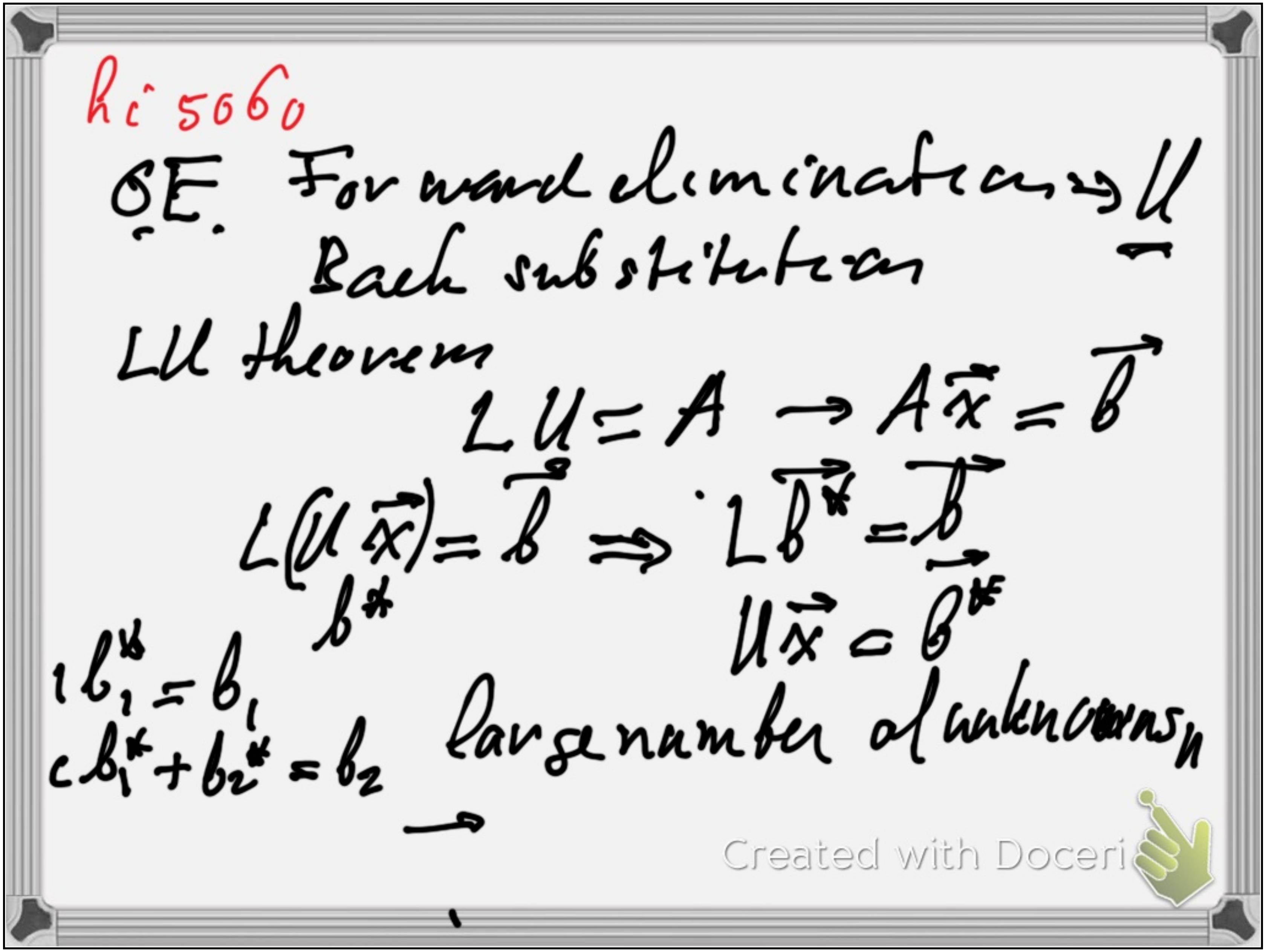


hi 5060

GE. Forward elimination ⇒ $\underline{U}$

Back substitution

LU theorem

$$LU = A \to A\vec{x} = \vec{b}$$

$$L(\underbrace{U\vec{x}}_{b^*}) = \vec{b} \Rightarrow L\vec{b}^* = \vec{b}$$

$$U\vec{x} = \vec{b}^*$$

$1b_1^* = b_1$

$cb_1^* + b_2^* = b_2$

large number of unknowns, n

→

solve $L U \vec{x} = \vec{b}$: ~ n^2 operations

1 op = 1* and 1+

$A_{mn} \to A_{nn}$

Find LU ~ $\frac{1}{3} n^3$ ops

Gauss-Jordan

$$\begin{pmatrix} c & \cdots \\ 0 & c \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ \vdots & & \vdots & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} b_1^* \\ b_2^* \\ b_3^* \end{pmatrix}$$

ops to find the diagonal matrix ~ $\frac{1}{2} n^3$ ops

~~Find the inverse matrix~~

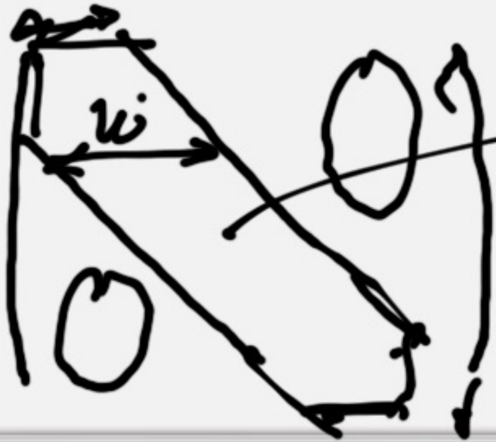
~~$A \rightarrow A^{-1}$ then if $A\vec{x} = \vec{b}$~~

~~$\vec{x} = A^{-1}\vec{b}$~~

~~n^3 ops~~

~~n^2 ops same as solving $LU\vec{x} = \vec{b}$~~

Band matrices



nonzero elements

→ save time with the calculations.

Band matrix LU → ops = $O(n, w^2)$

solve → $O(nw)$

In~~verse matrix~~ become full

→ ~~computational time~~ explode

~~Determinants~~ ~~Cramer's rule~~

↳ ~~straight~~ multiply out a ~~determinant~~ $n!$

$$|A| = \begin{cases} 0 : & \text{singular, no unique solutions} \\ \neq 0 : & \text{nonsingular unique solution} \end{cases}$$

$$\underbrace{\begin{pmatrix} 0.1 & & 0 \\ & 0.1 & \\ 0 & & 0.1 \\ & & & \ddots \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\downarrow$$

$$\underline{|A| = 10^{-n}} \rightarrow |A| = 0 \text{ on computer}$$

if n is large enough

Numerical methods

→ condition number C determines whether the system is solvable ~~accurat~~ accurately

$\frac{1}{C} \approx$ machine epsilon → solution no good

$\frac{1}{C} = O(1)$ maximum accuracy

Partial Pivoting

Modified example — pivot

−2 5

$\textcircled{1}x + 1y + 14z = -5$ (1)

$2x + 2y - 11z = -6$ (2)

$-5x + y + 9z = 12$ (3)

$\textcircled{1}x + 1y + 14z = -5$ (1)

$\textcircled{0}y - 39z = 4$ (2')

$6y + 79z = -13$ (3')

partial pivoting