$$x + y + 14z = -5$$
$$2x + 2y - 11z = -6 \Rightarrow$$
$$-5x + y + 9z = 12$$

$$1x + 1y + 14z = -5$$
$$0y - 39z = 4$$
$$6y + 79z = -13$$

partial pivoting

must partial pivot if the pivot is zero

otherwise

- on a computer pp to make the absolute value of the pivot as big as possible
- in this class: partial pivot to $\boxed{\text{avoid fractions}}$

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scaling: use decent units for unknowns and equations

To avoid fractions in this class: if pp does not help consider using a nonunit amount of the given equation

but not for LU Theorem.

$$\begin{matrix} - & \cdots & 2 & \cdots & -3 \\ & & 3 & \cdots & 2 \end{matrix}$$

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Echelon form:

A matrix is in echelon form if the first nonzero element in each row (if it exists) is always to the right of a nonzero element in the previous row

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 2 & \cdots & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & \cdots \end{pmatrix}$$

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$$\begin{matrix} 1 & 1 & 14 & -5 \\ 0 & -1 & -39 & 4 \\ 0 & 0 & -155 & 11 \end{matrix}$$

echelon yes

$$\begin{pmatrix} 1 & 1 & 14 & -5 \\ 0 & -1 & -39 & 4 \\ 0 & 2 & 0 & 0 \end{pmatrix} \text{ no} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ yes} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ yes}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ no} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The first nonzero element in each row is called the pivot

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A matrix is in "row canonical" or "row-reduced echelon form" if it is in echelon form and in addition, the pivots are all 1 and the elements above the pivot are also zero

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

zero matrix yes

$$\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{pmatrix} \text{ yes}$$

unit matrix yes $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

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Every matrix can be reduced to echelon form (yes) } → G.E.

and to ~~row canonical~~ → only if you are told so or in a special case (later) ~~[illegible]~~ RIP

Proof by ~~a~~ the procedure that does it

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By example

$$\begin{bmatrix} 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 2 & 7 & 8 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

look at entire matrix for an acceptable pivot:

if none: done!

... search

if found: pivot it to the first row

Then create zeros below that pivot

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$$\begin{pmatrix} 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 2 & 7 & 8 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} -1 \\ \\ \\ \\ \end{array} \qquad \begin{pmatrix} 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{array}{l} \\ -\frac{2}{3} \\ \\ \\ \end{array}$$

Trick: Now do the same for the submatrix below and to the right of the just created pivot

$$\begin{pmatrix} 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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To proceed to row canonical

→ create zeros above the pivols

→ divide ~~equa~~ rows by the pivot

$$\begin{pmatrix} 0 & 0 & 0 & 2 & 5 & 6 & 2 \\ 0 & 0 & 0 & 0 & 3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2 3

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