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The row space of a matrix A is the space "spanned" by its rows

Example:

$$A = \begin{pmatrix} 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & \boxed{2} & 5 & 6 & 2 \\ 0 & 0 & 2 & 7 & 8 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \vec{r}_1^{\,T} \\ \vec{r}_2^{\,T} \\ \vec{r}_3^{\,T} \\ \vec{r}_4^{\,T} \end{pmatrix}$$

$$\bar{x} = \alpha_1 \begin{pmatrix} 0 \\ 0 \\ 3 \\ 4 \\ 4 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 0 \\ 2 \\ 5 \\ 6 \\ 2 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 2 \\ 7 \\ 8 \\ 3 \end{pmatrix} + \cancel{\alpha_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}$$

Would like the simplest basis of the row space

Row space is unaffected by G.E.

e.g. replace $\vec{r}_3^{\,*T} \to \vec{r}_3^{\,T} - \vec{r}_1^{\,*T}$

$$\alpha_3 [\vec{r}_3^{\,T*}] = \alpha_3 [\vec{r}_3^{\,T}] - \alpha_3 [\vec{r}_2]$$

So simplify row space by G.E.

$$\begin{pmatrix} \quad & \quad & \quad \end{pmatrix} \xrightarrow{GE} \begin{pmatrix} 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

→ row is 3 D

want simplify further:

go to row canonical

$$\rightarrow \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

} simplest basis of row space

row space

$$\vec{r} = \alpha_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -\frac{1}{2} \end{pmatrix}$$

The column space of a matrix is spanned by its columns: here

$$\cancel{\alpha_1 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}} + \alpha_3 \begin{pmatrix} 0 \\ 2 \\ 2 \\ 0 \end{pmatrix} + \alpha_4 \begin{pmatrix} 3 \\ 5 \\ 7 \\ 0 \end{pmatrix} +$$

$$\alpha_5 \begin{pmatrix} 4 \\ 6 \\ 8 \\ 0 \end{pmatrix} + \alpha_6 \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}$$

at most 3D space

Theorem: the dimension of the row space = dimension of column space = "rank" of A

To Find simplest column space use G.E. to row canonical on $A^T$

$$A^T = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 \\ 3 & 5 & 7 & 0 \\ 4 & 6 & 8 & 0 \\ 1 & 2 & 3 & 0 \end{pmatrix} \xrightarrow[\text{to R.C.}]{\text{G.E.}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 2 & 7 & 8 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix}$$

$|-2| = 2$ ??

$|-2| = -2$

Zill chap 8 $|A|$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

4x4: no easy trick

Theorem:

if $|A| = 0$, $A\vec{x} = \vec{b}$ $\begin{cases} \text{has } \infty \text{ many solutions} \\ \underline{\underline{\text{or}}} \\ \text{no solution} \end{cases}$

e.g.: $\vec{b} = 0$, $A = 0$

if $|A| \neq 0$ $\quad A\vec{x} = \vec{b}$ has exactly one unique solution

("solution" → solution vector")

Do not use on a computer