

hisob6 Co-factor expansion

$$|A| = \begin{vmatrix} -5^{+} & 0^{-} & 1^{+} & 6^{-} \\ 2^{-} & -1^{+} & 3^{-} & 7^{+} \\ 4^{+} & 4^{-} & -5^{+} & -8^{-} \\ 1^{-} & -1^{+} & 6^{-} & 2^{+} \end{vmatrix}$$

- 1) put in a checker board sign pattern
- 2) select a row or column to expand to → lots of zeros! → here second column
- 3) expand in terms of cofactors
 $- 0 \text{ C.F.} - 1 \text{ C.F.} - 4 \text{ C.F.} - 1 \text{ C.F.}$

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$$\begin{vmatrix} -5 & 0 & 1 & 6 \\ 2 & -1 & 3 & 7 \\ 4 & 4 & -5 & -8 \\ 1 & -1 & 6 & 2 \end{vmatrix} = -0 \begin{vmatrix} 2 & 3 & 7 \\ 4 & -5 & -8 \\ 1 & 6 & 2 \end{vmatrix}$$

minors

$$-1 \begin{vmatrix} -5 & 1 & 6 \\ 4 & -5 & -8 \\ 1 & 6 & 2 \end{vmatrix} - 4 \begin{vmatrix} -5 & 1 & 6 \\ 2 & 3 & 7 \\ 1 & 6 & 2 \end{vmatrix}$$

$$-1 \begin{vmatrix} -5 & 1 & 6 \\ 2 & 3 & 7 \\ 4 & -5 & -8 \end{vmatrix}$$

$$= -1 \left\{ \begin{matrix} -5 \begin{vmatrix} -5 & -8 \\ 6 & 2 \end{vmatrix} - 1 \begin{vmatrix} 4 & -8 \\ 1 & 2 \end{vmatrix} + 6 \begin{vmatrix} 4 & -5 \\ 1 & 6 \end{vmatrix} \\ -4 \begin{vmatrix} -5 & 3 & 7 \\ 6 & 2 \end{vmatrix} - 1 \begin{vmatrix} 2 & 7 \\ 1 & 2 \end{vmatrix} + 6 \begin{vmatrix} 2 & 3 \\ 1 & 6 \end{vmatrix} \end{matrix} \right\}$$

$-5 \times 2 + 8 \times 6$

$$\begin{aligned}
 & -1 \left\{ -5 \begin{vmatrix} 3 & 7 \\ -5 & -8 \end{vmatrix} - 1 \begin{vmatrix} 2 & 7 \\ 4 & -8 \end{vmatrix} + 6 \begin{vmatrix} 2 & 3 \\ 4 & -5 \end{vmatrix} \right\} \\
 & = 1 \left\{ 5(-10+40) + 1(8+8) - 6(24+12) \right\} \\
 & \quad + 4 \left\{ 5(6-42) + 1(9-7) - 6(12-3) \right\} \\
 & \quad + 1 \left\{ 5(-24+35) + 1(-16-28) - 6(-10-12) \right\} \\
 & = -773
 \end{aligned}$$

n! operations

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Other way: using G.E. to echelon form

$$|A| = \begin{vmatrix} -5 & 0 & 16 \\ 2 & -1 & 3 & 7 \\ 4 & 4 & -5 & -8 \\ 1 & -1 & 6 & 2 \end{vmatrix} \Rightarrow |A| = - \begin{vmatrix} 1 & -1 & 6 & 2 \\ 2 & -1 & 3 & 7 \\ 4 & 4 & -5 & -8 \\ -5 & 0 & 1 & 6 \end{vmatrix}$$

(Sign changes if you swap 2 rows or 2 columns)

$$-|A| = \begin{vmatrix} 1 & -1 & 6 & 2 \\ 0 & 1 & -9 & 3 \\ 0 & 8 & -29 & -16 \\ 0 & -5 & 31 & 16 \end{vmatrix}$$

$\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 - 2\vec{r}_1 \\ \vdots \end{vmatrix}$
 $\begin{vmatrix} \vec{r}_1 \\ \vec{r}_2 & -2 \\ \vdots \end{vmatrix}$

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$$-|A| = \begin{vmatrix} 1 & -1 & 6 & 2 \\ 0 & 1 & -9 & 3 \\ 0 & 0 & 43 & -40 \\ 0 & 0 & -14 & 31 \end{vmatrix}$$

$\begin{matrix} \leftarrow 24 \\ \leftarrow 43 \end{matrix}$

$$-43|A| = \begin{vmatrix} 1 & -1 & 6 & 2 \\ 0 & 1 & -9 & 3 \\ 0 & 0 & 43 & -40 \\ 0 & 0 & 0 & 773 \end{vmatrix} = 1 \times 1 \times 43 \times 773$$

For a triangular matrix, the determinant is just the product of the elements on the main diagonal
 $O_p s = \prod_{i=1}^n n_i \ll n!$

$$|A| = -773$$



A whiteboard with a grey border and black corner protectors. In the top-left corner, the expression $|A|$ is written in black ink. Below it, the expression A^{-1} is written and enclosed in a hand-drawn black circle. In the bottom-right corner, the text "Created with Doceri" is written in a light grey font, followed by a green hand icon pointing upwards.