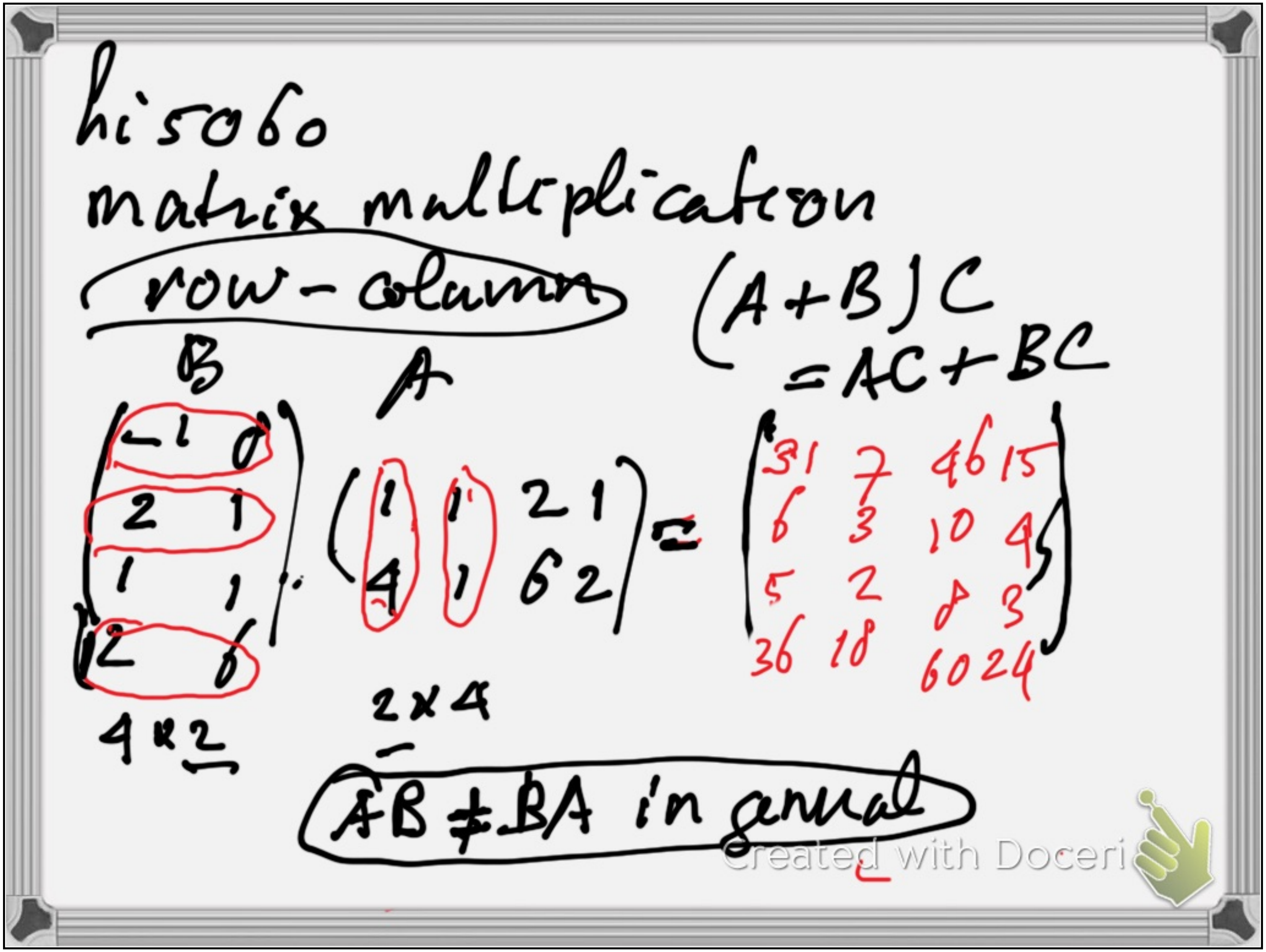
hi 5060
matrix multiplication
row-column
(A+B)C
=AC+BC
B
A
-1 0
2 1
1 1
2 6
1 1 2 1
4 1 6 2
=
31 7 46 15
6 3 10 4
5 2 8 3
36 18 60 24
4×2
2×4
AB ≠ BA in general
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$A = \{a_{ij}\}$ $\quad B = \{b_{ij}\}$

$C = AB$ then

$n_A = m_B$

$$c_{ij} = \sum_{k=1}^{n_A} a_{ik} b_{kj} \quad \text{for } i = 1, m_A, \; j = 1, n_B$$

```
for i = 1 : m_A
  for j = 1 : n_B
    c_ij = 0
    for k = 1 : n_A      (n_A ← m_B)
      c_ij = c_ij + a_ik * b_kj
    end end end
```

$c_{ij} = a_{ij} + b_{ij}$

```
for i = 1, m
   for j = 1, n
      c_ij = a_ij + b_ij
   end
end
```

Transpose

The transpose of A, A^T, has A's rows as columns and A's columns as rows

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

Index notation

$$a^T_{ij} = a_{ji}$$

Note: $(AB)^T = B^T A^T$

Special matrices

Zero matrix: all elements zero

$A + Z = A \qquad AZ = Z'$

Identity matrix

$$I_{3\times3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(main diagonal)

I → square matrix

→ zero except on the main diagonal $i=j$ where it is 1

$1 \times 1 \quad I = (1)$

$2\times 2 \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad 3\times 3 \quad I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$AI = A \quad IA = A$

if they exist

<u>Diagonal matrix</u> : All elements off the main diagonal are zero

e.g. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad a_{ij} = 0$ if $i \neq j$

Upper triangular matrix:
all elements below the main diagonal are zero

e.g.: $\begin{pmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad a_{ij} = 0$ if $j < i$

Lower T.M. " " above " "
" " "

e.g. $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 0 & 0 \end{pmatrix}$ $\quad a_{ij} = 0$ if $j > i$

Symmetric matrix : $A^T = A$

$$\begin{pmatrix} 1 & 3 & 6 \\ 3 & 0 & 2 \\ 6 & 2 & 5 \end{pmatrix}$$

$$a^T_{ij} = a_{ji}$$

Gaussian elimination (Simple) ↓ no "partial pivoting"

$$x + y + 14z = -5 \quad (1)$$
$$2x + y - 11z = -6 \quad (2)$$
$$-5x + y + 9z = 12 \quad (3)$$

First, without using matrices

Use (1) to eliminate x from (2) and (3)

$(2') = (1)(-2) + (2)$

$$0x - y - 39z = 4$$

$(3') = (3) + 5x$ (3')

$$0x + 6y + 79z = -13$$

$(3'') = (3') + 6(2') \rightarrow 155z = 11$