt{6} & \sqrt{6} \end{pmatrix} = E$$

$\lambda_1 = 1+\sqrt{6}$

$\lambda_2 = 1-\sqrt{6}$

$$E^{-1} = \frac{1}{3\sqrt{6}+3\sqrt{6}} \begin{pmatrix} \sqrt{6} & +3 \\ -\sqrt{6} & 3 \end{pmatrix}$$

$$A' = E^{-1} A E = \begin{pmatrix} 1+\sqrt{6} & 0 \\ 0 & 1-\sqrt{6} \end{pmatrix}$$

$\vec{w} = A\vec{v}$

$\vec{w}' = A'\vec{v}'$

$$A = \begin{pmatrix} 5 & 1 & 0 & 9 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

$\boxed{\text{defective} = \text{non-diagonalizable}}$

Note: numerical problems unless
A is symmetric
(can make the matrix
upper triangular → using coordinate system rotation
—Schur transform)

For defective matrices you
can make the matrix bidiagonal
(nonzero first superdiagonal
Jordan form.)

Hermitian matrices have

$$(A^T)^* = A$$

replace $i=\sqrt{-1}$ with $-i$

Real symmetric matrices (or Hermitian matrices) have

1) Real eigenvalues
2) Never defective
3) The eigenvectors ~~can~~ must be chosen to be orthogonal and of unit length

$\underbrace{\text{orthogonal and of unit length}}_{\text{orthonormal}}$

e.g. $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$

→ any symmetric matrix can be turned into a diagonal matrix by simply rotating the coordinate system

Since E consists of orthonormal eigenvectors, it is called orthonormal.

For orthonormal matrices only

$$E^{-1} = E^T \quad (\text{or } E^{-1} = (E^T)^*)$$

must be used

Examples

Inertia matrix of a solid body
is symmetric

$$\begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_3 \end{pmatrix} \longrightarrow \text{principal moments of inertia}$$

$$E_{kin} = E_{kin\,cg} + \frac{1}{2}\vec{\omega}^T I \vec{\omega}$$

$$\frac{d I_{cg} \vec{\omega}}{dt} = \vec{M}_{cg}$$

2) Stress tensor in fluids or solid mechanics → principal stresses

3) Strain rate tensor → principal strain rates

4) Modes in Dynamics: Lagrangian equations → natural frequencies and modeshapes

5) Principal axes in beam bending

6) quadratic forms

7) Orthogonality property of Fourier transformations

8) real measurable from
Hermitian matrices

9) Expectation values

10) "Separation of variables"
in solving P.D.E.s

11) Classification of P.D.E.s