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$$A = \begin{pmatrix} 5 & 1 & 0 & 9 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 \end{pmatrix} \leftarrow$$

$\lambda_1 = 5 \quad \lambda_2 = 1$

$\lambda_3 = \lambda_4 = 0$

If all eigenvalues are different then there are n independent eigenvectors → matrix is not ~~d~~ defective

$\lambda_3 = \lambda_4 = 0 \quad A - \lambda I = A$

1St → $e_1 = 0$

2nd row : $e_2 = 0$

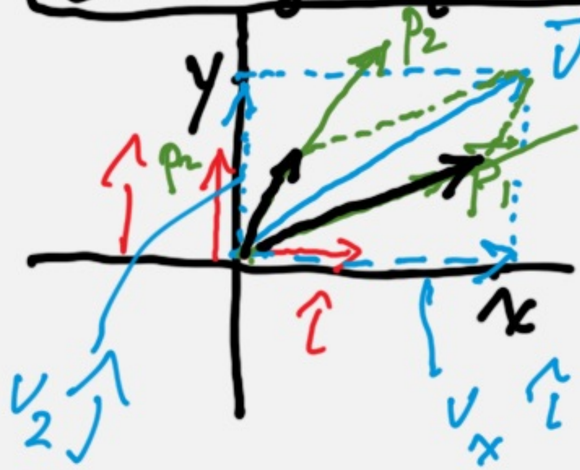
3th row → $e_4 = 0$

eigenvectors are a basis of the nullspace

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} e_3 \quad \vec{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

A is defective.

Change of basis (2D example)



$\vec{v} = v_1\hat{i} + v_2\hat{j}$

v_1, v_2 are components of $\vec{v}$ (in Cartesian coordinates)

$v_x\hat{i} = v_1\hat{i}$

In the new coordinate system with basis $\vec{p_1}$ and $\vec{p_2}$,

$\vec{v} = v_1'\vec{p_1} + v_2'\vec{p_2}$

v_1', v_2' are components of $\vec{v}$ in the new $\vec{p_1}, \vec{p_2}$ system.

Converting components between systems: $\vec{v} = v_1'\vec{p}_1 + v_2'\vec{p}_2$

Let's assume we write $\vec{p}_1$ and $\vec{p}_2$ in the original cartesian coordinate system

$$\vec{v} = (\vec{p}_1 \ \ \vec{p}_2)\begin{pmatrix} v_1' \\ v_2' \end{pmatrix}.$$

If $\vec{p}_1, \vec{p}_2$ described in Original ~~Cartesia~~ system then this gives $\vec{v}$ in the original system $\rightarrow$ $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = P\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} \qquad P = (\vec{p}_1 \ \ \vec{p}_2)$$

$\vec{v} = P\vec{v}'$ $\qquad \vec{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ $\vec{v}' = \begin{pmatrix} v_1' \\ v_2' \end{pmatrix}$

P takes vector in the new system to the old system.

P is called the transformation matrix from old to new

$$\boxed{\vec{v} = P\vec{v}' \longleftrightarrow \vec{v}' = P^{-1}\vec{v}}$$

What about matrices.

old coordinates

$AP\vec{v}' = P\vec{w}'$ $\quad P^{-1}$ *

$A\vec{v} = \vec{w} \Leftrightarrow A'\vec{v}' = \vec{w}'$, A'

$\overbrace{P^{-1}AP}^{A'}\vec{v}' = \vec{w}'$

$$\boxed{\begin{array}{ll} \vec{v} = P\vec{v}' & \vec{v}' = P^{-1}\vec{v} \\ A = PA'P^{-1} & A' = P^{-1}AP \end{array}}$$

One way to use this: eigenvectors of a non-defective matrix as new basis.

Then $A\vec{v} = \vec{w}$ in original coordinates

$$A(v_1'\vec{e}_1 + v_2'\vec{e}_2 + v_3'\vec{e}_3 + \dots)$$

$$= \underbrace{\lambda_1 v_1'}_{w_1'}\vec{e}_1 + \underbrace{\lambda_2 v_2'}_{w_2'}\vec{e}_2 + \underbrace{\lambda_3 v_3'}_{w_3'}\vec{e}_3 + \dots$$

$$A' = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 & 0 & \cdots \\ 0 & \lambda_2 & 0 & 0 & 0 & \cdots \\ 0 & 0 & \lambda_3 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \end{pmatrix}$$

diagonal with eigenvalues on the main diagonal

$A'\vec{v}' = \vec{w}'$ solve for v'

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad A^2 = \begin{pmatrix} a_{11}^2 & a_{12}^2 \\ a_{21}^2 & a_{22}^2 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11}^2 + a_{12}a_{21} & \\ & \end{pmatrix}$$

$$A'^2 = \begin{pmatrix} \lambda_1^2 & & 0 \\ & \lambda_2^2 & \\ 0 & & \lambda_3^2 \end{pmatrix}$$

$$A'^{\frac{1}{2}} = \begin{pmatrix} \sqrt{\lambda_1} & & 0 \\ & \sqrt{\lambda_2} & \\ 0 & & \sqrt{\lambda_3} \end{pmatrix}$$