


Prisob

$$\begin{pmatrix} 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\substack{2 \\ 3}} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\times -\frac{1}{2}}$$

RC, RREF

$$\begin{pmatrix} 0 & 0 & 2 & 5 & 0 & 5 \\ 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\substack{3 \\ \frac{1}{3}}} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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Do not do this when just
solving a system

The null space of a matrix

= set of all vectors \vec{x} so
that $A\vec{x} = 0$

$\vec{x} = 0$ is always in the null space
(vector)

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Example

$$\begin{matrix} 2 \\ \rightarrow \end{matrix} \begin{pmatrix} 1 & -3 & 2 \\ -2 & 1 & -3 \end{pmatrix} \xrightarrow{\text{E.F.}} \begin{pmatrix} 1 & -3 & 2 \\ 0 & -5 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$


~~$\begin{pmatrix} 1 & -3 & 2 & | & 0 \\ 0 & -5 & 1 & | & 0 \end{pmatrix}$~~ must solve backwards

For the pivot unknown

$$-5x_2 + x_3 = 0 \rightarrow x_2 = +\frac{1}{5}x_3 //$$

$$1x_1 - 3\left(+\frac{1}{5}x_3\right) + 2x_3 = 0$$

$$x_1 = -\frac{7}{5}x_3 //$$

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$$x_2 = \frac{1}{5} x_3 \quad x_1 = -\frac{7}{5} x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{7}{5} \\ \frac{1}{5} \\ 1 \end{pmatrix} x_3$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 2 & 3 & 0 \end{pmatrix} = A_{EF}$$

$$x_2 = -2x_4 - 3x_5$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} x_4 + \begin{pmatrix} 0 \\ -3 \\ 0 \\ 0 \\ 1 \end{pmatrix} x_5 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_6$$

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Inhomogeneous \rightarrow solution space

Example: $A\vec{x} = \vec{b}$

$$\left(\begin{array}{cccc|c} 0 & 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right)$$

Example

$$\left(\begin{array}{cccccc|c} 0 & 0 & -4 & 0 & 0 & 10 & 1 \\ 0 & 0 & 1 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -3 & 2 & 0 \end{array} \right)$$

Solution space is empty set already echelon solve for pivot unknowns

$$x_4 = 3x_5 - 2x_6 \quad x_3 = -x_5 + x_6 + 2$$

$$0x_2 = +4x_3 - 10x_6 + 1 = -4x_5 + 4x_6 - 10x_6 + 1$$

$$x_2 = \frac{1}{2}x_5 - \frac{3}{4}x_6 + \frac{1}{2}$$

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$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} x_4 + \begin{pmatrix} 0 \\ 1/2 \\ -1 \\ 3 \\ 1 \\ 0 \end{pmatrix} x_5 + \begin{pmatrix} 1 \\ 3/4 \\ 1 \\ -2 \\ 0 \\ -1 \end{pmatrix} x_6 + \begin{pmatrix} 0 \\ 9/8 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

~~Vector space?~~

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