hi 506 03

Inverse matrices using minors

minor of a_{11} (cross out row and column)

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} +|A_{11}| & -|A_{12}| & +|A_{13}| & \cdots \\ -|A_{21}| & +|A_{22}| & -|A_{23}| & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}^T$$

My way to do this for A^{-1}

Example:

$$A = \begin{pmatrix} 2 & 5 \\ -7 & -3 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 2 & -7 \\ 5 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{29}\begin{pmatrix} +(-3) & -5 \\ -(-7) & 2 \end{pmatrix}$$

$$= \frac{1}{29}\begin{pmatrix} -3 & -5 \\ 7 & 2 \end{pmatrix}$$

$$|A| = 2(-3) - 5(-7)$$
$$= 29$$

$$A^{-1}A = I$$
$$= AA^{-1}$$

$$x\begin{pmatrix} 2 & 5 \\ -7 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$A^{-1}_{6\times 6} = 36\times 5! + 6! = 5040$

Alternate procedure : use G.E

Idea $\quad (A \,|\, I) \xrightarrow[\text{to row canonical}]{G.E} (I \,|\, A^{-1})$

Exa

Example:

$$\left(\begin{array}{ccc|ccc} 4 & 1 & -7 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 2 & 2 & 0 & 0 & 1 & 0 \\ 4 & 1 & -7 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \;\; -2$$

$$\left(\begin{array}{ccc|ccc} 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & -3 & -7 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \qquad \left(\begin{array}{ccc|ccc} 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & -3 & -7 & 1 & -2 & 0 \end{array}\right) \;\; 3$$

$$\left(\begin{array}{ccc|ccc} 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -7 & 1 & -2 & 3 \end{array}\right) \;\; -2 \qquad \begin{array}{c} \frac{1}{2} \\ \\ -\frac{1}{7} \end{array} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & 1 & -2 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -7 & 1 & -2 & 3 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{2} & -1 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -\frac{1}{7} & \frac{2}{7} & -\frac{3}{7} \end{array}\right)$$

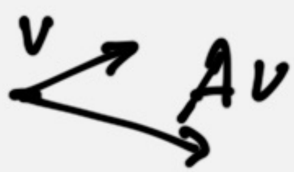
$$A^{-1}$$

$$A\left(\vec{a}_1^{\,-1}\ \ \vec{a}_2^{\,-1}\ \ \vec{a}_3^{\,-1}\right) = I$$

Chapter 9 Zill: eigenvalues and eigenvectors.

Definition:

If $A\vec{e} = \lambda \vec{e}$ with $\vec{e} \neq 0$ then $\vec{e}$ is an eigenvector of A, and λ is the corresponding eigenvalue



Note: if $A\vec{e} = \lambda\vec{e}$ then

$$A(2\vec{e}) = \lambda(2\vec{e})$$

$\rightarrow$ **must** normalize eigenvectors.

(For symmetric A, make the length 1)

Finding them $A\vec{e} = \lambda\vec{e}$

$$A\vec{e} - \lambda\vec{e} = 0 \quad \underbrace{(A - \lambda I)}_{\text{square matrix}}\vec{e} = 0$$

$\rightarrow$ determinant must be zero

Example

independent

$A = \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$ want eigenvectors and eigen values

$|\lambda I - A|$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} \rightarrow$ do not use 6.B

$= (1-\lambda)(1-\lambda) - 2*3 \rightarrow (\lambda-1)^2 = 6$

$= \lambda^2 - 2\lambda - 5 = 0$

warning: do not multiply out too quickly

$\lambda - 1 = \pm\sqrt{6}$

$\lambda_1 = 1+\sqrt{6}$ $\lambda_2 = 1-\sqrt{6}$

$(A - \lambda I)\vec{e} = 0$